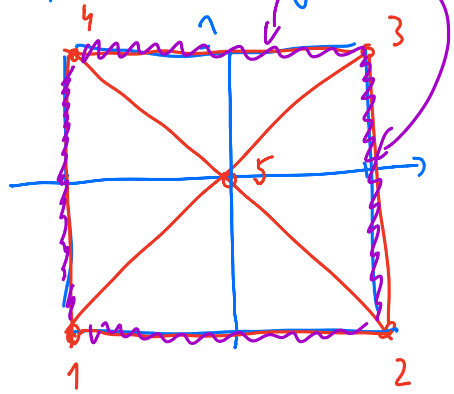
Primer: $\Omega = (-1,1) \times (-1,1)$

$-\Delta u = 1$ na Ω

$u\big|_{\partial\Omega} = 0 = g_0$

T_h:



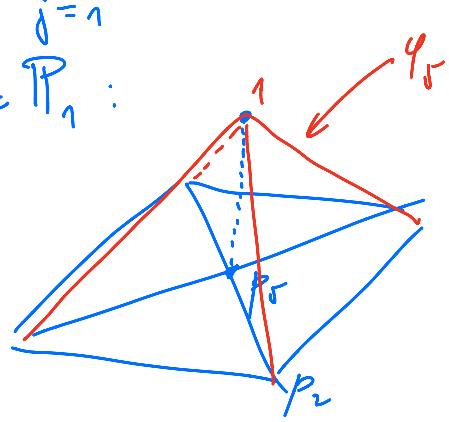
Trikotniki: Δ_{154}, Δ_{125}, Δ_{235}, Δ_{345}

Rešitev iščemo v obliki

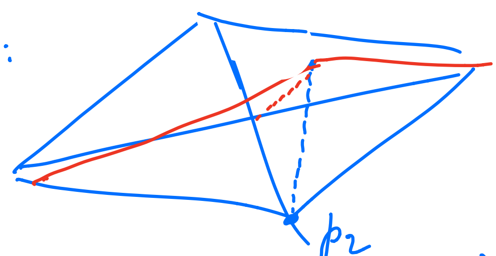
$$u_h = \sum_{j=1}^{5} \alpha_j \varphi_j$$

$\varphi_j \in \mathbb{P}_1$:

φ_5:



φ_2:



Zaradi robnih pogojev je

$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$:

$$\boxed{u_h = \alpha_5 \varphi_5}$$

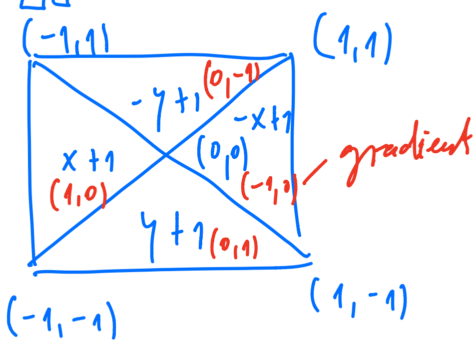
Edina testna f. je $v=\varphi_5$.

Zadostiti moramo enačbi

$$\int_{\Omega} \nabla u_h \cdot (\nabla \varphi_5)^T \, d\Omega = \int_{\Omega} 1\, \varphi_5 \, d\Omega$$

$$\Rightarrow \int_{\Omega} \alpha_5 \, \nabla \varphi_5 \cdot (\nabla \varphi_5)^T \, d\Omega$$

$$= \int_{\Omega} \varphi_5 \, d\Omega$$



Na Δ_{154} je φ_5 enaka:

$\alpha x + \beta y + \gamma$:

$$\left.\begin{aligned} -\alpha - \beta + \gamma &= 0 \\ 0 + 0 + \gamma &= 1 \\ -\alpha + \beta + \gamma &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \gamma &= 1 \\ \alpha &= 1 \\ \beta &= 0 \end{aligned}$$

$\Rightarrow \quad x+1$

$$\int_{\Omega} \alpha_5 \cdot 1 \, d\Omega = \int_{\Omega} 1 \cdot \varphi_5 \, d\Omega$$

$$\alpha_5 \cdot 4 = 4/3 \Rightarrow \boxed{\alpha_5 = \frac{1}{3}}$$

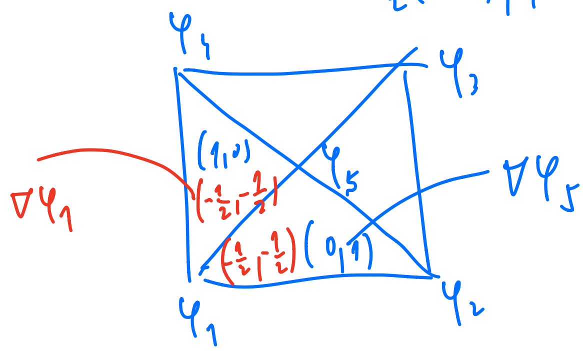
$$\boxed{u_h = \frac{1}{3} \varphi_5}$$

② $\Omega = (-1,1) \times (-1,1)$

$-\Delta u = 1$ na Ω

$u\big|_{\partial\Omega} = g_0$

$$g_0(x,y) = \begin{cases} 0; & x=1,\ -1 \le y \le 1 \\ 0; & y=1;\ -1 \le x \le 1 \\ \frac{1}{2}(1-y); & x=-1 \\ \frac{1}{2}(1-x); & y=-1 \end{cases}$$



Iz robnih pogojev sledi

$\alpha_2 = \alpha_3 = \alpha_4 = 0$ in

$\alpha_1 = 1.$

Kakšen je predpis za φ_1?

$\varphi_1(x,y) = -\frac{1}{2}(x+y)$

$\nabla\varphi_1 = (-\frac{1}{2}, -\frac{1}{2})$ na /

$u_h = \varphi_1 + \overset{?}{\alpha_5}\varphi_5$

$$\int_\Omega \nabla u_h (\nabla\varphi_5)^T \, d\Omega$$

$$= \int_\Omega 1 \cdot \varphi_5 \, d\Omega$$

$$\int_\Omega \left(\nabla\varphi_1 (\nabla\varphi_5)^T + \alpha_5 \nabla\varphi_5 (\nabla\varphi_5)^T\right) d\Omega = 4/3$$

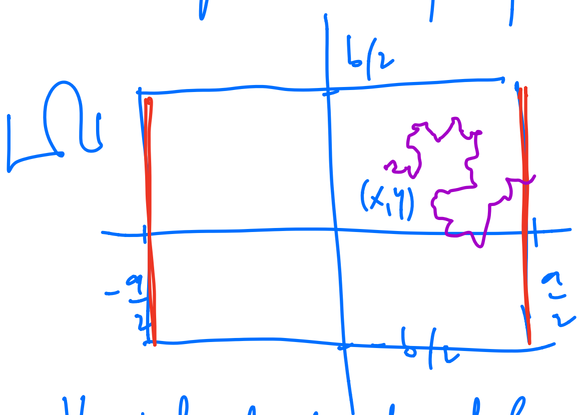
$$\alpha_5 \cdot 4 + \int_{\Omega} \nabla \varphi_1 (\nabla \varphi_5)^T d\Omega = 4/3$$

$$4\alpha_5 + \left(-\frac{1}{2} - \frac{1}{2}\right) = 4/3$$

$$\alpha_5 = \frac{7}{12} \approx 0.58$$

$$\boxed{u_h = \varphi_1 + \frac{7}{12}\varphi_5}$$

Gibanje delca po pravokot.



Verjetnost, da bo delec zapustil pravokotnik skozi vdeči stranici, je $u(x,y)$:

$\Delta u = 0$ na Ω

$u|_{\Gamma_1} = 0$, Γ_1 - spodnja in zg. stranica

$u|_{\Gamma_2} = 1$, Γ_2 - leva in desna stranica