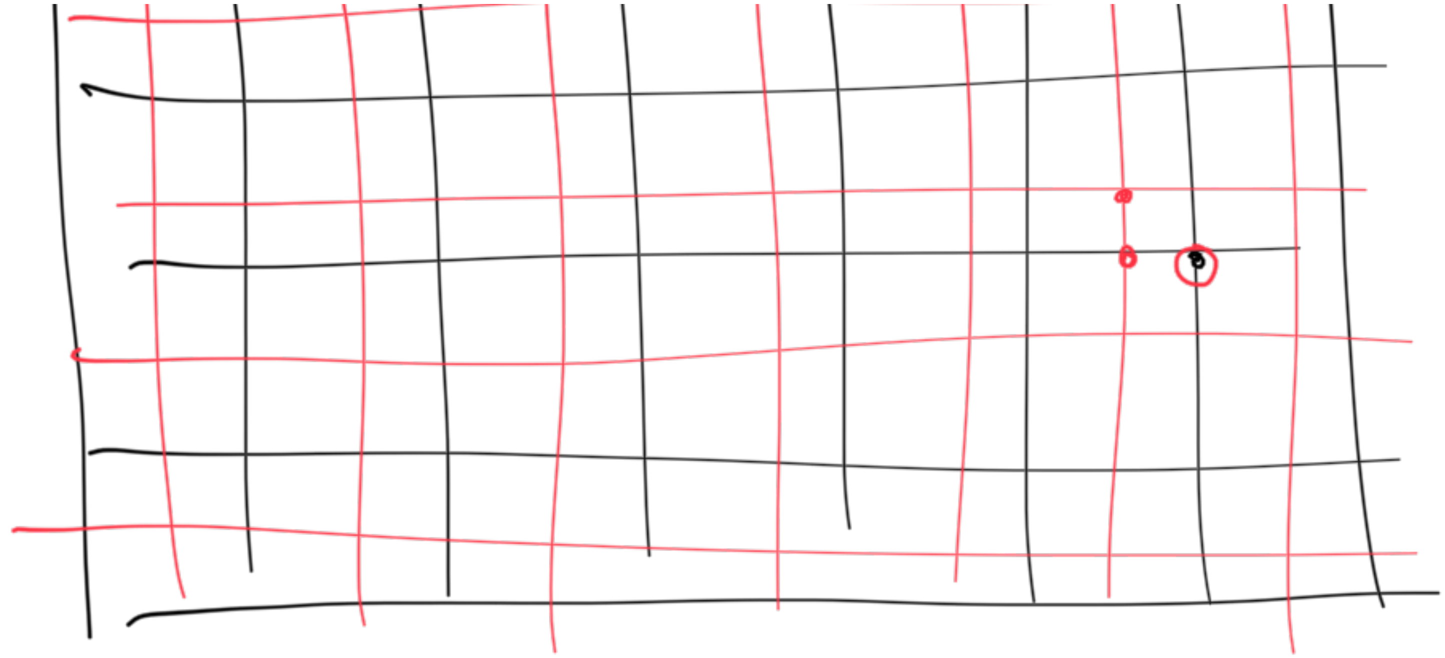
$$\mu = \frac{\gamma_w + w - 1}{w\sqrt{\gamma_w}}$$

$$\gamma_w - \mu w \sqrt{\gamma_w} + w - 1 = 0 \; ; \quad x = \sqrt{\gamma_w}$$

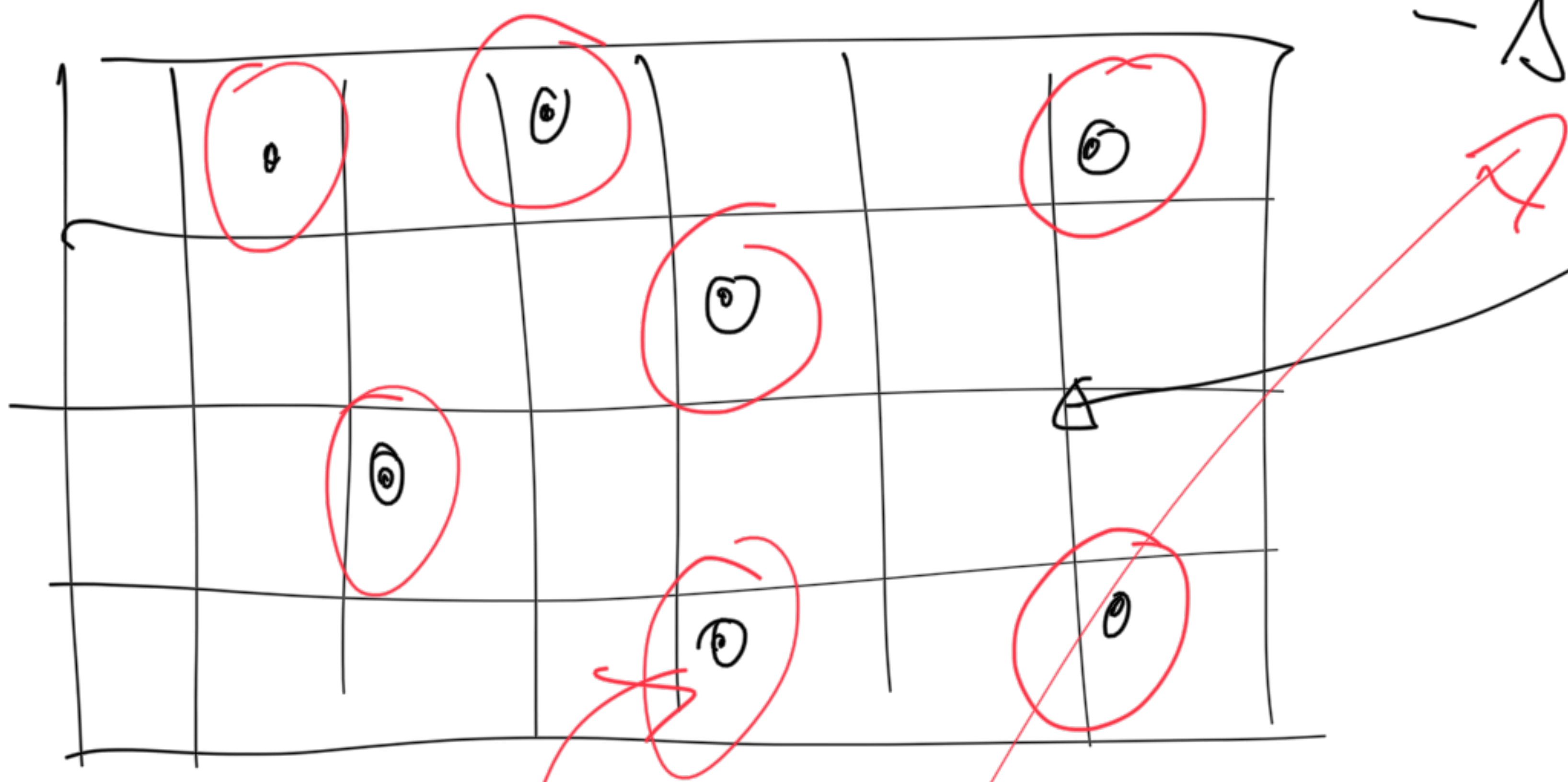
$$x^2 - \mu w x + w - 1 = 0$$

$$\boxed{x_{1,2} = \frac{\mu w}{2} \pm \sqrt{\left(\frac{w\mu}{2}\right)^2 + 1 - w}}$$

$$\boxed{\gamma_w = x^2}$$



$$-\Delta u = f$$



– brezmrežne metode

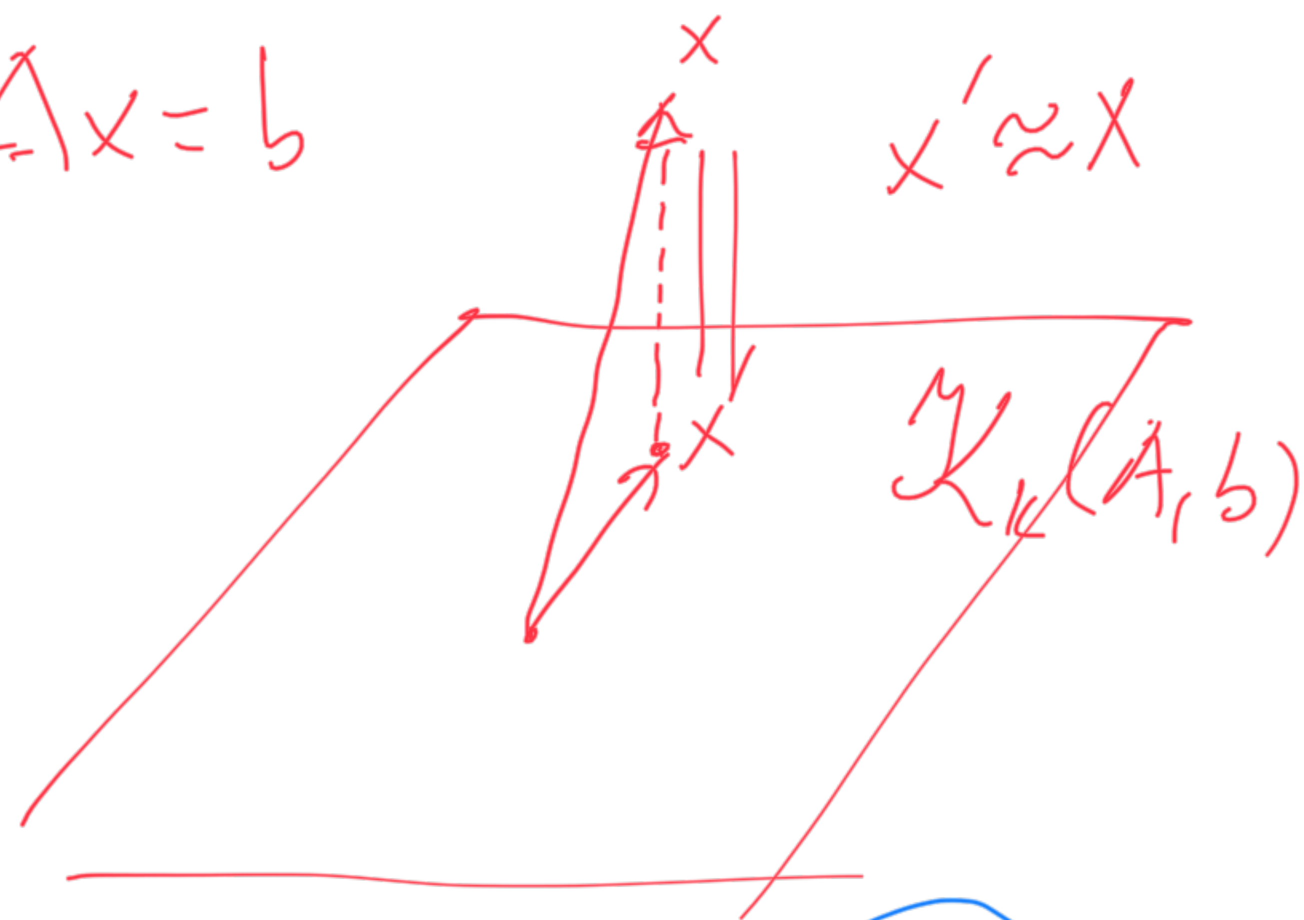
(RBF – radial basis functions)

(RBF - radial basis functions)

$$u = \sum_j \boxed{\alpha_j} \phi_j$$

$$\phi_j(x) = f(\|x\|)$$

$$Ax = b$$

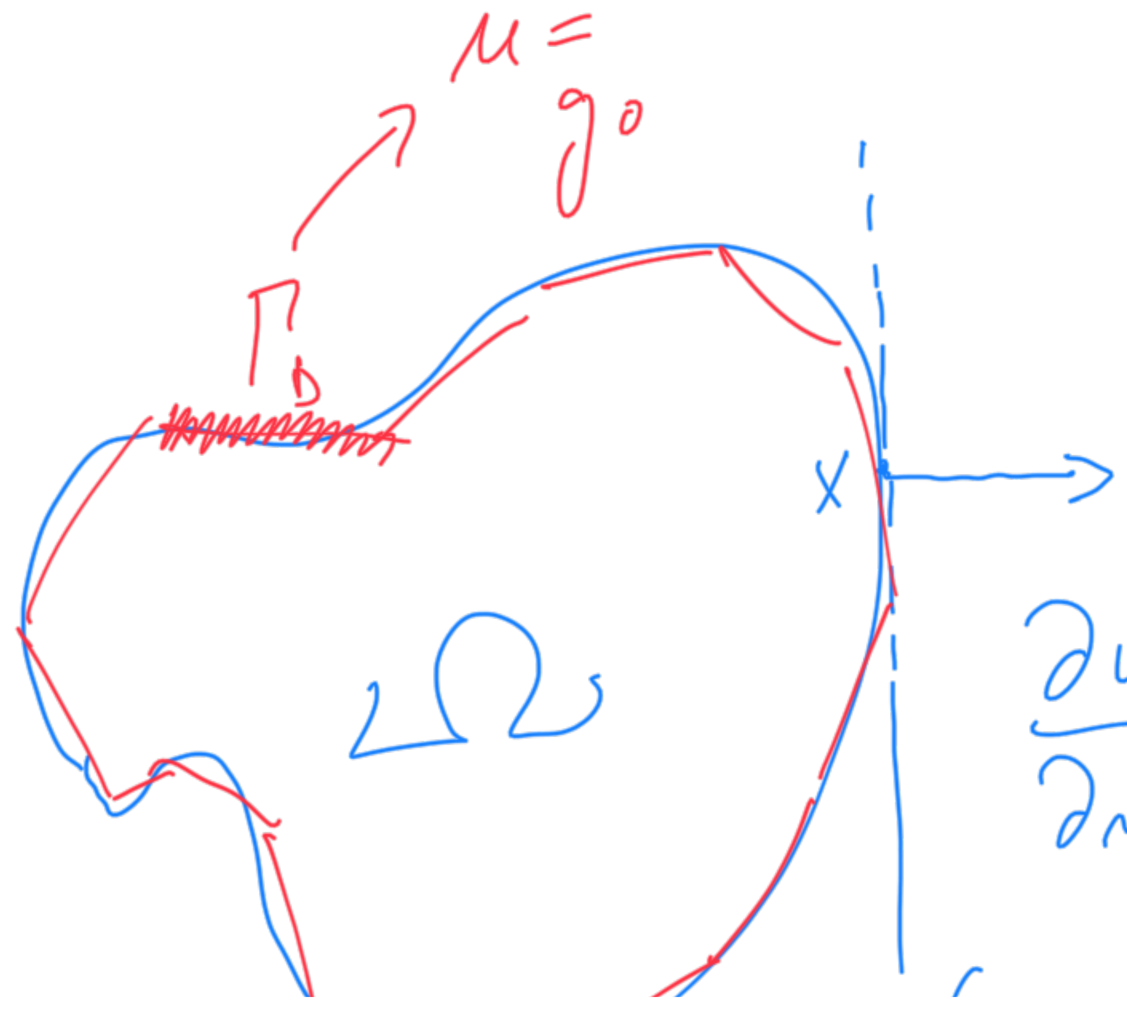


$-\Delta u = f$ na Ω spójna

$$\partial\Omega = \Gamma_1 \cup \Gamma_2$$

$$u\big|_{\Gamma_1} = g_0 \qquad \underline{\text{Dirichlet}}$$

$$\frac{\partial u}{\partial n}\bigg|_{\Gamma_2} = g_1 \qquad \underline{\text{Neumann}}$$



$n, \ \|n\| = 1$

$$\frac{\partial u}{\partial n} = \nabla u \cdot n$$

$$\begin{bmatrix} n_x \end{bmatrix}$$

$= (u_x, u_y) \cdot \begin{bmatrix} \; \\ n_y \end{bmatrix}$

Γ_N:

$$\frac{\partial u}{\partial n} = g_1 = \underline{\underline{u_x \cdot n_x + u_y n_y}}$$

· mešani pogoji:

$$\boxed{\frac{\partial u}{\partial n} + \alpha u = g, \quad x \in \partial D}$$

$$\boxed{-\operatorname{div}(\underset{=c(\vec{x})}{c} \operatorname{grad} u) + \underset{=a(\vec{x})}{a} u = f} = f(\vec{x})$$

$$n = n(\vec{x}), \quad \vec{x} \in \mathbb{R}^2 \quad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$-\operatorname{div}(c \operatorname{grad} u) + a u = f$$

$$\operatorname{grad} u = \nabla u = (u_x, u_y)$$

$$\operatorname{div} \varphi = \varphi_x + \varphi_y$$

$$\operatorname{div}(\operatorname{grad} u) = \Delta u$$

$c = 1$, $a = 0$, f poljuben

$$-\operatorname{div}(\operatorname{grad} u) = f$$

$$\boxed{-\Delta u = f} \quad \text{Poisson}$$

$g: I \to \mathbb{R}$ ene spremenljivke

$$J_g(x) = \; ? \; = [g'(x)]$$

$$\rho(J_g(x)) \le q < 1$$

$$\rho([g'(x)]) = |g'(x)| \le q < 1$$

$$\overset{\text{vektor}}{x^{(k+1)}} = \overset{\text{vektor}}{x^{(k)}} - \overset{\text{matrika}}{J_F^{-1}(x^k)} \cdot F(\overset{\text{vektor}}{x^{(k)}})$$

Newton v eni dim.:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \boxed{f'(x_k)^{-1}}\,\boxed{f(x_k)}$$

$$x^{(k+1)} = x^{(k)} - J_F(x^{(k)})^{-1} \cdot F(x^{(k)})$$

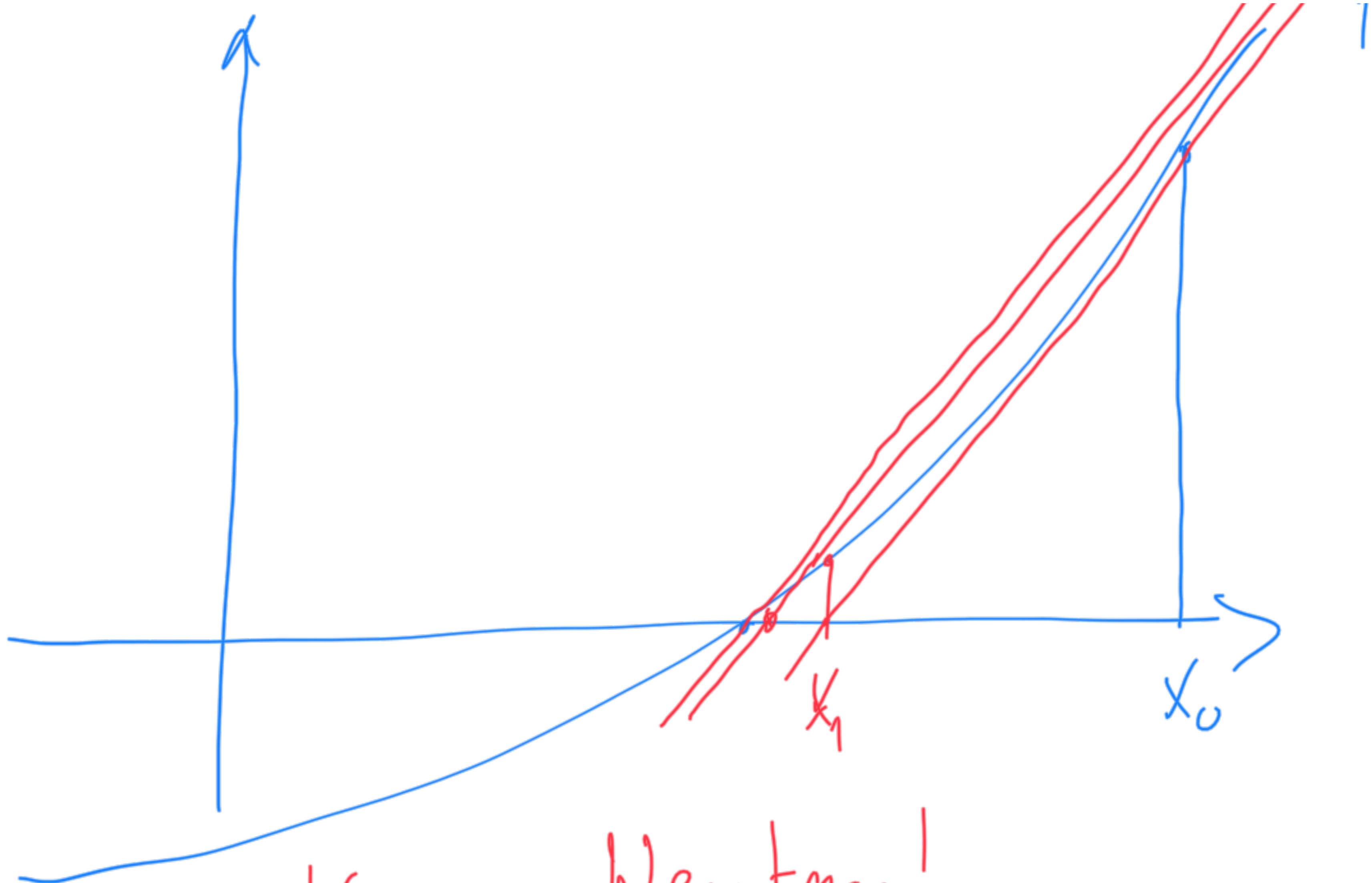
Ne želimo računati inverza

$$J_F\left(x^{(k)}\right)\underbrace{\left(x^{(k+1)}-x^{(k)}\right)}=-F\left(x^{(k)}\right)$$

$$\boxed{A\cdot\Delta x^{(k)} = b}$$

$\Rightarrow$ reševanje sistema lin. enačb

Za konstrukcijo $x^{(k+1)}$ moramo na vsakem koraku rešiti sistem lin. enačb!

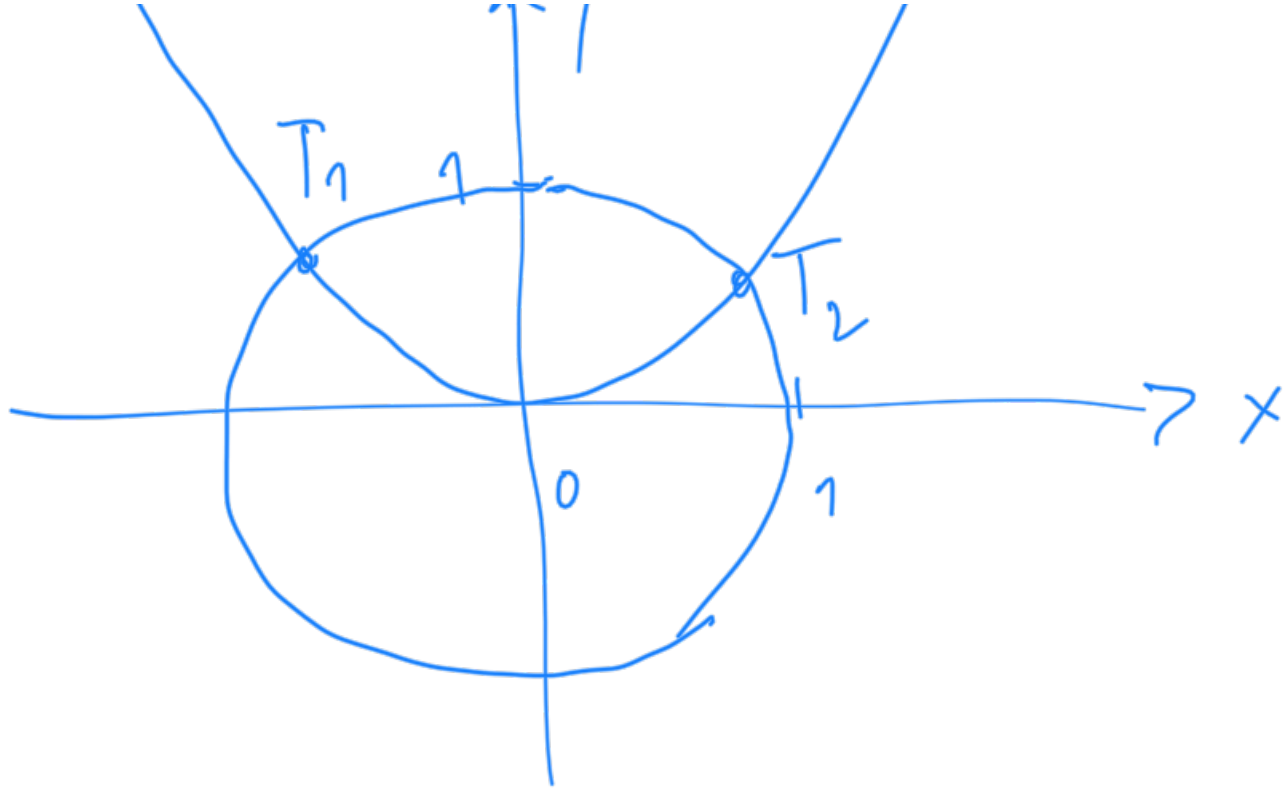


Kvazi Newton!

Rešujemo sistem

$$x^2 + y^2 - 1 = 0 \quad \leftarrow \text{krožnica}$$

$$x^2 - y = 0 \quad \leftarrow \text{parabola}$$



$$z^3 + 1 = 0, \quad z \in \mathbb{C}.$$

$$z = x + iy \Rightarrow z^3 + 1 = x^3 + 3x^2(iy)$$

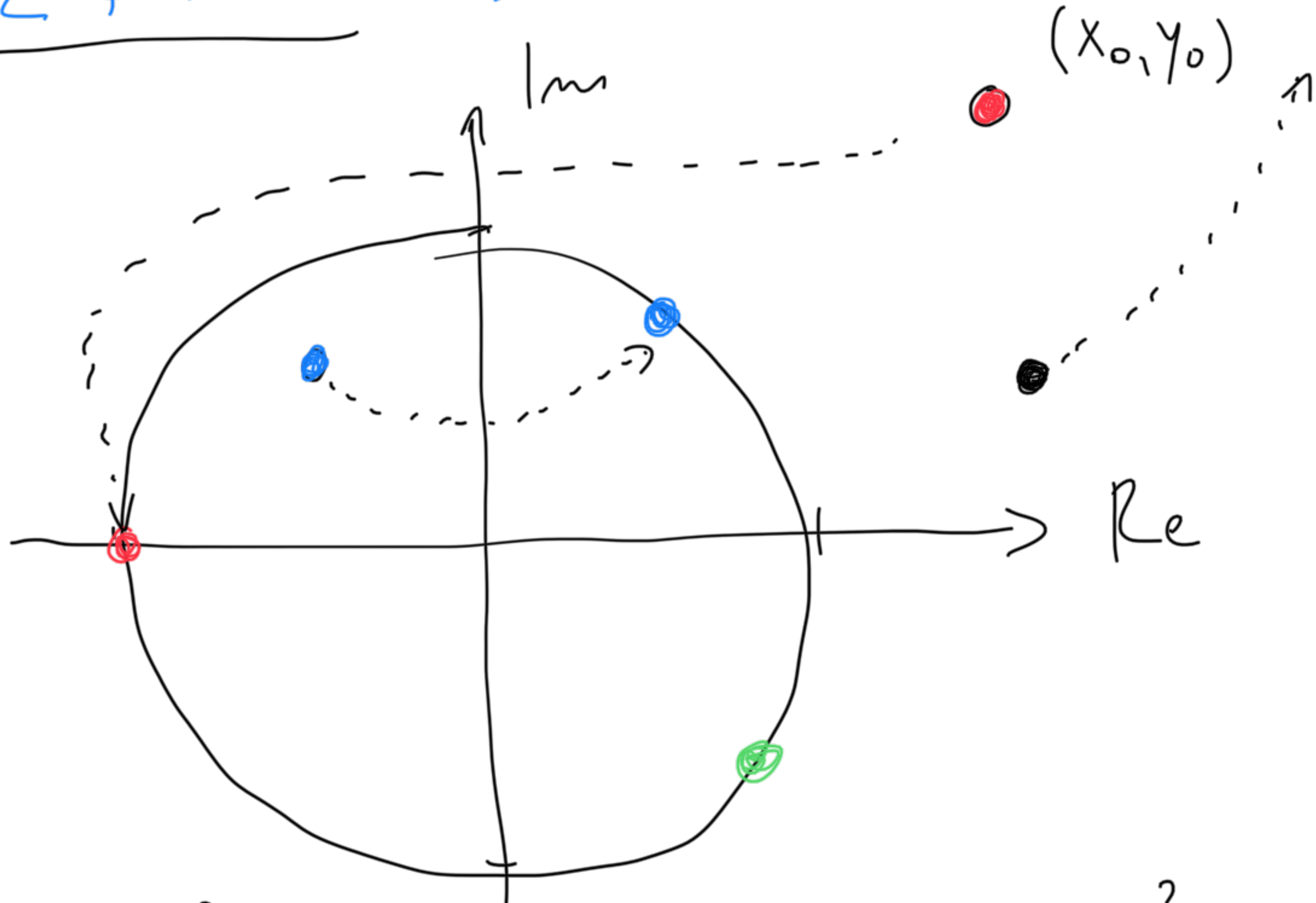
$$-3xy^2 - iy^3 + 1 = 0$$

$$\mathrm{Re}(z^3+1) = \boxed{x^3 - 3xy^2 + 1 = 0}$$

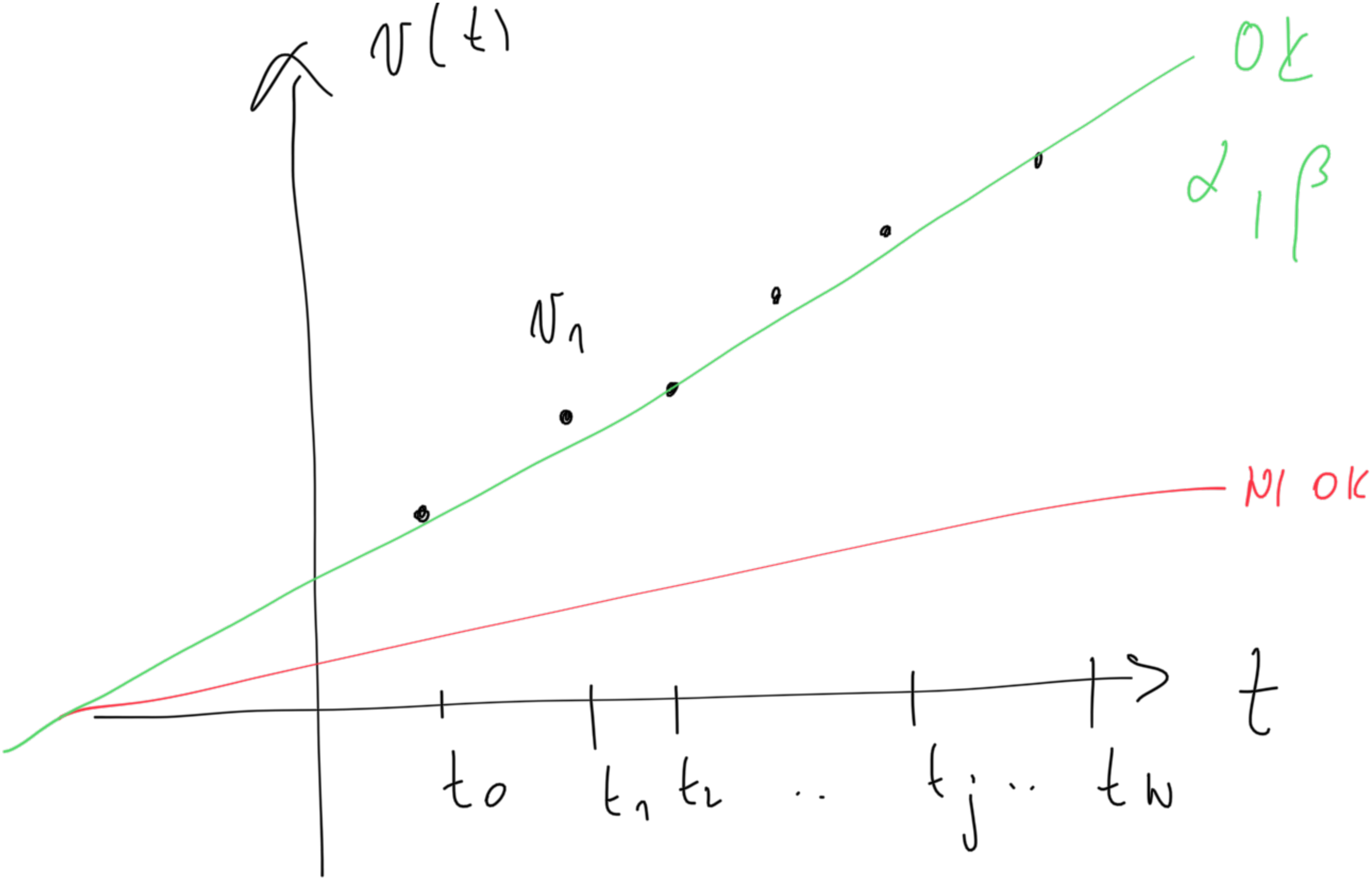
$$\mathrm{Im}(z^3+1) = \boxed{3x^2y - y^3 = 0}$$

Rešujemo z Newtonovo metodo!

$z^3 + 1 = 0 \Rightarrow$ tri rešitve



Kako se pobarva ravnina?



$$Ax = b, \quad A \in \mathbb{R}^{n\times m}, \quad b \in \mathbb{R}^{n}, \quad n \gg m$$

$\mathrm{rang}(A) \leq n$

Če je $\mathrm{rang}(A) < n \Rightarrow$

$\exists\, \underset{\neq 0}{z} \in \ker(A) : \boxed{Az = 0} = \sum_j z_j a_j = 0$

x je rešitev po metodi min. kvad $\Rightarrow \|Ax - b\|_2$

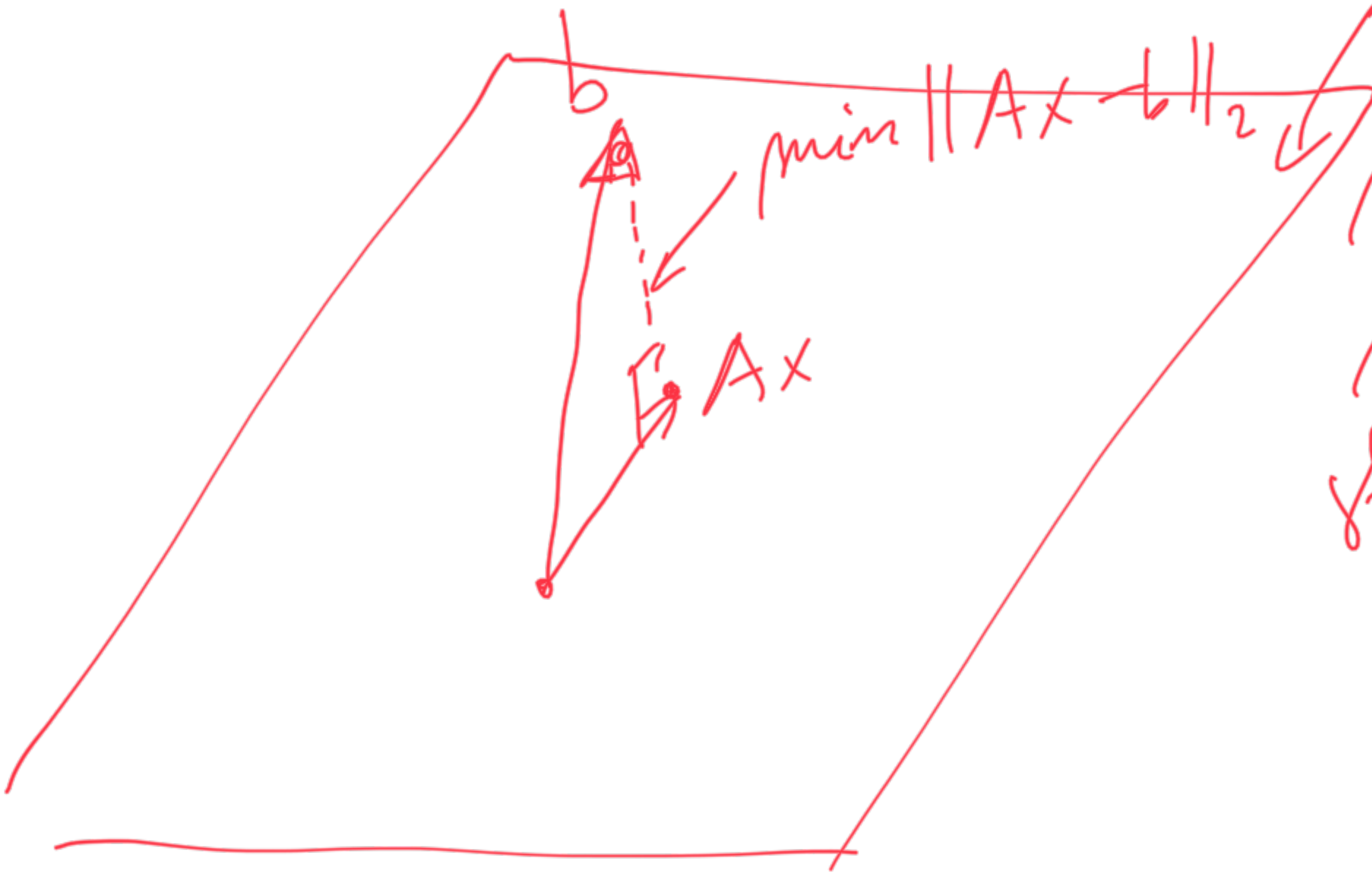
$\tilde{x} = x + z \Rightarrow \|A(x+z) - b\|_2$

$$= \|Ax + A''z - b\|_2 = \|Ax - b\|_2$$

Zahtevamo, da je

A polnega ranga: $\operatorname{rang}(A) = n$

$$Ax = b$$



podprost. napet na stolpce $A \in \mathbb{R}^{n\times m}$

$$Ax = b$$

$$A^T A x = A^T b$$

je simetrična!

$$(A^T A)^T = A^T (A^T)^T = A^T A \checkmark$$

<u>Pozitivna definitnost:</u>

B je poz. def., če je

$$x^T B x > 0 \quad \forall x \neq 0$$

$$B = A^T A: \quad x^T (A^T A) x > 0$$

$$(x^T A^T)(Ax) = (Ax)^T (Ax)$$

$$(x^T A^T)(Ax) = (Ax)^T (Ax)$$
$$\qquad y^T \cdot y$$

$$= \|Ax\|_2^2 \geq 0$$

Če bi bilo $\|Ax\|_2^2 = 0 \Rightarrow$

$Ax = 0 \Rightarrow x \in \ker(A)$ in $x \neq 0$

(nemogoče, $\ker$ je $\operatorname{rang}(A) = n$)

$\operatorname{rang}(A) = n$

$A = QR$, $Q \in \mathbb{R}^{m \times n}$ ortonorm. stolpci

$$R = \begin{bmatrix} \ddots & \\ 0 & \ddots \end{bmatrix} \in \mathbb{R}^{n \times n}$$

diagonalci so $\neq 0$

$\Downarrow$

$$A^T A x = A^T b$$

$$(QR)^T (QR) = (QR)^T b$$

$$(QR)^T(QR)x = (QR)^T b$$

$$R^T \underbrace{Q^T Q}_{I} R x = R^T Q^T b$$

$$R^T R x = R^T (Q^T b) \quad / \cdot (R^T)^{-1}$$

$\exists R^{-1} \Downarrow (R^T)^{-1}$

$(R^T)^{-1}$?

lev

$$\boxed{Rx = Q^T b}$$

direktno rješavanje