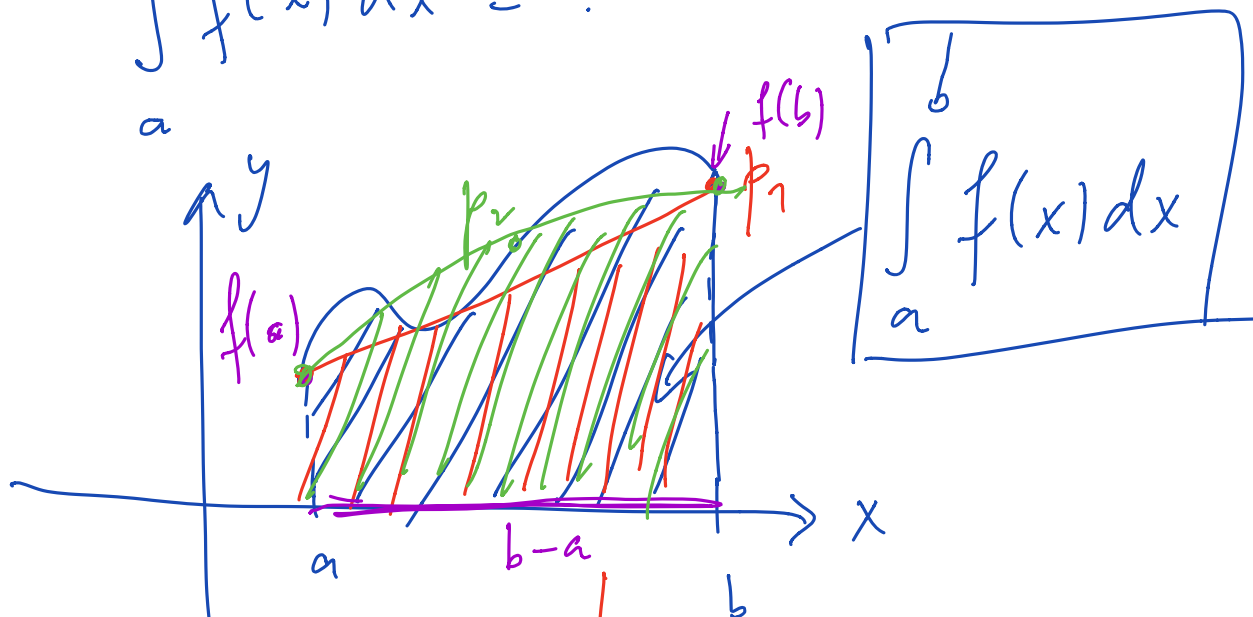


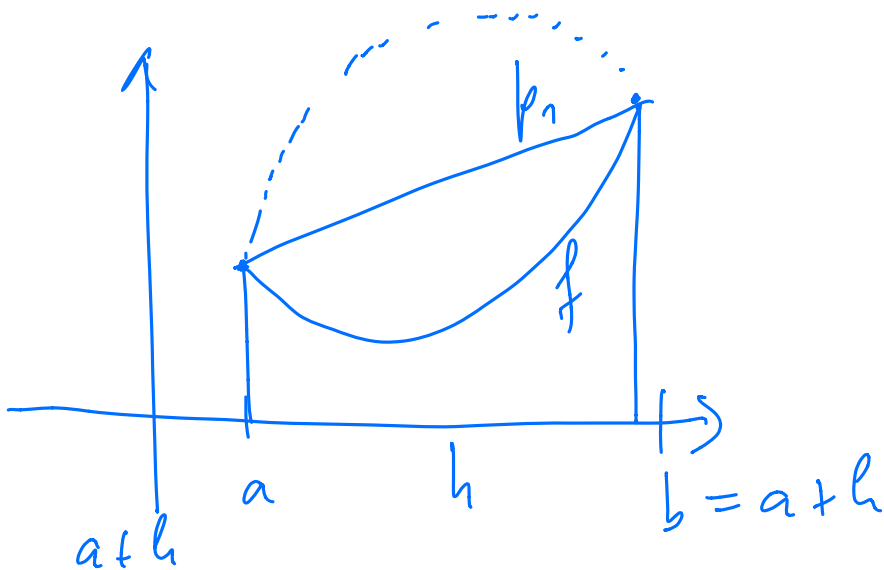
Numerical integration

$$\int_a^b f(x) dx = ?$$



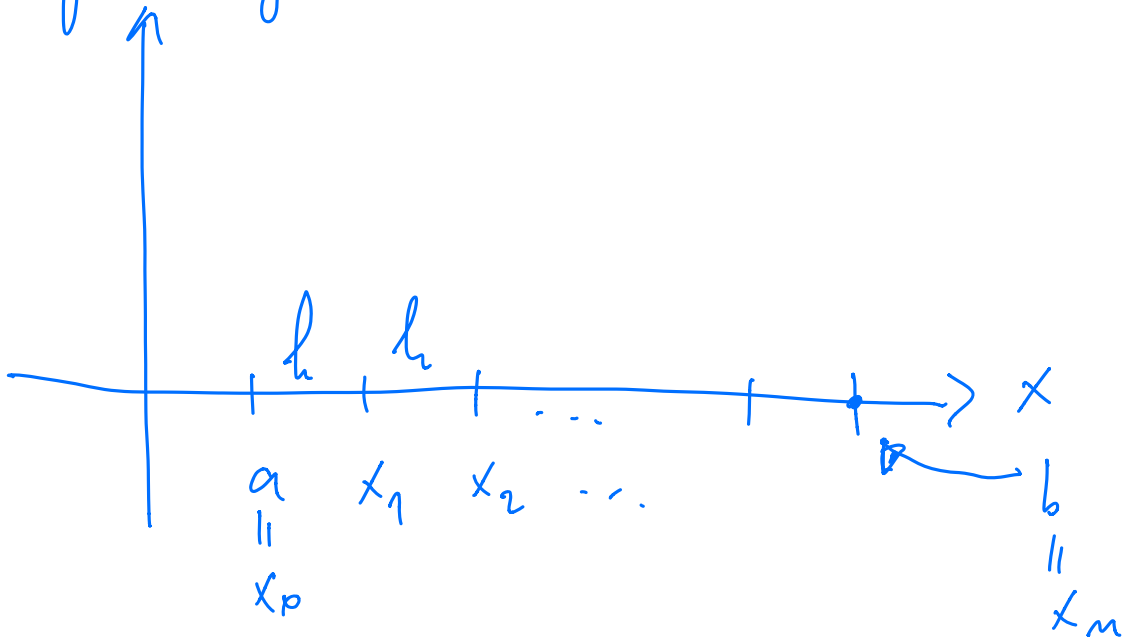
$$\int_a^b f(x) dx \approx \int_a^b p_1(x) dx = \dots =$$

$$\frac{b-a}{2} (f(a) + f(b)) : \text{Trapezium formula}$$



$$\int_a^{a+h} f(x) dx = \frac{h}{2} (f(a) + f(a+h)) - \frac{h^3}{12} f''(\xi); \quad a < \xi < a+h$$

Can we get $h \gg 1$?



$h_i := x_i - x_{i-1} =: h$ constant

$$\int_a^b f(x) dx \stackrel{?}{=} \int_{x_0}^{x_1} f(x) dx + \int_{x_1}^{x_2} f(x) dx + \dots +$$

$$\int_{x_{m-1}}^{x_m} f(x) dx = \sum_{i=1}^m \int_{x_{i-1}}^{x_i} f(x) dx =$$

$$\sum_{i=1}^m h (f(x_{i-1}) + f(x_i)) \stackrel{?}{=} \quad \swarrow f(x_i) = f_i$$

$$\sum_{i=1}^n \frac{h}{2} (f_{i-1} + f_i) = \frac{h}{2} (f_0 + f_1 + f_1 + f_2 + \dots + f_{n-1} + f_n)$$

$$f_{n-1} + f_n = h \left(\frac{1}{2} f_0 + f_1 + \dots + f_{n-1} + \frac{1}{2} f_n \right)$$

Trapezno pravilo

Napaka: $O(h^2)$

Numerično reševanje računalniškega problema
za eno navadno dif. enačbo

ODE : ordinary diff. equation

Primer:
$$\begin{cases} y'(x) = y(x) & (y' = y) \\ y(0) = 1 \end{cases}$$

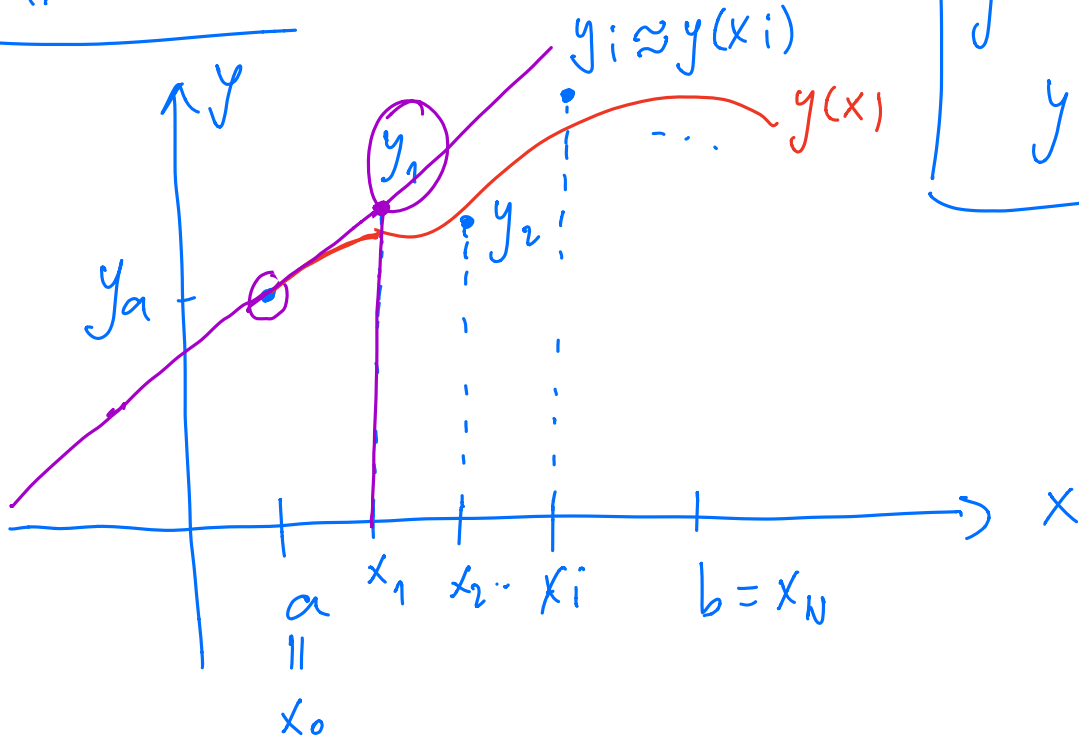
$$\frac{dy}{y} = dx \Rightarrow \log y = x + C / \exp$$

$$y = e^{x+C} = e^x \cdot e^C = D e^x$$

$$y(0) = D \cdot 1 = 1 \Rightarrow D = 1$$

Prüfer: $y(x) = e^x$

Numerično:



$$\begin{aligned} y' &= f(x, y) \\ y(a) &= y_a \end{aligned}$$

✓ x_0 potremmo: ① $y(x_0) = y(a) = y_a$

$$\textcircled{2} \quad y'(x_0) = f(x_0, y(x_0))$$

$$y_1 = y_0 + h f(x_0, y_0)$$

$$y_i = y_{i-1} + h f(x_{i-1}, y_{i-1}); \quad i = 1, 2, \dots, N$$

Enderejever metoda (eksplisitna)