Modelni problem v 1-D:

$$-u''(x) = \sin x, \quad \Omega = (0, 3\pi/2)$$

$$u(0) = 0, \quad u(3\pi/2) = -1$$

$$-\Delta u + \overset{=0}{c}\, u = f$$

$$\int_{\Omega} \nabla u (\nabla v)^T \, d\Omega = \int_{\Omega} f \cdot v \, d\Omega$$

$$\Rightarrow \int_0^{3\pi/2} u'(x)\, v'(x)\, dx = \int_0^{3\pi/2} \sin x \, v(x)\, dx$$

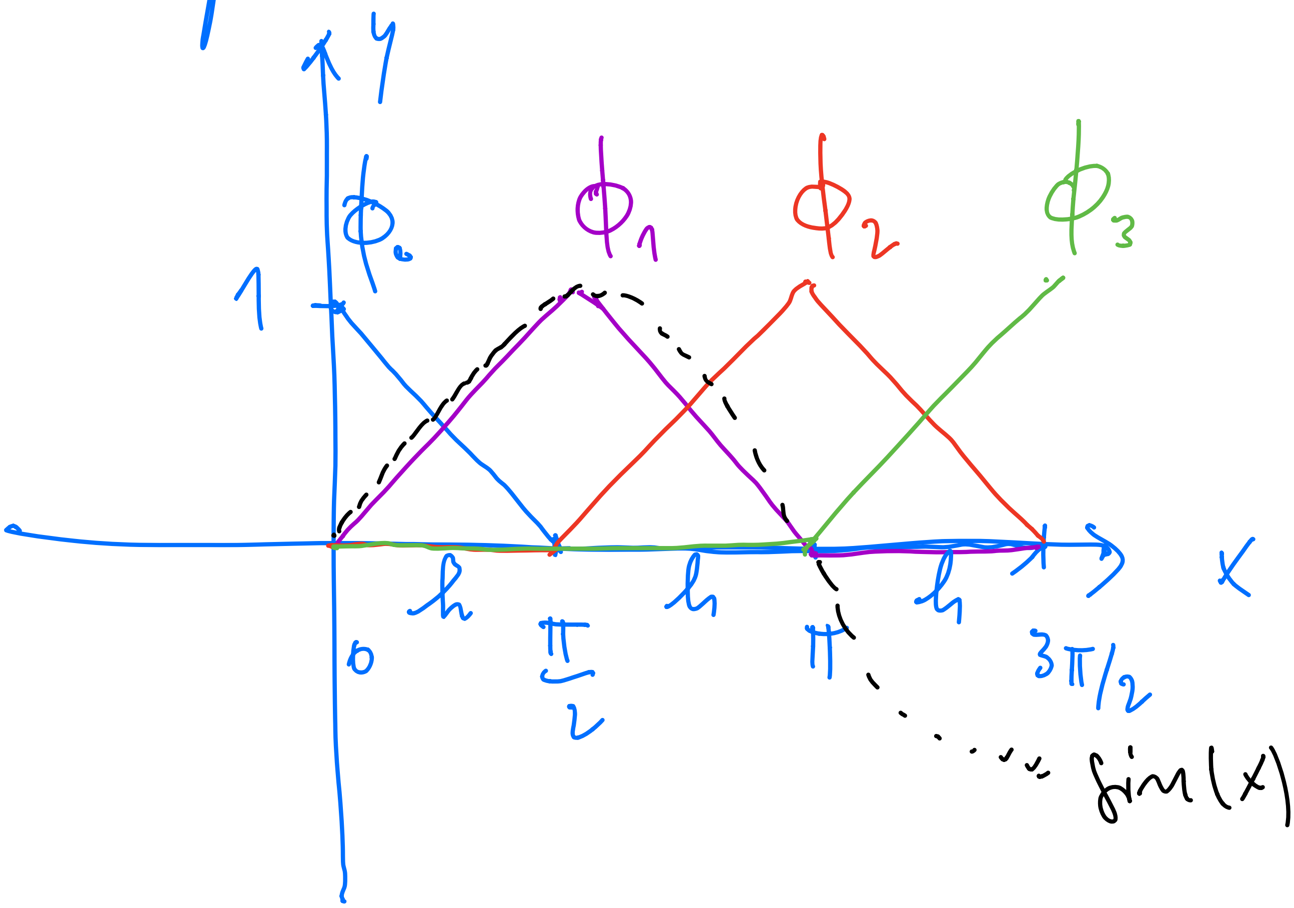
Kaj je rešitev (analitično)?

$$\boxed{u(x) = \sin x}$$

Kaj vzamemo za v?

Vzamemo odsekoma lin. fun.!

Ideja :



Rešitev iščemo v obliki

$$u_h = \sum_{j=0}^{3} \alpha_j \phi_j \;,\; \alpha_j \in \mathbb{R}.$$

Robni pogoji: $u_h(0) = 0$ in

$u_h(3\pi/2) = -1$:

$$u_h(0) = \sum_{j=0}^{3} \alpha_j \phi_j(0) = \alpha_0 \phi_0(0)$$

$$= \alpha_0 \cdot 1 = 0 \Rightarrow \boxed{\alpha_0 = 0}$$

$$u_h(3\pi/2) = \sum_{j=0}^{3} \alpha_j \phi_j(3\pi/2)$$

$$= \alpha_3 \phi_3(3\pi/2) = \alpha_3 \cdot 1 = -1 \Rightarrow$$

$$\boxed{\alpha_3 = -1}$$

ϕ_1 şi ϕ_2 sta tectni (spaţimo, da şi $\phi_1(0) = 0$ şi $\phi_1(3\pi/2) = 0$ ter $\phi_2(0) = 0$ in $\phi_2(3\pi/2) = 0$)

(1) $$\int_0^{3\pi/2} u_h'(x)\, \phi_1'(x)\, dx = \int_0^{3\pi/2} \sin x \cdot \phi_1(x)\, dx$$

② $\int_0^{3\pi/2} u_u'(x)\,\phi_2'(x)\,dx = \int_0^{3\pi/2} \sin x \cdot \phi_2(x)\,dx$

Dobili smo sistem dveh lin. enačb za α_1 in α_2:

$$\int_0^{3\pi/2} \left(\sum_{j=0}^{3} \alpha_j \phi_j'(x) \right) \cdot \phi_1'(x)\,dx$$

$$= \int_0^{3\pi/2} \Big(\alpha_1 \phi_1'(x)^2 + \alpha_2 \phi_2'(x)\,\phi_1'(x) + (-1)\underbrace{\phi_3'(x)\,\phi_1'(x)}_{=0,\ \text{ker se nosilca ne sekata}} \Big)\,dx$$

$$= \alpha_1 \int_0^{3\pi/2} \phi_1'(x)^2\,dx + \alpha_2 \int_0^{3\pi/2} \phi_2'(x)\,\phi_1'(x)\,dx$$

$$= \alpha_1 \int_0^\pi \phi_1'(x)^2 \, dx + \alpha_2 \int_{\pi/2}^\pi \phi_2'(x)\, \phi_1'(x) \, dx$$

$$= \alpha_1 \int_0^\pi \left(\frac{2}{\pi}\right)^2 dx + \alpha_2 \int_{\pi/2}^\pi \left(-\frac{4}{\pi^2}\right) dx$$

$$= \alpha_1 \frac{4}{\pi^{\cancel{2}}} \cdot \cancel{\pi} - \alpha_2 \frac{\cancel{4}\,2}{\pi^{\cancel{2}}} \cdot \frac{\cancel{\pi}}{\cancel{2}}$$

$$= \frac{4}{\pi}\alpha_1 - \frac{2}{\pi}\alpha_2$$

$$= \int_0^{3\pi/2} \sin x \cdot \phi_1(x) \, dx$$

per parter

$$= 2 \int^{\pi/2} \frac{2}{\pi} x \sin x \, dx \overset{\text{per parter}}{=} \frac{4}{\pi}$$

Tony:
$$\boxed{\frac{4}{\pi}\alpha_1 - \frac{2}{\pi}\alpha_2 = \frac{4}{\pi}}$$

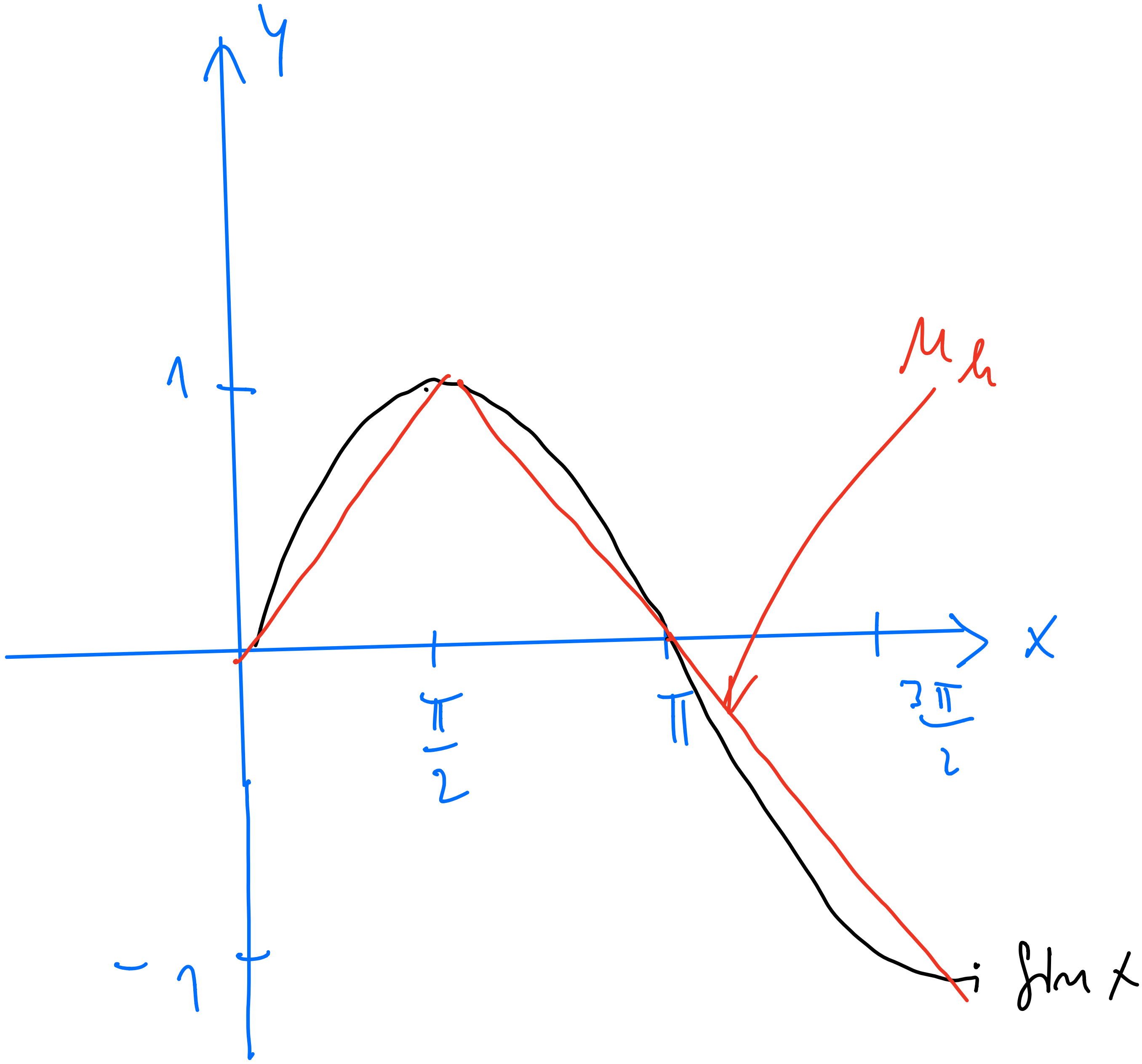
$$\int_0^{3\pi/2} \left(\sum_{j=0}^{3} \alpha_j \phi_j'(x)\right) \phi_2'(x)\,dx$$

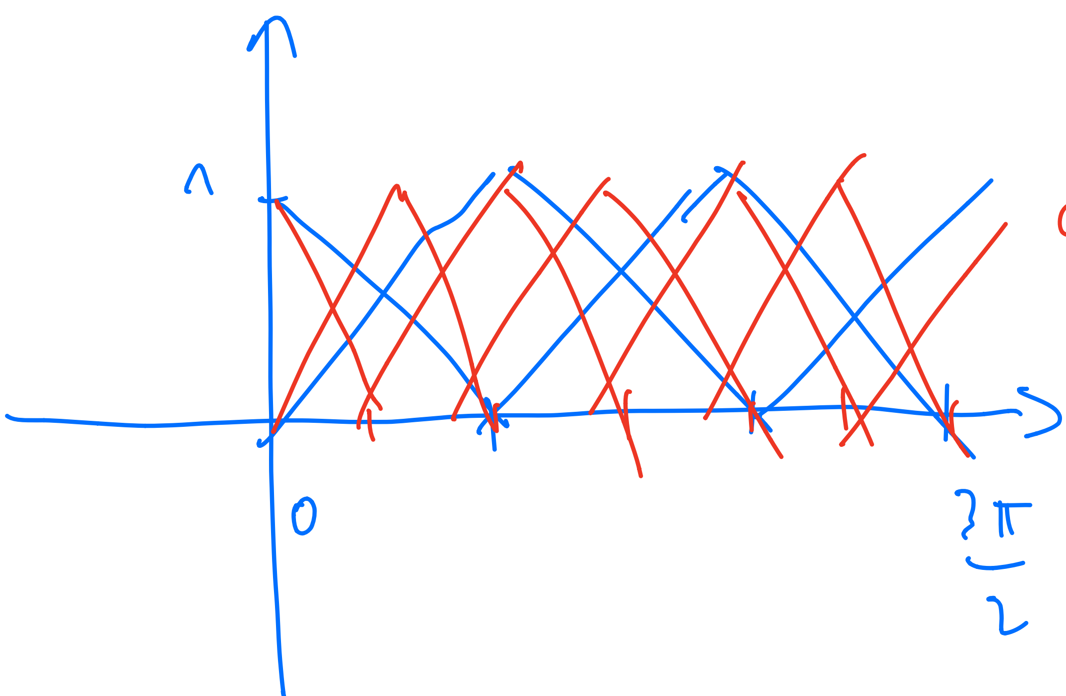
$$= \int_0^{3\pi/2} \sin(x)\,\phi_2(x)\,dx \;\ldots$$

$$\boxed{-\frac{2}{\pi}\alpha_1 + \frac{4}{\pi}\alpha_2 = -\frac{2}{\pi}}$$

$$\Rightarrow \alpha_1 = 1, \quad \alpha_2 = 0.$$

$$\boxed{u_h(x) = \phi_1(x) - \phi_3(x)}$$

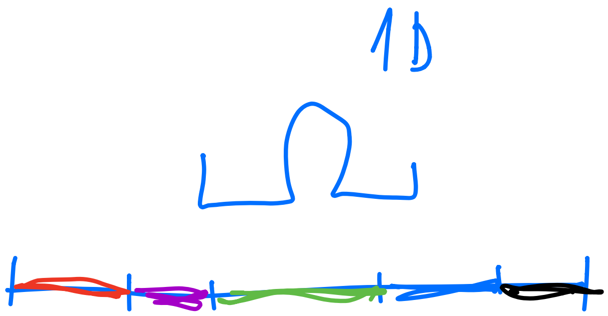


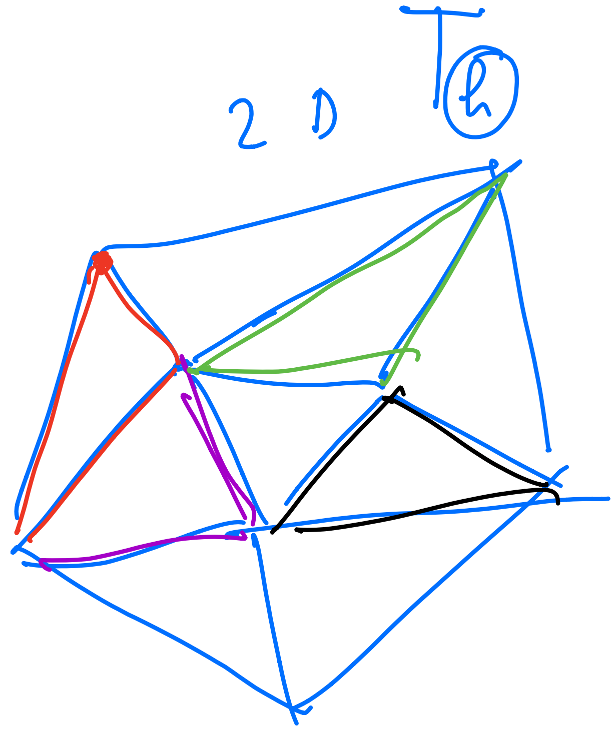


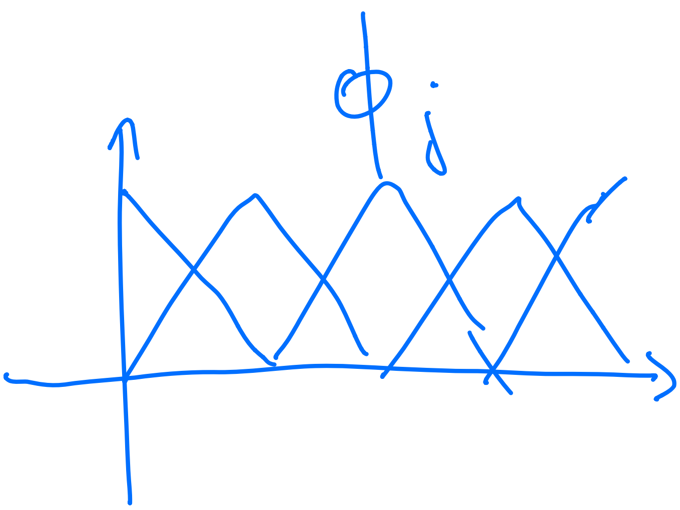
ottimo:
$h \to 0$

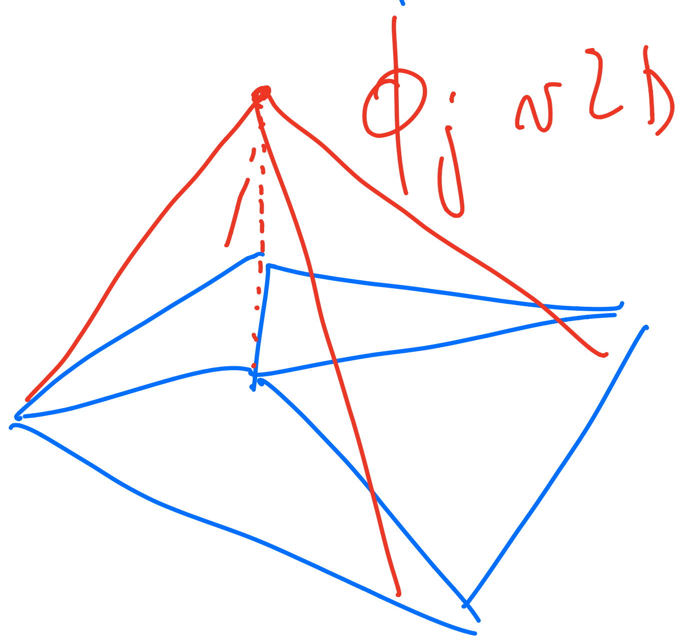
Ali je: $\lim_{h \to 0} u_h = u = \text{sm}$?

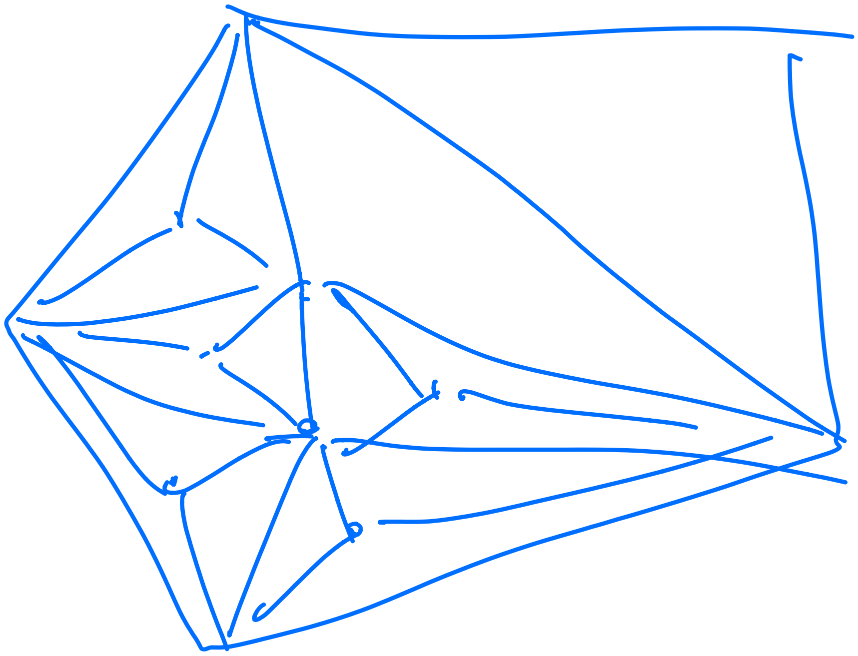
Kako je v 2D?



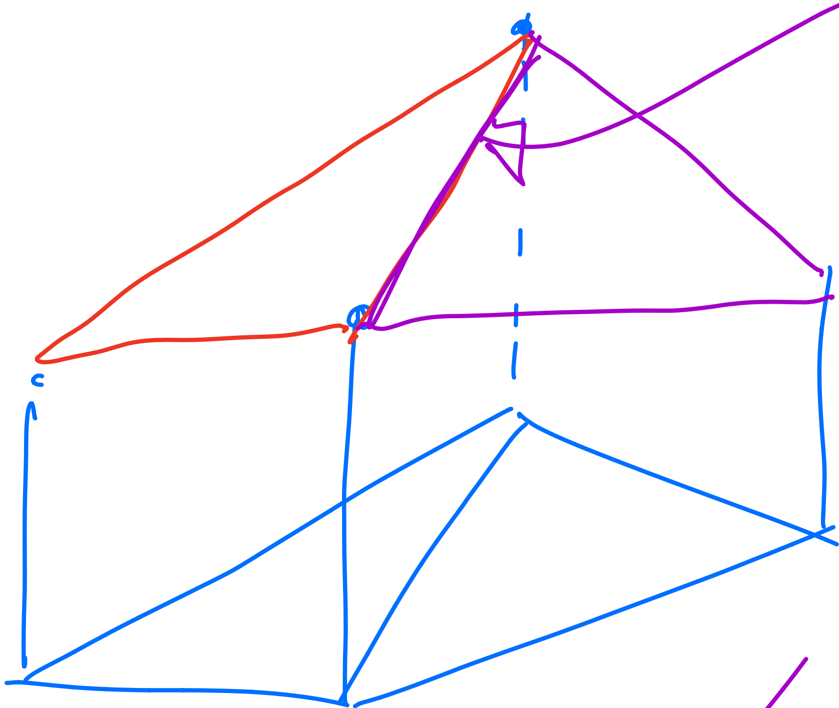




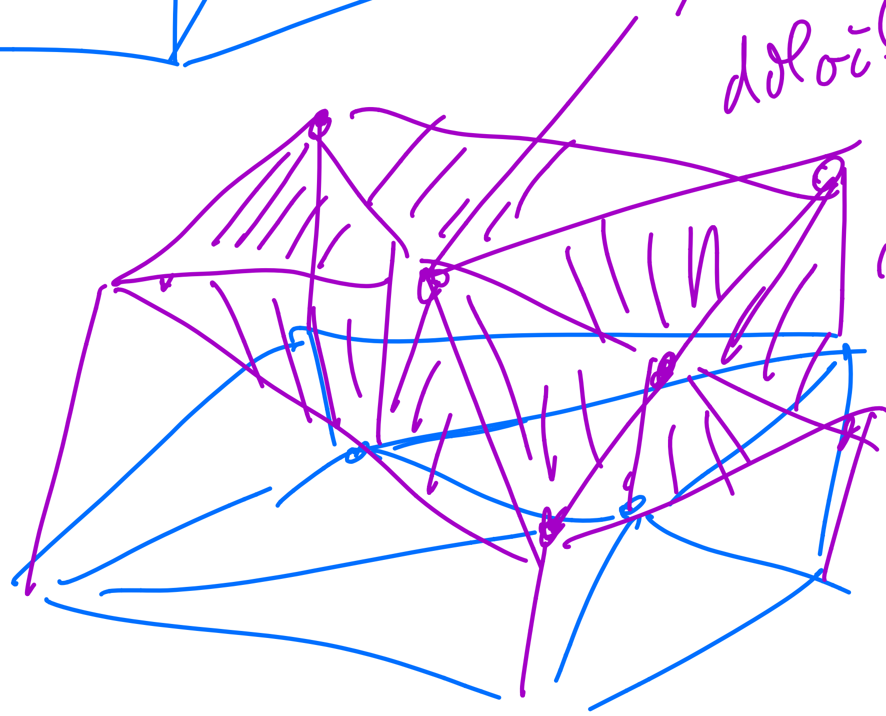




NI DOBRO, če ne dvolimo na homeomo na celi domeni!



Zvezna!



Ta je enolično določena z vred. v ogliščih