

ANALIZA 1
22. domača naloga

(1) Izračunaj naslednje integrale.

(a) $\int e^{\sin^2 x} \sin 2x \, dx$	(b) $\int (x^2 - 2x + 5)e^{-x} \, dx$	(c) $\int \ln(x + \sqrt{1+x^2}) \, dx$
(d) $\int x \sin x \cos x \, dx$	(e) $\int \sin(\ln x) \, dx$	(f) $\int \sin^3 x \ln(\cos x) \, dx$
(g) $\int \frac{\sin^3(\ln x) \cos(\ln x)}{x} \, dx$	(h) $\int \frac{1 + \cos^2 x}{1 + \cos 2x} \, dx$	(i) $\int x^2 \operatorname{arctg} x \, dx$
(j) $\int \frac{\operatorname{arctg} \sqrt{x}}{\sqrt{x}(1+x)} \, dx$	(k) $\int e^{\sqrt{x}} \, dx$	(l) $\int (\arcsin x)^2 \, dx$
(m) $\int \frac{1}{2 \operatorname{ch} x + \operatorname{sh} x + 2} \, dx$	(n) $\int x^5 e^{-x^2} \, dx$	(o) $\int (3x^2 - 2x + 1) \ln x \, dx$

(a) $e^{\sin^2 x}$	(b) $-(x^2 + 5)e^{-x}$	(c) $x \ln(x + \sqrt{1+x^2}) - \sqrt{1+x^2}$
(d) $\frac{1}{8} \sin 2x - \frac{x}{4} \cos 2x$	(e) $\frac{x}{2} (\sin(\ln x) - \cos(\ln x))$	(f) $\frac{\cos^3 x}{9} (3 \ln(\cos x) - 1) + \cos x (1 - \ln(\cos x))$
(g) $\frac{1}{4} \sin^4(\ln x)$	(h) $\frac{1}{2} (\operatorname{tg} x + x)$	(i) $\frac{x^3}{3} \operatorname{arctg} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2)$
(j) $\operatorname{arctg}^2 \sqrt{x}$	(k) $2(\sqrt{x} - 1)e^{\sqrt{x}}$	(l) $x(\arcsin^2 x - 2) + 2\sqrt{1-x^2} \arcsin x$
(m) $\ln \frac{3e^x + 1}{e^x + 1}$	(n) $-(\frac{x^4}{2} + x^2 + 1)e^{-x^2}$	(o) $(x^3 - x^2 + x) \ln x - \frac{x^3}{3} + \frac{x^2}{2} - x$

(2) Izračunaj integrale racionalnih funkcij.

(a) $\int \frac{x^2 + 2x + 4}{x^2 + x + 1} \, dx$	(b) $\int \frac{3x + 2}{x^3 + x} \, dx$	(c) $\int \frac{x^2 + x + 1}{x^4 + 4x^2} \, dx$
(d) $\int \frac{3x + 5}{(x^2 + 2x + 2)^2} \, dx$	(e) $\int \frac{4 - x^4}{4 + x^4} \, dx$	(f) $\int \frac{1}{x^3(1+x^2)^2} \, dx$
(g) $\int \frac{1}{x^4 + x^2 + 1} \, dx$	(h) $\int \frac{x^3 + 4x^2 - 2x + 1}{x^4 + x} \, dx$	(i) $\int \frac{x^2 - x + 14}{(x-4)^3(x-2)} \, dx$
(j) $\int \frac{1}{(x^4 - 1)^2} \, dx$	(k) $\int \frac{x + 1}{x^3 - 4x^2 + 5x - 2} \, dx$	(l) $\int \frac{x^6 - 2x^4 + 3x^3 - 9x^2 + 4}{x^5 - 5x^3 + 4x} \, dx$
(m) $\int \frac{x^4}{(1 - x^5)^7} \, dx$	(n) $\int \frac{x^3 + 1}{(x+1)(x^2+1)^2} \, dx$	(o) $\int \frac{x^4 + 1}{x^3 - x^2 + x - 1} \, dx$
(p) $\int \frac{x^2}{1 + x^6} \, dx$	(q) $\int \frac{4x^3 + 6x^2 - 15x - 5}{(x^2 - 4x + 4)(x^2 + x + 1)} \, dx$	

(a) $x + \frac{1}{2} \ln(x^2 + x + 1) + \frac{5}{\sqrt{3}} \operatorname{arctg} \frac{2x+1}{\sqrt{3}}$	(b) $\ln \frac{x^2}{x^2+1} + 3 \operatorname{arctg} x$	(c) $\frac{3}{8} \operatorname{arctg} \frac{x}{2} + \frac{1}{8} \ln \frac{x^2}{x^2+4} - \frac{1}{4x}$
(d) $\frac{x-\frac{1}{2}}{x^2+2x+2} + \operatorname{arctg}(x+1)$	(e) $\frac{1}{2} \ln \frac{x^2+2x+2}{x^2-2x+2} + \operatorname{arctg} \frac{2x}{2-x^2} - x$	(f) $-\frac{1}{2}(\frac{1}{x^2} + \frac{1}{x^2+1}) + \ln \frac{1+x^2}{x^2}$
(g) $\frac{1}{4} \ln \frac{x^2+x+1}{x^2-x+1} + \frac{1}{2\sqrt{3}} \operatorname{arctg} \frac{x\sqrt{3}}{1-x^2}$	(h) $\log \frac{x^3-x^2+x}{(1+x)^2} + \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2x-1}{\sqrt{3}}$	(i) $2 \ln \left \frac{x-4}{x-2} \right + \frac{3}{x-4} - \frac{13}{2(x-4)^2}$
(j) $\frac{3}{16} \ln \left \frac{x+1}{x-1} \right + \frac{3}{8} \operatorname{arctg} x - \frac{x}{4(x^4-1)}$	(k) $\frac{2}{x-1} + 3 \ln \left \frac{x-2}{x-1} \right $	(l) $\frac{x^2}{2} + \frac{1}{2} \ln \left \frac{x^2(x+1)^3(x-1)}{(x+2)^2} \right $
(m) $\frac{1}{30} (x^5 - 1)^{-6}$	(n) $\operatorname{arctg} x + \frac{1}{2(x^2+1)}$	(o) $\frac{(x+1)^2}{2} + \frac{1}{2} \ln \frac{(x-1)^2}{x^2+1} - \operatorname{arctg} x$
(p) $\frac{1}{3} \operatorname{arctg} x^3$	(q) $-\frac{3}{x-2} + \frac{4}{\sqrt{3}} \operatorname{arctg} \frac{2x+1}{\sqrt{3}} + \ln \frac{(x-2)^6}{x^2+x+1}$	

Opomba: V zgornjih nedoločenih integralih je povsod treba dodati splošno konstanto. V nekaterih integralih je uporabljen naslednji “adicijski izrek” za funkcijo arctg :

$$\operatorname{arctg} x \pm \operatorname{arctg} y = \operatorname{arctg} \frac{x \pm y}{1 \mp xy}.$$

Namen tega je le, da so formule krajše; vam rezultatov ni treba preoblikovati na ta način.