

1. Let $A_n(x) = \sum_k A(n, k)x^k$ be an Eulerian polynomial. Prove that $A_n(1) = n!$ and that

$$A_n(-1) = \begin{cases} 1; & n = 0, \\ 0; & n > 0, n \text{ even}, \\ (-1)^{\frac{n+1}{2}} E_n; & n \text{ odd}. \end{cases}$$

2. We are throwing a coin. The probability for a number is p . Determine the mean and variance of when the first number appears.
3. Determine the inverse of an ogf $F(x) = x - x^n$.
4. The k -Catalan number C_k^n is the number of rooted trees with $(k-1)n+1$ leaves, where every vertex has 0 or k descendants. They are determined as $C_k^n = \frac{1}{k(n+1)} \binom{k(n+1)}{n} = \frac{1}{(k-1)n+1} \binom{kn}{n}$. Prove that C_k^n equals the number of generalized Dyck paths of length kn , where a generalized Dyck path is a path from $(0,0)$ to $(kn,0)$ with steps $(1, k-1)$ and $(1, -1)$, never going below the x -axis.
5. Let $F_q(n)$ denote the number of permutations in S_n without cycles of length $\leq q$. Determine the asymptotic behavior of this sequence.