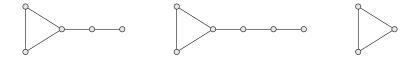
- 1. Find a recurrence equation for  $c_n$ , the number of connected graphs on n vertices.
- 2. A group of n children is divided into groups (each having at least 2 members). In each group, one child stands in the middle, and the others form a circle around him. Let  $a_n$  denote the number of such arrangements. Find the egf of this sequence. (We distinguish between children.)
- 3. Let  $d_n$  denote the number of ways a group of n children can separate into an odd number of subgroups, where each subgroup consists of at least three children, who stand in a circle, and all other children in this group stand behind one of the children in the circle. Note that the circle always consists only of three children.

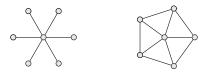
Determine the exponential generating function  $D(x) = \sum_{n} d_{n} \frac{x^{n}}{n!}$ .

Example (without names of the 14 children):



4. Let  $d_n$  denote the number of labeled graphs on n vertices which have connected components isomorphic to a star or a wheel. Determine the exponential generating function for the sequence  $(d_n)_n$ .

Remember that a star  $K_{1,n}$ ,  $n \geq 3$ , is a graph with n+1 vertices. One of them has n neighbors, all others have just one. A wheel  $W_n$ ,  $n \geq 3$ , is a graph with n+1 vertices, n of them form a cycle, and one (central) vertex is connected to all vertices on this cycle. The figure below shows unlabeled graphs  $K_{1,6}$  and  $W_5$ .



5. Find a linear recursion for Motzkin numbers.