Števila

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$
$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$
$$a^2 - b^2 = (a-b)(a+b)$$
$$a^3 \pm b^3 = (a \pm b)\left(a^2 \mp ab + b^2\right)$$

Funkcije

D_f in Z_f

$f(x)$	D_f	Z_f
e^x	$\mathbb{R}$	$(0,\infty)$
$\log x$	$(0,\infty)$	$\mathbb{R}$
$\arcsin x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\arccos x$	$[-1,1]$	$[0,\pi]$
$\arctan x$	$\mathbb{R}$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Kvadratna funkcija

$f(x) = ax^2 + bx + c$

Diskriminanta: $D = b^2 - 4ac$

Ničle: $x_{1,2} = \frac{-b\pm\sqrt{D}}{2a}$

Teme: $T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

Kotne funkcije

Adicijski izreki

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Defaktorizacija

$$\sin x \cos y = \frac{1}{2}\left(\sin(x-y) + \sin(x+y)\right)$$
$$\sin x \sin y = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$$
$$\cos x \cos y = \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$$

Faktorizacija

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$
$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x - \cos y = 2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$$

Dvojni in polovični koti

$$\cos(2x) = \cos^2 x - \sin^2 x \quad \sin(2x) = 2\sin x \cos x$$
$$\cos^2 x = \frac{1+\cos(2x)}{2} \qquad \sin^2 x = \frac{1-\cos(2x)}{2}$$

Vrednosti kotnih funkcij

$\theta \rightarrow$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/

$$\sin(\pi - x) = \sin x, \quad \cos(\pi - x) = -\cos x$$
$$\sin(\pi + x) = -\sin x, \quad \cos(\pi + x) = -\cos x$$
$$\sin(2\pi - x) = -\sin x, \quad \cos(2\pi - x) = \cos x$$

Odvodi

Tabela odvodov

$$(x^r)' = rx^{r-1} \quad \text{za } r \in \mathbb{R}$$
$$(e^x)' = e^x, \qquad (\ln x)' = \frac{1}{x}$$
$$(a^x)' = a^x \ln a \quad \text{za } a > 0$$
$$(\log_a x)' = \frac{1}{x \ln a} \quad \text{za } a > 0$$
$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x$$
$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\arctan x)' = \frac{1}{1+x^2}$$

Tangenta, linearen približek

$y = f(x_0) + f'(x_0)(x - x_0)$

Integrali

Tabela integralov

$$\int x^r\,dx = \frac{1}{r+1}x^{r+1} + C, \;\; r \neq -1$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$
$$\int e^x\,dx = e^x + C$$
$$\int a^x\,dx = \frac{1}{\ln a}a^x + C \quad \text{za } a > 0$$
$$\int \sin x\,dx = -\cos x + C$$
$$\int \cos x\,dx = \sin x + C$$
$$\int \frac{1}{\cos^2 x}\,dx = \tan x + C$$
$$\int \frac{1}{\sin^2 x}\,dx = -\cot x + C$$
$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$
$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$
$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \ln(x + \sqrt{x^2+1}) + C$$

Prostornina vrtenine

$$V = \pi \int_a^b f(x)^2 \, dx$$

Površina plašča vrtenine

$$P = 2\pi \int_a^b f(x)\sqrt{1+f'(x)^2} \, dx$$

Dolžina loka

$$l = \int_a^b \sqrt{1+f'(x)^2} \, dx$$

Diferencialne enačbe

Ločljivi spremenljivki

Enačba: $y' = f(x)g(y)$

Reševanje: $\int \frac{dy}{g(y)} = \int f(x)\,dx$

Linearna 1. reda

Enačba: $y' + p(x)y = q(x)$

Reševanje: **HD:** $y' + p(x)y = 0$, $y = Ce^{-\int p(x)\,dx}$

VK: $y = C(x)e^{-\int p(x)\,dx}$

Bernoulli

Enačba: $y' + p(x)y = q(x)y^r$; $r \in \mathbb{R} \setminus \{0,1\}$

Reševanje: $z = y^{1-r} \Longrightarrow z' = (1-r)y^{-r}y'$

Linearna 2. reda s konst. koef.

Enačba: $ay'' + by' + cy = f(x)$

Reševanje: **HD:** $a\lambda^2 + b\lambda + c = 0$, $D = b^2 - 4ac$

$D > 0$**:** $y_1 = e^{\lambda_1 x}$, $y_2 = e^{\lambda_2 x}$

$D = 0$**:** $y_1 = e^{\lambda x}$, $y_2 = xe^{\lambda x}$

$D < 0$**:** $\lambda_{1,2} = \alpha \pm \beta i$; $y_1 = e^{\alpha x}\cos(\beta x)$, $y_2 = e^{\alpha x}\sin(\beta x)$

VK:

$$y = u_1 y_1 + u_2 y_2, \quad \Delta = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & y_2 \\ \frac{f}{a} & y_2' \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & \frac{f}{a} \end{vmatrix}$$

$$u_1' = \frac{\Delta_1}{\Delta} \qquad u_2' = \frac{\Delta_2}{\Delta}$$

PR:

$$f(x) = p(x)e^{\mu x} : \quad y_P = x^s q(x) e^{\mu x};$$
$$\deg q = \deg p, \quad s = \text{ stopnja ničle } \mu \text{ v KE}$$

PR:

$$f(x) = p(x)\cos(\mu x) \quad \text{ali} \quad p(x)\sin(\mu x) :$$
$$y_P = x^s q(x) \left(A\cos(\mu x) + B\sin(\mu x)\right);$$
$$\deg q = \deg p, \quad s = \text{ stopnja ničle } \mu i \text{ v KE}$$

Parcialni odvodi

Tangentna ravnina, linearen približek

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Stacionarne točke in lokalni ekstremi

Stac. tč.: $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$

Lok. ekstr.: $D(x_0, y_0) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}(x_0, y_0)$

- $D < 0$: (x_0, y_0) ni lok. ekstr.,
- $D > 0$ in $f_{xx} < 0$: (x_0, y_0) je lok. maks.
- $D > 0$ in $f_{xx} > 0$: (x_0, y_0) je lok. min.

Masa in težišče

$$m = \iint_D \rho(x, y)\, dx\, dy$$

$$x_T = \frac{1}{m}\iint_D x\,\rho(x, y)\, dx\, dy \quad y_T = \tfrac{1}{m}\iint_D y\,\rho(x, y)\, dx\, dy$$

Polarne koordinate

$$x = r\cos\varphi, \quad y = r\sin\varphi, \quad J(r, \varphi) = r$$
$$r = \sqrt{x^2 + y^2}, \quad \tan\varphi = \tfrac{y}{x}$$

Cilindirčne koordinate

$$x = r\cos\varphi, \qquad y = r\sin\varphi, \qquad z = z$$
$$J(r, \varphi, z) = r, \quad r = \sqrt{x^2 + y^2}, \quad \tan\varphi = \frac{y}{x}$$

Sferične koordinate

$$x = r\cos\varphi\cos\theta, \qquad y = r\sin\varphi\cos\theta, \qquad z = r\sin\theta$$
$$J(r, \varphi, \theta) = r^2\cos\theta, \quad r = \sqrt{x^2 + y^2 + z^2}, \quad \tan\varphi = \frac{y}{x},$$
$$\tan\theta = \frac{z}{\sqrt{x^2 + y^2}}, \qquad \varphi \in [0, 2\pi), \qquad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$