Naj bo $p_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$, $n \geq 1$. Matrika

$$A_n = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & & & & & & \\ 0 & \dots & & & & 0 & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & \dots & & & -\frac{a_{n-2}}{a_n} & -\frac{a_{n-1}}{a_n} \end{bmatrix} \in \mathbb{R}^{n \times n}$$

ima lastne vrednosti, ki so ravno ničle p_n.

Rešitev: (z indukcijo na n)

① $n = 1$; $p_1(x) = a_1 x + a_0$

$$A_1 = \left[-\frac{a_0}{a_1}\right] \Rightarrow \det(A_1 - \lambda I)$$

$$= -\frac{a_0}{a_1} - \lambda = 0 \Rightarrow -\frac{1}{a_1}\left(a_0 + a_1\lambda\right) = 0 \,/ \cdot a_1$$

$$\cancel{\Rightarrow -(a_0 + a_1\lambda) = 0 \Rightarrow} \; \lambda \text{ ničla } p_1$$

Trdimo lahko več:

$n=1$ OK ✓

$$\det(A_n - \lambda I) = \boxed{(-1)^n \frac{1}{a_n} p_n(\lambda)}$$

$$\det(A_n - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 & 0 & \cdots & 0 \\ 0 & -\lambda & 1 & 0 & \cdots & 0 \\ \vdots & & & & & \\ & & & & & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & \cdots & & -\frac{a_{n-1}}{a_n} & -\lambda \end{vmatrix}$$

razvoj

$$= (-1)^{n+1}\left(-\frac{a_0}{a_n}\right) \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ -\lambda & 1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & & -\lambda & 1 \end{vmatrix}$$

$$+ (-\lambda) \begin{vmatrix} -\lambda & 1 & 0 & \cdots & 0 \\ 0 & -\lambda & 1 & \cdots & 0 \\ \vdots & & & & \\ -\frac{a_1}{a_n} & \cdots & & -\frac{a_{n-1}}{a_n} & -\lambda \end{vmatrix}$$

$$=(-1)^n \frac{a_0}{a_n} + (-\lambda)(-1)^{n-1}\frac{1}{a_n}\left(a_n\lambda^{n-1}+\dots+a_1\right)$$

$$=(-1)^n\left(\frac{a_0}{a_n}+\frac{1}{a_n}\left(a_n\lambda^{n}+\dots+a_1\lambda\right)\right)$$

$$=(-1)^n\frac{1}{a_n}\left(a_n\lambda^{n}+\dots+a_1\lambda+a_0\right)$$

roots v

$$=(-1)^n\frac{1}{a_n}p_n(\lambda) \quad \square$$

Matlabu

<u>Naloga</u>: Modelirajte širjenje COVID-19 v SLO v obdobju od 4. 3. 2020 do 12. 3. 2020!

<u>Rešitev</u>: Predpostavili bomo model <u>eksponentnega širjenja</u>.

$$f(t) = \boxed{a}\cdot e^{\boxed{b}t}$$; t - čas, $f(t)$ št. okuženih

neznanki

Podatki:

t	4	5	6	7	8	9	10	11	12
$f(t)$	1	3	7	12	16	23	31	57	131

kumulativno

Linearizacija:

$$\log f(t) = \log(a e^{bt}) = \underbrace{\log a}_{\tilde{a}} + bt$$

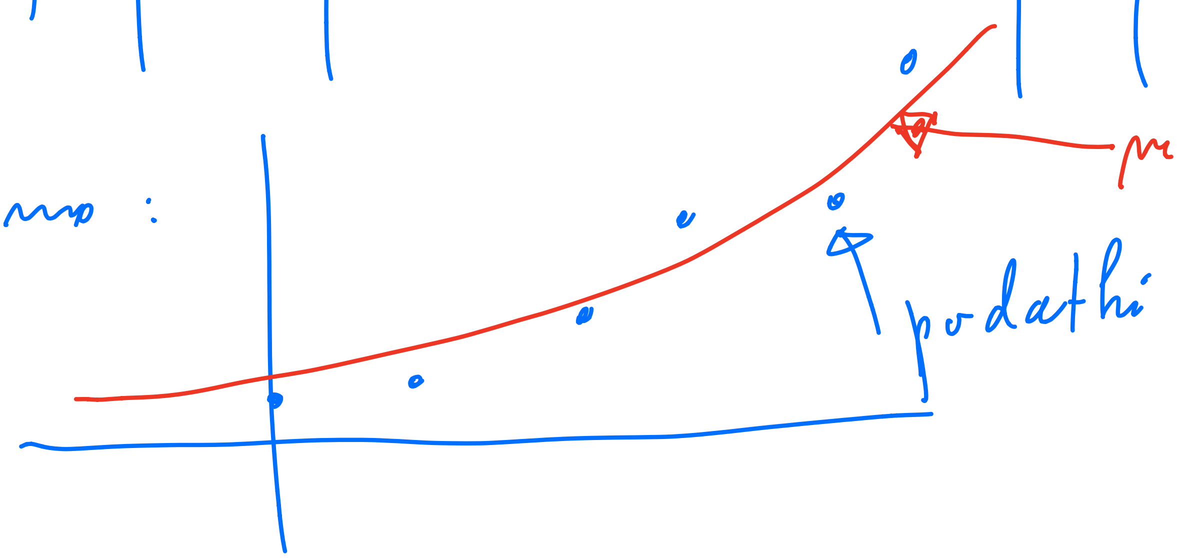
$$= \tilde{a} + bt \; ; \; a = e^{\tilde{a}}$$

Normiranje časa: $u = \frac{t-4}{\cancel{12}\,8}$

$\Rightarrow$

u	u_0		u_8
$\log f(t)$	F_0	. . .	F_8

Želimo:



$$\boxed{\begin{aligned} F_0 &= \tilde{a} + b u_0 \\ F_1 &= \tilde{a} + b u_1 \\ &\vdots \\ F_8 &= \tilde{a} + b u_8 \end{aligned}}$$

Predoločen sistem za $\tilde{a}$ in b.

$$A = \begin{bmatrix} 1 & u_0 \\ 1 & u_1 \\ \vdots & \vdots \\ 1 & u_8 \end{bmatrix}; \quad c = \begin{bmatrix} F_0 \\ F_1 \\ \vdots \\ F_8 \end{bmatrix}; \quad x = \begin{bmatrix} \tilde{a} \\ b \end{bmatrix}$$

Rešujemo $\boxed{Ax = c}$!

$$\min_x \| Ax - c \|_2^2 :$$

Normalni sistem:

$$\boxed{A^T A x = A^T c}$$

$\Rightarrow$ a im b!!!