1. Naloga. Dani so vektorji $v_1=(1,0,1)$, $v_2=(0,2,2)$, $v_3=(1,1,0)$.

(a) Ali so vektorji lin. neodvisni? DA

(b) Določi prostornino paralelepipeda, napetega na v_1, v_2, v_3. 4

(c) Določi površino paralelograma, določenega z v_1, v_2. $2\sqrt{3}$

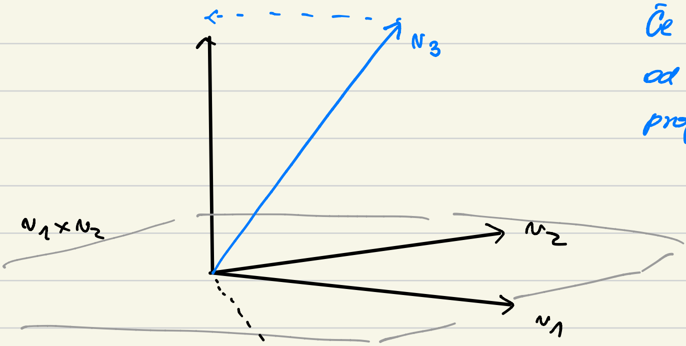
(d) Ali so v_1, v_2, v_3 poz. orientirani? Ne.

(e) Izrazi vektor $(2,3,2)$ kot lin. kombinacijo v_1, v_2, v_3

(f) Izračunaj kot med v_2 in v_3.

(a)
(b), (c), (d)

$$v_1 \times v_2 = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 2 & 2 \end{vmatrix} = (-2,-2,2)$$



Če naj bo v_3 lin. neodvisen od v_1 in v_2, mora biti projekcija v_3 na $v_1 \times v_2$ neničelna.

$$\underline{\langle v_1 \times v_2, v_3 \rangle} = \langle (-2,-2,2),(1,1,0) \rangle = -4 = (v_1, v_2, v_3).$$

Negativni skal. produkt pove, da je v_3 usmerjen „nasprotno" kot $v_1 \times v_2$: komponenta v_3 v smeri $v_1 \times v_2$ je negativna.

(f) kot med v_1 in v_2:

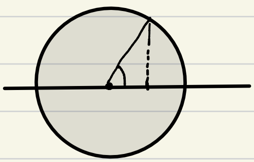
$$\langle v_2, v_3 \rangle = |v_2| \cdot |v_3| \cos\varphi$$

$$\langle (0,2,2), (1,1,0) \rangle = 2 \cdot \sqrt{2} \cdot \sqrt{2} \cdot \cos\varphi = 0 \cdot 1 + 2 \cdot 1 + 2 \cdot 0 = 2$$

$$4 \cdot \cos\varphi = 2 \Rightarrow \cos\varphi = 1/2 \Rightarrow \varphi =$$

$$\varphi = 60^\circ = \pi/3$$

$$(\sin\varphi = \sqrt{3}/2)$$



$$v_1 = (1,0,1), \quad v_2 = (0,2,2), \quad v_3 = (1,1,0) \; ; \; v = (2,3,2)$$

$$v = a_1 v_1 + a_2 v_2 + a_3 v_3$$

$$\begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} = a_1 \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + a_2 \cdot \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + a_3 \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} 2 &= a_1 \cdot 1 + a_2 \cdot 0 + a_3 \cdot 1 = 2 \quad \cdot(-1) \\ 3 &= a_1 \cdot 0 + a_2 \cdot 2 + a_3 \cdot 1 = 3 \\ 2 &= \underline{a_1 \cdot 1} + a_2 \cdot 2 + a_3 \cdot 0 = 2 \quad + \end{aligned}$$

$$\begin{aligned} a_1 \cdot 1 &+ a_2 \cdot 0 + a_3 \cdot 1 = 2 \\ a_1 \cdot 0 &+ a_2 \cdot 2 + a_3 \cdot 1 = 3 \quad \cdot(-1) \\ a_1 \cdot 0 &+ \underline{a_2 \cdot 2} + a_3 \cdot (-1) = 0 \quad + \end{aligned}$$

$$\begin{aligned} a_1\cdot 1 + a_2\cdot 0 + a_3\cdot 1 &= 2 \\ a_1\cdot 0 + a_2\cdot 2 + a_3\cdot 1 &= 3 \\ a_1\cdot 0 + a_2\cdot 0 + a_3\cdot(-2) &= -3 \quad \cdot(1/2) / \cdot(1/2) \end{aligned}$$

$+$ $+$ a_3 vstavimo v prejšnje enačbe

$$\begin{aligned} a_1\cdot 1 + a_2\cdot 0 + a_3\cdot 0 &= 2 - 3/2 = 1/2 \\ a_1\cdot 0 + a_2\cdot 2 + a_3\cdot 0 &= 3 - 3/2 = 3/2 \\ a_1\cdot 0 + a_2\cdot 0 + a_3\cdot 1 &= 3/2 \end{aligned}$$

$a_1 = 1/2$
$a_2 = 3/4$
$a_3 = 3/2$

Sistem zapišemo v matrični obliki

$$\begin{array}{c} (-1) \\ + \end{array} \begin{bmatrix} 1 & 0 & 1 & \vdots & 2 \\ 0 & 2 & 1 & \vdots & 3 \\ 1 & 2 & 0 & \vdots & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & \vdots & 2 \\ 0 & 2 & 1 & \vdots & 3 \\ 0 & 2 & -1 & \vdots & 0 \end{bmatrix} \begin{array}{c} (-1) \\ + \end{array} \sim$$

$$\begin{array}{ccccc} v_1 & v_2 & v_3 & & v \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & \vdots & 2 \\ 0 & 2 & 1 & \vdots & 3 \\ 0 & 0 & -2 & \vdots & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & \vdots & 2 \\ 0 & 2 & 1 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & 3/2 \end{bmatrix} \sim \begin{array}{cc} & + \\ + & \\ (-1) & (-1) \end{array}$$

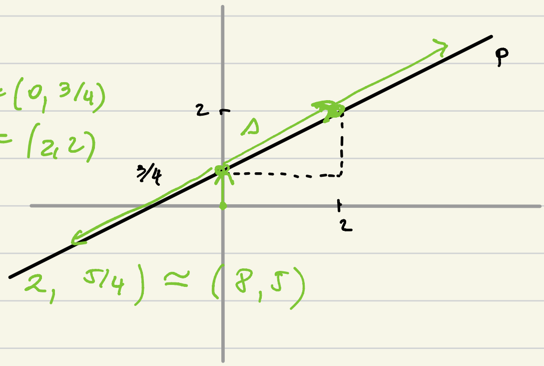
$$\sim \begin{bmatrix} 1 & 0 & 0 & \vdots & 1/2 \\ 0 & 2 & 0 & \vdots & 3/2 \\ 0 & 0 & 1 & \vdots & 3/2 \end{bmatrix} \sim \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & \vdots & 1/2 \\ 0 & 1 & 0 & \vdots & 3/4 \\ 0 & 0 & 1 & \vdots & 3/2 \end{bmatrix} \end{array}$$

Opisani postopek se imenuje Gaussova eliminacija.

Enačba premice v $\mathbb{R}^2$

$A = (0, 3/4)$

$B = (2, 2)$



$\vec{s} = (2, 5/4) \approx (8, 5)$

$y = kx + n$ (eksplicitna oblika)

$n = 3/4$

$k = \dfrac{5/4}{2} = 5/8$

$y = 5/8\, x + 3/4$

$(x, y) = (0, 3/4) + t \cdot (2, 5/4)$ ← vektorska oblika enačbe premice

$$\left.\begin{aligned} x &= 0 + t \cdot 2 \\ y &= 3/4 + t \cdot 5/4 \end{aligned}\right\}$$ parametrična oblika

$(0, 3/4)$: točka na premici

Eliminiram t: $\dfrac{x-0}{2} = t$

$\dfrac{y - 3/4}{5/4} = t$

oz. $\dfrac{x-0}{2} = \dfrac{y - 3/4}{5/4}$ kanonična oblika

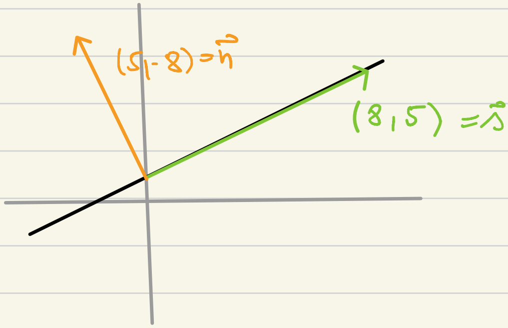
$(2, 5/4) = s$

$y = 3/4 + 5/8 \cdot x \quad / \cdot 8$

$\underbrace{5x - 8y}_{} + 6 = 0$ implicitna oblika

$\langle (x, y), (5, -8) \rangle = -6.$

Na premici p so tiste točke (x,y), ki imajo z normalo $\vec{n}$ konstanten skalarni produkt -6.



$(x_0,y_0) = (2,2) \in p$

$\langle (2,2), (5,-8) \rangle = 2\cdot 5 - 2\cdot 8 = -6$

$\langle (x,y), (5,-8) \rangle = -6.$

Napiši vse oblike za premico, ki gre skozi $A = (1,1)$ in je vzporedna vektorju $\vec{s} = (2,1)$.

$\hookrightarrow k = 1/2$ (y komp. / x komp.)

$$(x,y) = (1,1) + t\cdot(2,1)$$

$$x = 1+2t$$
$$y = 1+t$$

$$\frac{x-1}{2} = \frac{y-1}{1}$$

$$y = \frac{x}{2} - \frac{1}{2} + 1 = \frac{x}{2} + \frac{1}{2}$$

$$\langle (x,y), (1,-2) \rangle = \langle (1,1), (1,-2) \rangle = -1$$

dan

$\downarrow$

$$\langle (a,b), (\alpha,\beta) \rangle = 0$$

$$a\alpha + b\beta = 0$$

$$\alpha = b, \quad \beta = -a$$

Enačba premice v prostoru:

Premica je določena z točko A, ki leži na njej in smernim vektorjem s.

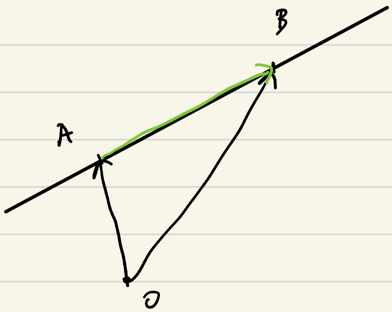
Zapiši vektorsko, parametrično in kanonično obliko enačbe premice, če je $A = (1,1,1)$ in $\vec{s} = (2,1,-1)$.

$$(x,y,z) = (1,1,1) + t\cdot(2,1,-1)$$

$x = 1+2t$
$y = 1+t$
$z = 1-t$

$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{-1}$$

Kako določimo smerni vektor, če imamo točki $A, B \in p$?



$\vec{s} = \vec{AB}$

Zapiši enačbo premice (v vektorski obliki) skozi točki $A = (1,0,1)$ in $B = (1,-1,0)$.

$$(x,y,z) = (1,0,1) + t \cdot \underbrace{(0,1,1)}_{= \vec{BA}}$$

$$\frac{x-1}{0} = \frac{y-0}{1} = \frac{z-1}{1}$$

$x = 1$
$y = t$
$z = 1+t$

DN: Iz spiska vektorji naredite 3. nalogo.