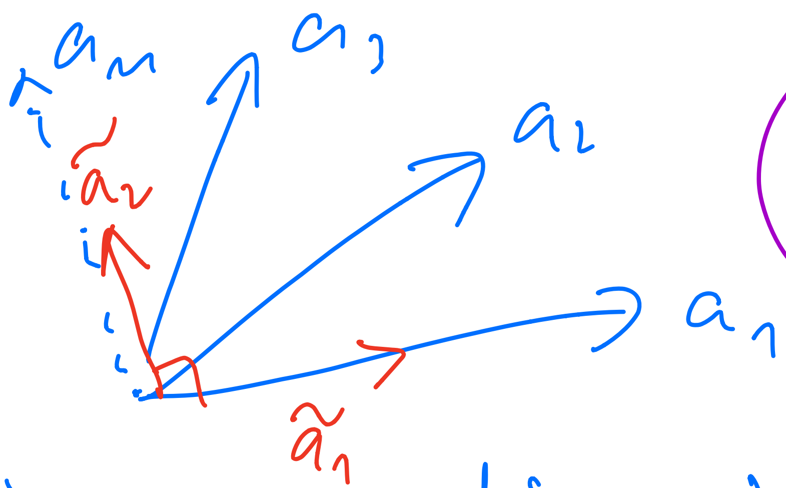
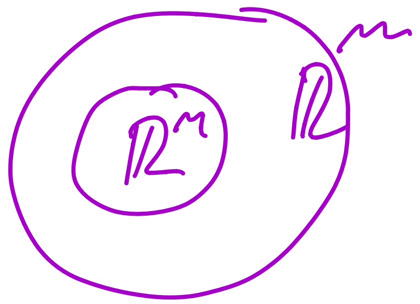
Gram-Schmidt:

$$A = [a_1, a_2, \dots, a_n] \in \mathbb{R}^{m\times n}$$





Im(A) je n-dim. podpr. $\mathbb{R}^m$

Ortogonalizacija:

$$a_1 \longmapsto \tilde{a}_1 = \frac{1}{\|a_1\|_2} a_1$$

$$a_2 \longmapsto \tilde{a}_2 \perp \tilde{a}_1$$

$a_2' = a_1 + \alpha a_2$ ni zahtevati:

$a_2' \perp \tilde{a}_1$, $\tilde{a}_2 = \frac{1}{\|a_2'\|} a_2'$

Postopek nadaljujemo za $\tilde{a}_3, \dots, \tilde{a}_n$!

Dobimo $\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$:

$\tilde{a}_i \perp \tilde{a}_j$; $\|\tilde{a}_i\|_2 = 1$:

$$[\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n]$$

matrika

$$= \underbrace{[a_1, a_2, \dots, a_n]} \cdot M$$

$$\Rightarrow A = [\tilde{a}_1, \dots, \tilde{a}_n] \cdot R$$

$$= \left[\tilde{a}_1 \,\middle|\, \cdots \,\middle|\, \tilde{a}_n\right] \cdot \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ 0 & r_{22} & \cdots & \\ 0 & & & \\ \vdots & & & \\ 0 & \cdots & & r_{nn} \end{bmatrix}$$

$$= Q \cdot R$$

Givensove rotacije:

želimo $\boxed{0}$

$$\underbrace{\begin{bmatrix} \cos\varphi_{12} & \sin\varphi_{12} & 0 & \cdots & 0 \\ -\sin\varphi_{12} & \cos\varphi_{12} & & 0 & \\ & & 1 & & \\ & & & \ddots & \\ & 0 & & & 1 \end{bmatrix}}_{R_{12}^T} \begin{bmatrix} * & * & \cdots & * \\ * & * & * & \cdots & * \\ * & & & & \\ \vdots & & A & & \\ * & & & & * \end{bmatrix}$$

se ne spremeni

$$R_{12}^T A = \begin{bmatrix} * & * & \cdots & * \\ \boxed{0} & & & \vdots \\ * & & & \vdots \\ \vdots & & & \vdots \\ * & \cdots & \cdots & * \end{bmatrix}$$

$$R_{13}^T\left(R_{12}^T A\right) = \begin{bmatrix} * & \cdots & * \\ 0 & & \vdots \\ 0 & & \\ * & & \vdots \\ \vdots & & \\ * & & * \end{bmatrix}$$

$$R_{14}^T\left(R_{13}^T R_{12}^T A\right) = \begin{bmatrix} * & \cdots & * \\ 0 & & \vdots \\ 0 & & \vdots \\ 0 & & \vdots \\ * & & \\ \vdots & & \\ * & & * \end{bmatrix}$$

$\cdots\,\cdot$

$$R_{1,m}^T R_{1,m-1}^T \cdots R_{12}^T A = \begin{bmatrix} * & * & \cdots & * \\ 0 & * & & \vdots \\ 0 & * & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & * & & * \end{bmatrix}$$

$$\underbrace{R_{m,m}^T R_{m,m-1}^T \cdots R_{12}^T}_{\begin{array}{c} \shortparallel \\ Q^T \end{array}} A = \begin{bmatrix} * & * & \cdots & * \\ 0 & * & & * \\ 0 & 0 & & \vdots \\ \vdots & \vdots & R & * \\ \vdots & \vdots & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 0 \end{bmatrix}$$

$$Q^T A = R$$

$$Q^T = \underbrace{R_{n,m}^T R_{n,m-1}^T \dots R_{12}^T}_{\text{ortogonalno}}$$

$\Rightarrow$ Q^T ortogonalna!

Res!: Q_1 in Q_2 ortogonalni

$\Rightarrow$ $Q_1 Q_2$ ortogonalna

$$Q_1^T Q_1 = I \qquad Q_2^T Q_2 = I :$$

$$(Q_1 Q_2)^T (Q_1 Q_2) = Q_2^T \underbrace{Q_1^T Q_1}_{I} Q_2$$

$$= I$$

$$Q^T A = R \;; \; Q^T \text{ ortogonalna}$$

$$Q(Q^T A) = QR$$

$$A = QR, \quad Q \text{ ortog.}$$

$$R = \begin{bmatrix} \ddots & \\ 0 & \end{bmatrix}$$

razširjeni QR razcep

Denimo, da imamo razširjeni QR razcep: $A = QR$

Min. kvad:

$$\min \|Ax-b\|_2^2 = \min \|Q^T(\overset{QR}{A}x-b)\|_2^2$$

$$= \min \|Rx - Q^T b\|_2^2$$

ni zg. trikotna ampak kvazi zg. trikotna

$$n \quad R = \begin{bmatrix} \tilde{R} \\ \hdashline 0 \end{bmatrix}$$

$$\|Rx - Q^T b\|_2^2 = \left\| \begin{array}{l} n\,\{ \\ m-n\,\{ \end{array} \begin{bmatrix} \tilde R \\ \hline 0 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} - \begin{bmatrix} c_1 \\ \hline c_2 \end{bmatrix} \right\|_2^2$$

$$= \left\| \begin{bmatrix} \tilde R x - c_1 \\ -c_2 \end{bmatrix} \right\|_2^2$$

$$= \|\tilde R x - c_1\|_2^2 + \|-c_2\|_2^2$$

$$\Downarrow$$

$$\tilde R x - c_1 = 0 \Rightarrow \boxed{\tilde R x = c_1}$$

zg. trik sistem