$$\hat{p} = p\underbrace{(1+\delta_1)(1+\delta_2)\dots(1+\delta_n)}_{(1+\gamma)}; \quad |\delta_i| \le u$$

$$(1-u)^n \le 1+\gamma \le (1+u)^n$$

(1) $(1-u)^n \ge 1 - nu$ $\qquad nu \ll 1$

Indukcija:

$n=1: \quad 1-u \ge 1-u$ ✓

$n \to n+1: \quad$ I.P. $(1-u)^n \ge 1-nu$

$$(1-u)^{n+1} = (1-u)(1-u)^n \ge (1-u)(1-nu)$$

$$= 1-(n+1)u + nu^2 \ge 1-(n+1)u$$

$$(1+u)^n = 1 + \binom{n}{1}u + \binom{n}{2}u^2 + \dots$$

$$= 1 + nu + \frac{n(n-1)}{2}u^2 + \dots$$

$$\cancel{1} - nu - O(u^2) \le (\cancel{1}+\gamma) \le \cancel{1} + nu + O(u^2)$$

$$|\gamma| \le nu + O(u^2)$$

Računanje članov rekurz. podanega zaporedja :

(1) $a_{n+1} - \frac{5}{2} a_n + a_{n-1} = 0$, $a_0 = 1$, $a_1 = 1/2$

$$\lambda^{n+1} - \frac{5}{2}\lambda^n + \lambda^{n-1} = 0 \quad / : \lambda^{n-1}$$

$$\lambda^2 - \frac{5}{2}\lambda + 1 = 0$$

$$\lambda_1 = 1/2 \; ; \; \lambda_2 = 2 \Rightarrow (1/2)^n, \; 2^n$$

$$a_n = A \cdot \left(\frac{1}{2}\right)^n + B\, 2^n = A \cdot 2^{-n} + B \cdot 2^n$$

$$\left.\begin{aligned} a_0 &= 1 = A + B \\ a_1 &= 1/2 = \frac{1}{2}A + 2B \end{aligned}\right\} \Rightarrow B = 0, \; A = 1$$

$$a_n = 2^{-n}$$

(2) $a_{n+1} - \frac{10}{3} a_n + a_{n-1} = 0$; $a_0 = 1$, $a_1 = 1/3$

$$\lambda^2 - \frac{10}{3}\lambda + 1 = 0 \Rightarrow \lambda_1 = \frac{1}{3}, \; \lambda_2 = 3$$

$$a_n = A \cdot 3^{-n} + B \cdot 3^n$$

$$\left.\begin{aligned} a_0 &= 1 = A + B \\ a_1 &= 1/3 = \frac{1}{3}A + 3B \end{aligned}\right. \Rightarrow A = 1, \; B = 0$$

$$\boxed{a_n = 3^{-n}}$$ a_n je padajoče

Ker $a_1 = 1/3$ ni predstavljivo:

$a_1 \to \bar{a}_1 \approx a_1 + 10^{-6}$ $\quad \bar{a}_1 = a_1(1+\mu)$

$|\mu| < 10^{-16}$

$a_0 = 1$; $a_1 = 1/3 + \varepsilon$; $|\varepsilon| \le \mu$

$$\begin{aligned} a_0 &= \boxed{1 = A + B} \\ a_1 &= \boxed{1/3 + \varepsilon = A \cdot \tfrac{1}{3} + B \cdot 3} \end{aligned}$$

$$A = 1 - B \Rightarrow$$

$$\frac{1}{3} + \varepsilon = (1-B)\frac{1}{3} + 3B \quad / \cdot 3$$

$$9B + \cancel{1} - B = \cancel{1} + 3\varepsilon$$

$$8B = 3\varepsilon$$

$$B = \frac{3}{8}\varepsilon$$

$$A = 1 - \frac{3}{8}\varepsilon = \frac{8 - 3\varepsilon}{8}$$

$$a_n = \frac{8 - 3\varepsilon}{8} 3^{-n} + \frac{3\varepsilon}{8} 3^{n}; \quad n = 0, 1, \ldots$$

Reševanje nelinearnih enač.

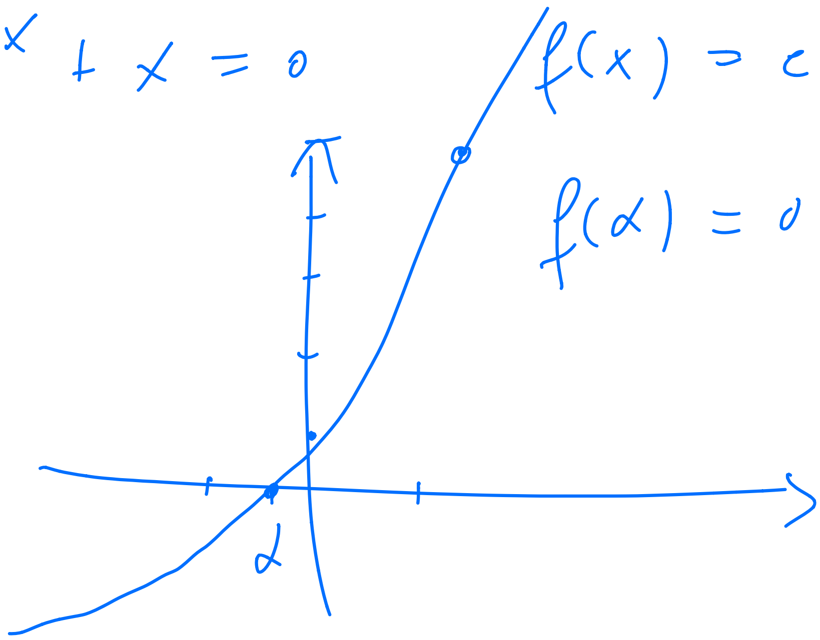
Primeri: ① $ax^2 + bx + c = 0$

nelinearni del

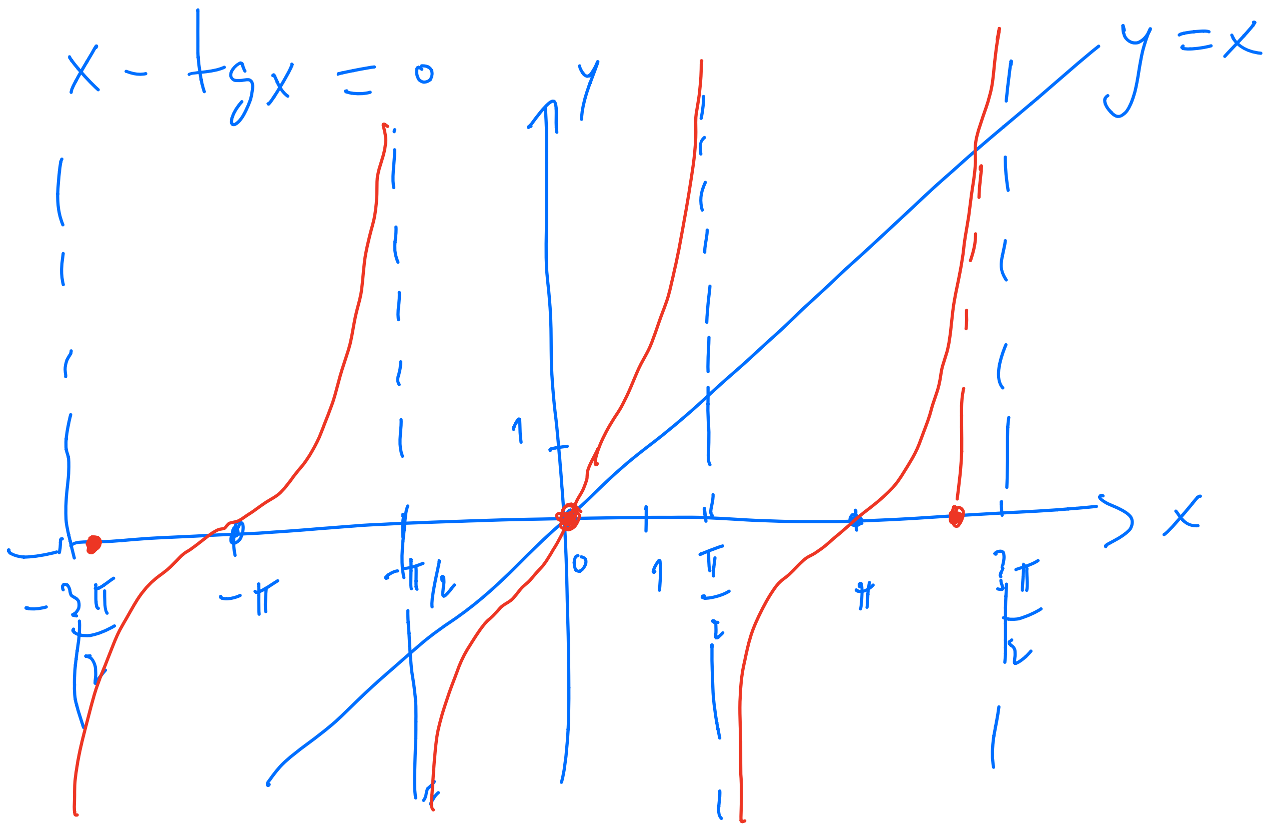
② $e^x + x = 0$

$f(x) = e^x + x$

$f(\alpha) = 0$



③ $x - \operatorname{tg} x = 0$



④ $p(x) = (x-1)(x-2)\ldots(x-2020)$

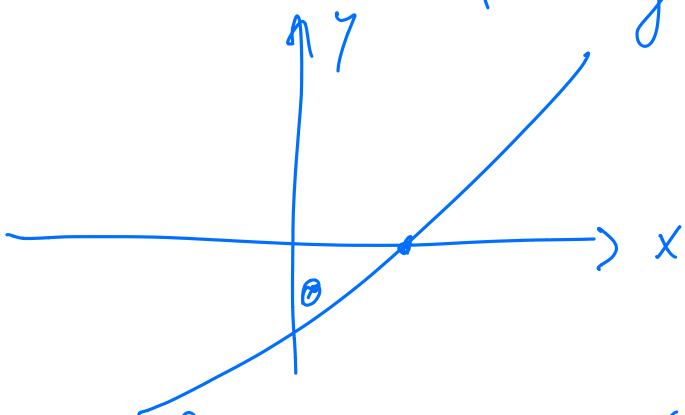
⑤ $x^2 + 1 = 0$ nima v $\mathbb{R}$, ima pa v $\mathbb{C}$.

Problem: $f: \mathbb{R} \to \mathbb{R}$ ali $f: D \subset \mathbb{R} \to \mathbb{R}$

Iščemo $\alpha \in D$: $f(\alpha) = 0$.

α je ničla funkcije f oz. α je rešitev enačbe $f(x) = 0$.

- α je enostavna, če je $f'(\alpha) \neq 0$



- α je večkratna, če je $f'(\alpha) = 0$:

m-kratna: $f'(\alpha) = f''(\alpha) = \dots = f^{(m-1)}(\alpha) = 0$

Občutljivost ničle:

Kako natančno lahko α izračunamo?

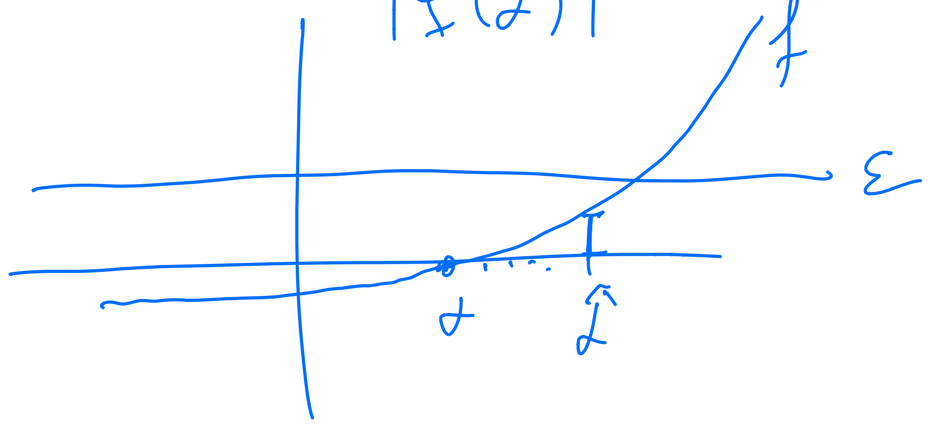
$\hat{\alpha}$ približek za ničlo α pri $f(\hat{\alpha}) = \varepsilon$, $|\varepsilon| \ll 1$.

$$f(\hat{\alpha}) = f(\alpha) + f'(\alpha)(\hat{\alpha}-\alpha) + \dots$$

$$\parallel$$

$$\varepsilon = 0 + f'(\alpha)(\hat{\alpha}-\alpha)$$

$$|\hat{\alpha}-\alpha| = \frac{\varepsilon}{|f'(\alpha)|} \quad ; \quad f'(\alpha) \neq 0 .$$



Če je α m-kratna ničla:

$$f(\hat{\alpha}) = \overset{=0}{f(\alpha)} + \overset{=0}{f'(\alpha)}(\hat{\alpha}-\alpha) + \frac{\overset{=0}{f''(\alpha)}}{2}(\hat{\alpha}-\alpha)^2$$

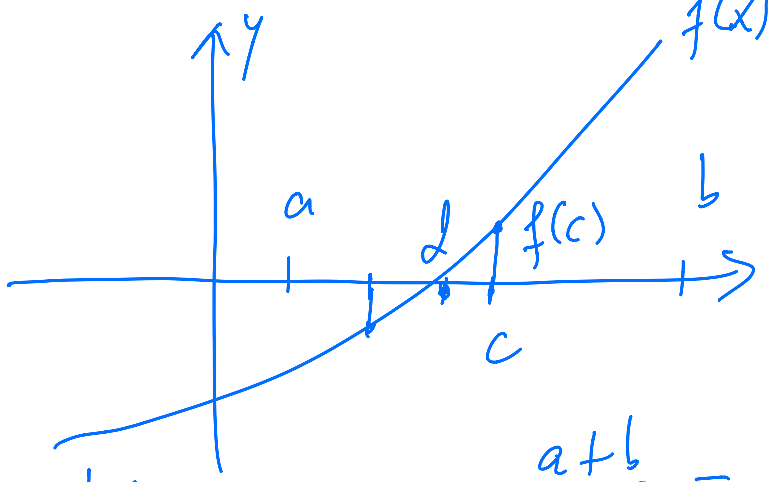
$$+ \dots + \frac{\overset{=0}{f^{(m-1)}(\alpha)}}{(m-1)!}(\hat{\alpha}-\alpha)^{m-1} + \frac{f^{(m)}(\alpha)}{m!}(\hat{\alpha}-\alpha)^{m}$$

$$+ \underbrace{f^{(m+1)}(\xi)\frac{(\hat{\alpha}-\alpha)^{m+1}}{(m+1)!}}_{\text{odrežemo}}$$

$$\varepsilon \doteq \frac{f^{(m)}(\alpha)}{m!}(\hat{\alpha}-\alpha)^{m} \Rightarrow$$

$$|\hat{\alpha}-\alpha| \doteq \sqrt[m]{\frac{m!}{f^{(m)}(\alpha)}} \cdot \sqrt[m]{\varepsilon}$$

BISEKCIJA

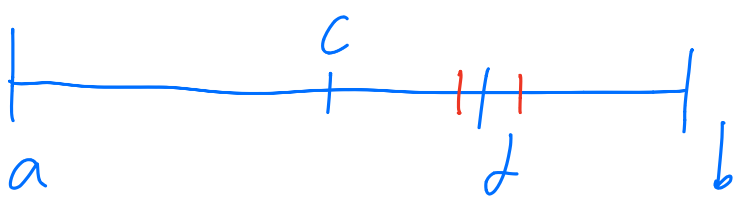


$f(a)\cdot f(b)<0$
ter f zvezna
$\Rightarrow$ f ima na (a,b) ničlo.

Ideja: $c = \frac{a+b}{2} = a + \frac{b-a}{2}$

Če je $f(c)\cdot f(a) < 0 \Rightarrow$ ničla je na (a,c), sicer je na (c,b)

Koliko korakov bisekcije potrebujemo, da ničlo izračunamo na ε natančno?



$$\frac{|b-a|}{2^k} < \varepsilon$$

$$k > \log_2 \frac{|b-a|}{\varepsilon}$$

Primer: $|b-a| = 1$, $\varepsilon = 10^{-10}$

$$k > \log_2 \frac{1}{10^{-10}} = 10 \cdot \log_2 10$$

$$\doteq 33.2$$