

LINEARNA ALGEBRA 2020/21  
5. VAJE: 2.11.2020

1. Reši sistem.

$$\begin{array}{ccccrc} & -x_2 & -x_3 & -x_4 & = & 2 \\ x_1 & +x_2 & +x_3 & +x_4 & = & 7 \\ 2x_1 & +4x_2 & +x_3 & -2x_4 & = & 1 \\ 3x_1 & +x_2 & -2x_3 & +2x_4 & = & 9 \end{array}$$

2. Prostor rešitev enačb zapiši parametrično.

a)  $\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & -1 & 0 \end{array} \right]$

b)  $\left[ \begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 6 & 2 & -1 & -2 & 2 \\ 3 & 1 & 2 & -1 & 1 \end{array} \right]$

3. Pokaži:

- a) Linearna kombinacija rešitev homogenega sistema enačb je rešitev istega sistema enačb.
- b) Razlika poljubnih dveh rešitev nehomogenega sistema enačb je rešitev homogenega dela sistema enačb.

4. Obravnavaj sistem.

$$\left[ \begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array} \right]$$

Za katere vrednosti  $a, b$  ima sistem eno/neskončno rešitev? Za katere vrednosti sistem nima rešitve.

5. Hkrati reši sistema  $2x_1 + 5x_2 = 1$ ,  $x_1 + 3x_2 = 0$  in  $2y_1 + 5y_2 = 0$ ,  $y_1 + 3y_2 = 1$ .

$$\left[ \begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array} \right].$$

6. Reši sistem enačb:

$$\left[ \begin{array}{ccc|c} 2 & 3 & -2 & -1 \\ 4 & -1 & 0 & 13 \\ 7 & 0 & -2 & 17 \\ 1 & 3 & -2 & -4 \end{array} \right]$$

7. Določi za katere vrednosti  $a$  je sistem rešljiv in ga za dobljene vrednosti reši.

$$\left[ \begin{array}{cccc|c} 2 & 0 & 2 & 0 & a+1 \\ -1 & 2 & 3 & 1 & -2a \\ 1 & -1 & -1 & 1 & 0 \\ 1 & 1 & 3 & 1 & a-1 \end{array} \right]$$

Izrazi še rešitev homogenega sistema.

8. Določi posplošeno rešitev predoločenega sistema enačb

$$\left[ \begin{array}{cc|c} 1 & 0 & 3 \\ -2 & 3 & 1 \\ 2 & -1 & 2 \end{array} \right]$$

9. Parametrično izrazi rešitev sistema enačb.

$$\left[ \begin{array}{ccccc|c} 1 & -1 & 0 & 1 & 2 & 0 \\ -2 & 2 & -1 & -4 & -3 & 0 \\ 1 & -1 & 1 & 3 & 1 & 0 \\ -1 & 1 & 1 & 1 & -3 & 0 \end{array} \right],$$

$$\left[ \begin{array}{cccccc|c} 1 & -1 & 1 & 3 & 3 & 4 & 2 \\ -2 & 2 & -1 & -3 & -4 & -5 & -1 \\ -1 & 1 & 0 & 0 & -1 & -4 & -5 \\ -1 & 1 & 1 & 3 & 1 & -1 & -2 \end{array} \right].$$

6. Točkam  $(1,0), (2,3), (-1,-2)$  priredi premico  $y = ax + b$ , ki se jim po metodi najmanjših kvadratov najbolj prilega. Najdi tudi parabolo  $y = ax^2 + bx + c$ , ki se točkam najbolj prilega (Izrazi le sistem s katerim dobiš koeficiente  $a, b$  in  $c$ .)

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad Y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

Iščemo  $a, b \in \mathbb{R}$ , da bo  $\sum_{i=1}^n (ax_i + b - y_i)^2$  najmanjša

$$X = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad Y = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$$

$$\|ax + b\bar{1} - \bar{y}\|^2$$

Iščemo projekcijo  $\bar{y}$  na ravnino, ki jo razpenjata  $X$  in  $\bar{1}$

$a$  in  $b$  določimo z enačbama

$$\langle ax + b\bar{1} - \bar{y}, X \rangle = 0$$

$$\langle ax + b\bar{1} - \bar{y}, \bar{1} \rangle = 0$$

Druge možnost: Posplošeno rešitev sistema enačb

če je  $z = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix} \quad w = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} \quad \langle z, w \rangle = w^T z = z^T w$

$$A = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{pmatrix}$$

Iščemo posplošeno rešitev sistema

$$A \begin{pmatrix} a \\ b \end{pmatrix} = \bar{y}$$

$$A^T A = \begin{pmatrix} \langle X, X \rangle & \langle X, \bar{1} \rangle \\ \langle X, \bar{1} \rangle & \langle \bar{1}, \bar{1} \rangle \end{pmatrix}$$

$$A^T A \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \langle X, X \rangle + b \langle X, \bar{1} \rangle \\ a \langle X, \bar{1} \rangle + b \langle \bar{1}, \bar{1} \rangle \end{pmatrix}$$

$$A^T \bar{y} = \begin{pmatrix} \langle X, \bar{y} \rangle \\ \langle \bar{1}, \bar{y} \rangle \end{pmatrix}$$

$$\begin{pmatrix} a \langle X, X \rangle + b \langle X, \bar{1} \rangle \\ a \langle X, \bar{1} \rangle + b \langle \bar{1}, \bar{1} \rangle \end{pmatrix} = \begin{pmatrix} \langle X, \bar{y} \rangle \\ \langle \bar{1}, \bar{y} \rangle \end{pmatrix}$$

2

Primerjaj z  $\star$

①

$$\begin{aligned} -x_2 - x_3 - x_4 &= 2 \\ x_1 + x_2 + x_3 + x_4 &= 7 \\ 2x_1 + 4x_2 + x_3 - 2x_4 &= 1 \\ 3x_1 + x_2 - 2x_3 + 2x_4 &= 9 \end{aligned}$$

① Zaposredno reševanje spremenljivk

$$x_2 = -2 + x_3 - x_4$$

$$x_1 + -2 - x_3 - x_4 + x_3 + x_4 = 7$$

$x_1 \dots$

② Gaussova eliminacija

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 7 \\ 0 + x_2 - x_3 - x_4 &= 2 \\ 2x_1 + 4x_2 + x_3 - 2x_4 &= 1 \\ 3x_1 + x_2 - 2x_3 + 2x_4 &= 9 \end{aligned}$$

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 7 \\ 0 + x_2 - x_3 - x_4 &= 2 \\ 0 \quad 2x_2 - x_3 - 4x_4 &= -13 \\ 0 \quad -2x_2 - 5x_3 - 1x_4 &= -12 \end{aligned}$$

$$\left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 1 & -2 \\ 0 & 2 & -1 & -4 & -13 \\ 0 & -2 & -5 & -1 & -12 \end{array} \right) \xrightarrow{\substack{-2 \\ +2}} \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 1 & -2 \\ 0 & 0 & -3 & -6 & -9 \\ 0 & 0 & -3 & -1 & -16 \end{array} \right) \xrightarrow{\substack{-3 \\ +3}} \left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right)$$

$$\left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right) \xrightarrow{\substack{-1 \\ -1 \\ -2}} \left( \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right)$$

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 7 \\ x_2 = 6 \quad x_2 + x_3 + x_4 &= -2 \\ x_3 = 5 \quad x_3 + 2x_4 &= 3 \\ x_4 &= -1 \end{aligned}$$

$$\left( \begin{array}{cccc|c} 1 & 1 & 1 & 0 & 8 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right) \xrightarrow{\substack{-1 \\ -1}} \left( \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right)$$

$$\left( \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right) \xrightarrow{-3} \left( \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right)$$

$$\left( \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -7 \end{array} \right)$$

$$\begin{aligned} x_1 &= 9 \\ x_2 &= -6 \\ x_3 &= 5 \\ x_4 &= -1 \end{aligned} \quad X = \begin{pmatrix} 9 \\ -6 \\ 5 \\ -1 \end{pmatrix}$$

②

a)  $\begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 1 & -1 & -1 & | & 0 \end{bmatrix}$  → Homogen (množica rešitev gre skozi 0)  
 b)  $\begin{bmatrix} 1 & 3 & 1 & 1 & | & 3 \\ 6 & 2 & -1 & -2 & | & 2 \\ 3 & 1 & 2 & -1 & | & 1 \end{bmatrix}$  → Nehomogen (ne gre skozi 0)

$$\begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 1 & -1 & -1 & | & 0 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 0 & -2 & -2 & | & 0 \end{bmatrix} \xrightarrow{/2} \begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 0 & -1 & -1 & | & 0 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & -1 & -1 & | & 0 \end{bmatrix}$$

$x_1 = 0$   
 $x_2 + x_3 = 0$   
 $x_3 = t$   
 $x_2 = -x_3 = -t$

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -t \\ t \end{pmatrix} = t \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$  Množica rešitev je premica  
 ↳ parameter

$$\begin{bmatrix} 1 & 3 & 1 & 1 & | & 3 \\ 6 & 2 & -1 & -2 & | & 2 \\ 3 & 1 & 2 & -1 & | & 1 \end{bmatrix} \xrightarrow{-2} \begin{bmatrix} 1 & 3 & 1 & 1 & | & 3 \\ 0 & 0 & -5 & 0 & | & 0 \\ 3 & 1 & 2 & -1 & | & 1 \end{bmatrix} \xrightarrow{-3} \begin{bmatrix} 1 & 3 & 1 & 1 & | & 3 \\ 0 & 0 & -5 & 0 & | & 0 \\ 0 & 0 & -1 & -4 & | & -8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 1 & 1 & | & 3 \\ 0 & -8 & -1 & -4 & | & -8 \\ 0 & 0 & -1 & -4 & | & -8 \end{bmatrix} \xrightarrow{+1} \begin{bmatrix} 1 & 3 & 0 & 1 & | & 3 \\ -8 & 0 & -4 & -8 & | & -8 \\ 1 & 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{+4} \begin{bmatrix} 1 & 3 & 0 & 1 & | & 3 \\ 0 & 2 & 0 & 1 & | & 2 \\ 1 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

→ Gaussova reducirana oblika

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & | & 0 \\ 0 & 2 & 0 & 1 & | & 2 \\ 0 & 0 & 1 & 0 & | & 0 \end{bmatrix}$$

$x_4$  - vzamemo za parameter

$x_1 - \frac{1}{2}x_4 = 0 \quad x_4 = t$   
 $2x_2 + x_4 = 2 \quad x_2 = -\frac{1}{2}x_4 + 1$   
 $x_3 = 0$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t \\ -\frac{1}{2}t + 1 \\ 0 \\ t \end{pmatrix}$$

Homogen del je rešitev →  $\begin{bmatrix} 1 & 3 & 1 & 1 & | & 0 \\ 6 & 2 & -1 & -2 & | & 0 \\ 3 & 1 & 2 & -1 & | & 0 \end{bmatrix}$

$X = t \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$   
 homogeni del      partikularni del

3. Pokaži:

- a) Linearna kombinacija rešitev homogenega sistema enačb je rešitev istega sistema enačb.  
 b) Razlika poljubnih dveh rešitev nehomogenega sistema enačb je rešitev homogenega dela sistema enačb.

a)  $(A|0)$   $x_1, x_2$  <sup>rešitvi</sup>  $\alpha x_1 + \beta x_2$  tudi reši sistem

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad x_1 = \begin{pmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1n} \end{pmatrix} \quad x_2 = \begin{pmatrix} x_{21} \\ x_{22} \\ \vdots \\ x_{2n} \end{pmatrix}$$

$$1. \quad a_{11}x_{11} + a_{12}x_{12} + \dots + a_{1n}x_{1n} = 0 \quad a_{11}x_{21} + a_{12}x_{22} + \dots + a_{1n}x_{2n} = 0$$

$$a_{11}(x_{11} + x_{21}) + a_{12}(x_{12} + x_{22}) + \dots + a_{1n}(x_{1n} + x_{2n}) = 0$$

$$a_{11}(x_{11} + x_{21}) + a_{12}(x_{12} + x_{22}) + \dots + a_{1n}(x_{1n} + x_{2n}) = 0$$

Enako velja v vseh vrsticah.

$$x_1 + x_2 = \begin{pmatrix} x_{11} + x_{21} \\ x_{12} + x_{22} \\ \vdots \\ x_{1n} + x_{2n} \end{pmatrix} \text{ je rešitev}$$

$(A 0)$	$AX=0$	$A(\alpha x_1 + \beta x_2)$
$\alpha x_1 = 0$	$\alpha AX_1 = 0$	$\alpha A(\alpha x_1 + \beta x_2)$
$\alpha x_2 = 0$	$\beta AX_2 = 0$	$\beta A(\alpha x_1 + \beta x_2) = 0 + 0$

$$\alpha (a_{11}x_{11} + a_{12}x_{12} + \dots + a_{1n}x_{1n}) = 0$$

$$\alpha a_{11}(\alpha x_{11}) + \alpha a_{12}(\alpha x_{12}) + \dots + \alpha a_{1n}(\alpha x_{1n}) = 0$$

$$\alpha x_1 = \begin{pmatrix} \alpha x_{11} \\ \vdots \\ \alpha x_{1n} \end{pmatrix} \text{ je rešitev}$$

b)  $AX=b$   $x_1, x_2$   $AX_1=b$   $AX_2=b$

$AX=0$  - homogeni del  $x_1 - x_2$  je rešitev hom. dela

$$A(x_1 - x_2) = \underline{Ax_1} - \underline{Ax_2} = b - b = 0$$

$$AX=b \quad \underline{Ax_1=b} \quad Ab=0, \text{ potem je } \underline{A(x_1+b)}=b$$

4. Obravnavaj sistem.

$$\begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 1 & 2 & 4 & | & 3 \\ 1 & 3 & a & | & b \end{bmatrix}$$

Za katere vrednosti  $a, b$  ima sistem eno/neskončno rešitev? Za katere vrednosti sistem nima rešitve.

$$\begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 1 & 2 & 4 & | & 3 \\ 1 & 3 & a & | & b \end{bmatrix} \xrightarrow{\substack{-R_1 \\ -R_1}} \begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 2 & a-3 & | & b-2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & a-5 & | & b-4 \end{bmatrix}$$

①  
 $a=5$

$$\begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & | & b-4 \end{bmatrix}$$

②  
 $a \neq 5$

$$\begin{bmatrix} 1 & 0 & 2 & | & 1 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 1 & | & \frac{b-4}{a-5} \end{bmatrix}$$

1.1

$b=4$

$$\begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

1.2

$b \neq 4$

$$\begin{bmatrix} 1 & 1 & 3 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & | & b-4 \end{bmatrix}$$

$0 = b-4 \quad \times$

ni rešitve ( $a=5 \quad b \neq 4$ )

$$\begin{bmatrix} 1 & 0 & 2 & | & 1 \\ 0 & 1 & 1 & | & 1 \end{bmatrix}$$

$$\begin{aligned} x_3 &= t \\ x_1 + 2x_3 &= 1 & x_1 &= 1 - 2t \\ x_2 + x_3 &= 1 & x_2 &= 1 - t \end{aligned}$$

$$x = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} t + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

( $a=5 \quad b=4$ )

neskončno rešitev

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 - 2\frac{b-4}{a-5} \\ 0 & 1 & 0 & | & 1 - \frac{b-4}{a-5} \\ 0 & 0 & 1 & | & \frac{b-4}{a-5} \end{bmatrix}$$

$$x = \begin{pmatrix} 1 - 2\frac{b-4}{a-5} \\ 1 - \frac{b-4}{a-5} \\ \frac{b-4}{a-5} \end{pmatrix}$$

imamo enolično rešitev

$a \neq 5 \quad b \text{ poljubno}$

5. Hkrati reši sistema  $2x_1 + 5x_2 = 1$ ,  $x_1 + 3x_2 = 0$  in  $2y_1 + 5y_2 = 0$ ,  $y_1 + 3y_2 = 1$ .

$$\left[ \begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} 2x_1 + 5x_2 = 1, \\ x_1 + 3x_2 = 0 \end{array} \quad \begin{array}{l} 2y_1 + 5y_2 = 0, \\ y_1 + 3y_2 = 1. \end{array}$$

$$\left[ \begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array} \right] \rightsquigarrow \left[ \begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 2 & 5 & 1 & 0 \end{array} \right] \rightsquigarrow \left[ \begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & -1 & 1 & -2 \end{array} \right] \rightsquigarrow \left[ \begin{array}{cc|cc} 1 & 3 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array} \right]$$

$$\rightsquigarrow \left[ \begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array} \right] \quad x = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad y = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} \text{ je inverz } \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

6. Reši sistem enačb:

$$\left[ \begin{array}{ccc|c} 2 & 3 & -2 & -1 \\ 4 & -1 & 0 & 13 \\ 7 & 0 & -2 & 17 \\ 1 & 3 & -2 & -4 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & -2 & -4 \\ 2 & 3 & -2 & -1 \\ 4 & -1 & 0 & 13 \\ 7 & 0 & -2 & 17 \end{array} \right] \xrightarrow{\substack{-2 \\ -4 \\ -7}} \left[ \begin{array}{ccc|c} 1 & 3 & -2 & -4 \\ 0 & -3 & 2 & 7 \\ 0 & -13 & 8 & 29 \\ 0 & -21 & 12 & 45 \end{array} \right] \xrightarrow{-7} \left[ \begin{array}{ccc|c} 1 & 3 & -2 & -4 \\ 0 & 3 & -2 & -7 \\ 0 & -13 & 8 & 29 \\ 0 & 0 & -2 & -4 \end{array} \right] \cdot 2$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & -2 & -4 \\ 0 & 3 & -2 & -7 \\ 0 & -13 & 8 & 29 \\ 0 & 0 & -2 & -4 \end{array} \right] \xrightarrow{\substack{-1 \\ -2 \\ -8}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 3 & 0 & -3 \\ 0 & -13 & 0 & 13 \\ 0 & 0 & -2 & 2 \end{array} \right] \xrightarrow{\substack{/3 \\ /13}} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & 2 \end{array} \right] \cdot (-1)$$

$$\rightsquigarrow \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

Če bi iskali posplošeno rešitev, bi dobili isto.  
(Ne bi vedeli, da je tudi degenerna rešitev.)