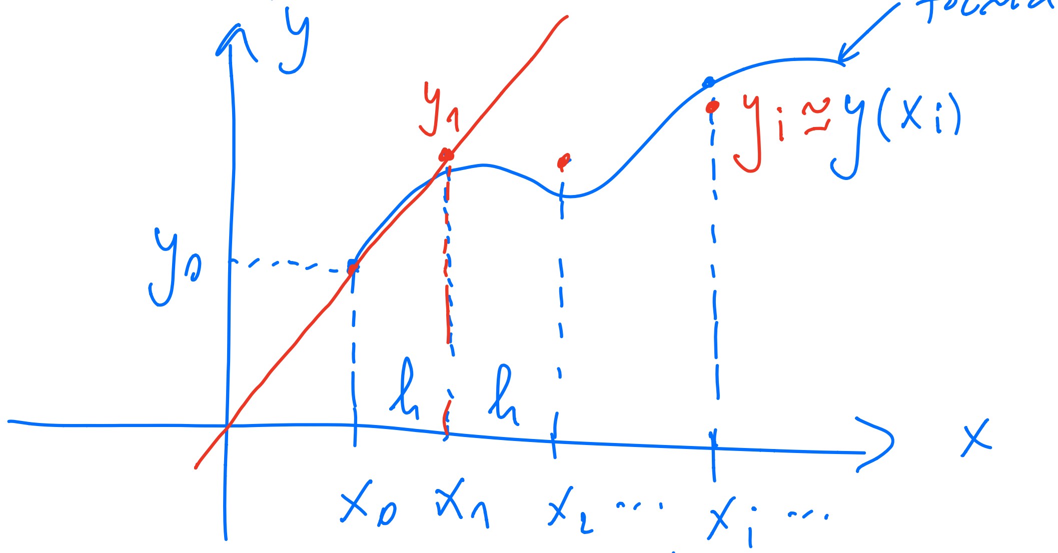
Reševanje NDE numerično:

$$y' = f(x,y), \quad y(x_0) = y_0$$



Eksplicitna Eulerjeva metoda:

$$y_{i+1} = y_i + h f(x_i, y_i), \quad i = 0, 1, \dots$$

Variante: implicitna Eulerjeva metoda

$$y_{i+1} = \underbrace{y_i + h f(x_{i+1}, y_{i+1})}, \quad i = 0, 1, \dots$$

$$y_{i+1} = g(y_{i+1})$$

$$|g'(y)| = h \cdot \left| \frac{\partial f}{\partial y}(x_{i+1}, y) \right| < 1,$$

če izberemo $h \ll 1$.

$$y_{i+1}^{(r+1)} = g\left(y_{i+1}^{(r)}\right)$$

Kako izberemo $y_{i+1}^{(0)}$?

Trapezna metoda:

$$y_{i+1} = y_i + \frac{h}{2}\left(f(x_i, y_i) + f(x_{i+1}, y_{i+1})\right)$$

Primer:

$$y' + 2xy - e^{-x^2} = 0, \quad y(0) = 0$$

Analitična rešitev: $y = x e^{-x^2}$

Rešujemo na $[0, 2]$.

Kako rešujemo sistem dif. enačb 1. reda?

$$y_1' = f_1(x, y_1, y_2, \dots, y_k)$$

$$y_2' = f_2(x, y_1, y_2, \dots, y_k)$$

$$\vdots$$

$$y_k' = f_k(x, y_1, y_2, \dots, y_k)$$

Začetni pogoj: $y_1(x_0) = y_{10}, y_2(x_0) = y_{20}, \dots$

$y_k(x_0) = y_{k0}$

Pišemo $Y = [y_1, y_2, \dots, y_k]^T$ ter

$$F(x, Y) = [f_1(x, Y), \dots, f_k(x, Y)]^T$$

ter uporabimo metodo za eno enačbo.

$$Y^{(r+1)} = Y^{(r)} + h \cdot F(x_r, Y^{(r)})$$

Reševanje dif. enačb višjega reda

$$y^{(k)} = f(x, y(x), y'(x), \dots, y^{(k-1)}(x))$$

$$y(x_0) = y_0,\ y'(x_0) = y'_0, \dots, y^{(k-1)}(x_0) = y^{(k-1)}_0$$

Uvedemo nove funkcije: $y_1 = y, y_2 = y', \dots, y_k = y^{(k-1)}$

Zapišemo sistem 1. reda:

$$y_1' = y_2 = f_1(x, y_1, y_2, \dots, y_k)$$

$$y_2' = y_3 = f_2(x, y_1, y_2, \dots, y_k)$$

$$\vdots$$

$$y_{k-1}' = y_k = f_{k-1}(x, y_1, \dots, y_k)$$

$$y_k' = f(x, y_1, y_2, \dots, y_k) = f_k(\dots)$$

<u>Primer</u>: $y'' = -y$, $y(0) = 0$, $y'(0) = 1$

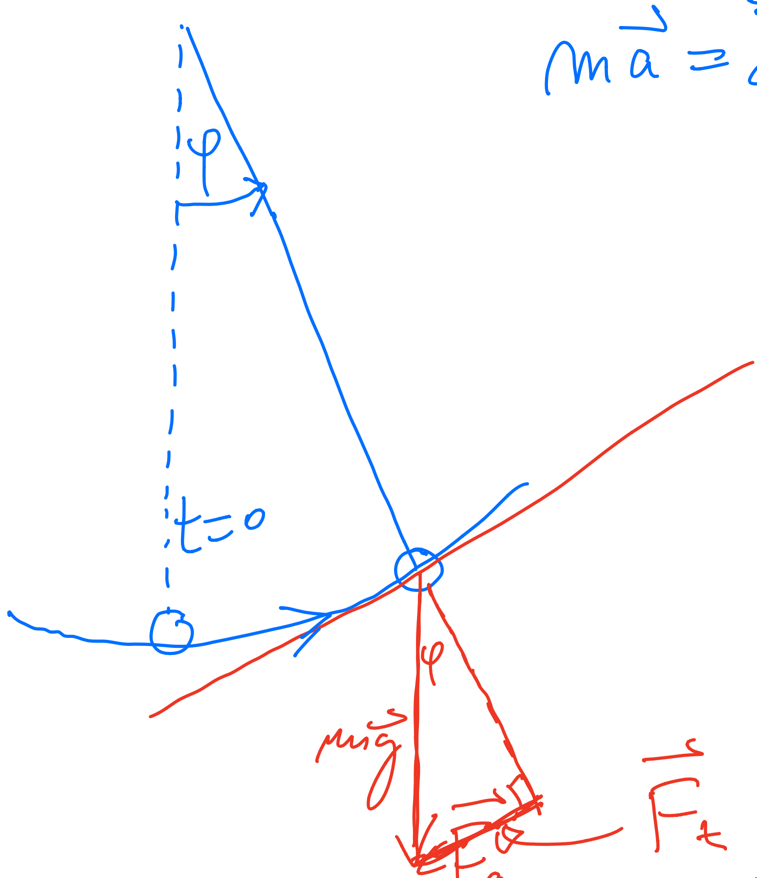
$\Rightarrow y(x) = \sin x$

$y_1 = y$, $y_2 = y'$ $\Rightarrow$

$$\boxed{\begin{aligned} y_1' &= y_2 \\ y_2' &= -y_1 \end{aligned}} \qquad \boxed{\begin{aligned} y_1(0) &= 0 \\ y_2(0) &= 1 \end{aligned}}$$

<u>Matematično nihalo</u>:

$$m\vec{a} = \sum_i \vec{F}_i$$



$$\frac{F_t}{F_g} = \sin\varphi$$

$$F_t = F_g \sin\varphi$$

g $\qquad F_t = mg\sin\varphi$

$$m a_t = \cancel{m} l \ddot{\varphi} = -\cancel{m} g \sin\varphi$$

$$\boxed{\ddot{\varphi} = -\frac{g}{l}\sin\varphi}$$

$\varphi(0) = 0$

$\dot{\varphi}(0) = \dot{\varphi}_0 = \omega_0$

$$f(t, \varphi(t)) = -\frac{g}{l}\sin(\varphi(t))$$

Primer: $g = 9.8\, m/s^2$

$l = 1m$

$\omega_0 = 1 s^{-1}$

Sistem: $\varphi_1 = \varphi$, $\varphi_2 = \dot{\varphi}$

$$\boxed{\begin{aligned} \dot{\varphi}_1 &= \varphi_2 \\ \dot{\varphi}_2 &= -\frac{g}{l}\sin\varphi_1 \end{aligned}}$$

Če je kot majhen $\varphi \ll 1$ $(\varphi \approx 0)$:

$$\ddot{\varphi} = -\frac{g}{l}\sin\varphi \overset{\approx}{\Rightarrow} \ddot{\varphi} = -\frac{g}{l}\varphi$$

Ker $\sin\varphi \approx \varphi$.

$$\boxed{\ddot{\varphi} = -\frac{g}{\ell}\varphi}$$

$$\ddot{\varphi} + \frac{g}{\ell}\varphi = 0$$

Kar. polinom: $\lambda^2 + \frac{g}{\ell} = 0$

$$\lambda_{1,2} = \pm i\sqrt{\frac{g}{\ell}}$$

$$\Rightarrow \varphi(t) = A\sin\sqrt{\frac{g}{\ell}}\,t + B\cos\sqrt{\frac{g}{\ell}}\,t$$

$$\varphi(0) = 0 \Rightarrow B = 0$$

$$\dot{\varphi}(0) = \omega_0 = A\cos\left(\sqrt{\frac{g}{\ell}}\cdot 0\right)\cdot\sqrt{\frac{g}{\ell}}$$

$$\Rightarrow A = \frac{\omega_0}{\sqrt{\frac{g}{\ell}}}$$

$$\Rightarrow \boxed{\varphi(t) = A\cdot\sin\sqrt{\frac{g}{\ell}}\,t}$$