DZ: 4(1): navedeni slučaj

4(2):
$$\begin{aligned} x+y+z-3&=0\\ x+2y+3z-6&=0\\ 2x-y+z&=0\end{aligned}$$

$$\begin{array}{r}(-2)\ (-1)\\ +\\ +\end{array}\left[\begin{array}{ccc|c}1&1&1&3\\1&2&3&6\\2&-1&1&0\end{array}\right]\sim\left[\begin{array}{ccc|c}1&1&1&3\\0&1&2&3\\0&-3&-1&-6\end{array}\right]\sim$$

$$\begin{array}{r}\\(3)\\+\end{array}\left[\begin{array}{ccc|c}1&1&1&3\\0&1&2&3\\0&0&5&3\end{array}\right]\sim\left[\begin{array}{ccc|c}1&0&-1&0\\0&1&0&9/5\\0&0&1&3/5\end{array}\right]\begin{array}{l}+\\ \uparrow +\\ (-2)\end{array}$$

$$-2\cdot\frac{3}{5}+\frac{15}{5}=\frac{9}{5}$$

$$\left[\begin{array}{ccc|c}1&0&0&3/5\\0&1&0&9/5\\0&0&1&3/5\end{array}\right]\qquad x=3/5,\ y=9/5,\ z=3/5$$

5(1)
$$\underbrace{\begin{aligned}x+2y-z&=1\\-2x-4y+2z&=1\end{aligned}}\quad /\cdot(-2)\ \leadsto\ \underbrace{-2x-4y+2z=-2}$$

$$\begin{array}{r}(-2)\\+\end{array}\left[\begin{array}{ccc|c}1&2&-2&1\\-2&-4&2&1\end{array}\right]\sim\left[\begin{array}{ccc|c}1&2&-2&1\\0&0&0&3\end{array}\right]$$

$0=3$! proti sl. sistem

5(2) $x+2y-2z=1$

$2x-y+z=2$

$y-z=0$

ta re → komb. 1. in 3. en.

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 2 & -1 & 1 & 2 \\ 0 & 1 & -1 & 0 \end{array}\right] \sim \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & -1 & 0 \end{array}\right] \end{array} \begin{array}{l} (-1) \\ \swarrow + \\ \end{array}$$

↓ igra vlogo parametra t

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\left.\begin{array}{l} x=1 \\ y=z \end{array}\right\}$ premica

ena enačba odveč

$x=1$

$y-t=0$

$z=t$

$(1, t, t)$! param. premica

6(1) $\frac{x-1}{2} = \frac{y+3}{5} = z-1 = t$; $2x+3y+z-14=0$

$x-1=2t$ $x=2t+1$ ↗ $2(2t+1)+3(5t-3)+t+1-14=0$

$y+3=5t$ $y=5t-3$ $20t-20=0$

$z=t+1$ $t=1$

$\Rightarrow x=3, y=2, z=2$

(x, y, z, t):

$$\begin{array}{c} (-2) \\ (-3) \\ (-1) \\ + \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & -5 & -3 \\ 0 & 0 & 1 & -1 & 1 \\ 2 & 3 & 1 & 0 & 14 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & -5 & -3 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 20 & 20 \end{array}\right]$$

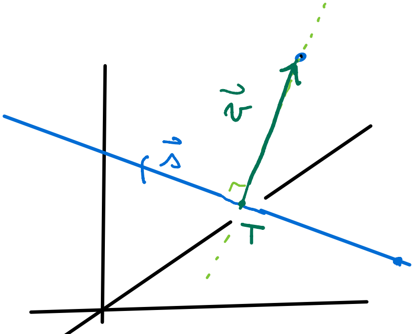
6 (2) $x+2 = \frac{y-3}{2} = z-1 = t \qquad x-y+z=5$

$x = t-2\,,\; y = 2t+3,\; z = t+1 \nearrow$

$t-2-2t-3+t+1 = 5$

$-4 = 5 \quad /\!/ \quad$ ni preseka

7 Izračunaj razdaljo točk A(0,0,0), B(7,2,0) in C(5,3,7) do $\frac{x-1}{4} = \frac{y}{3} = \frac{z-2}{5} : p$



$d(A,p)$

$\vec{s} = (4,3,5)$

(a)

$\underbrace{(1,0,2)+\lambda\cdot\vec{s}}_{\vec{OT}} + \vec{v} = (0,0,0)\,; \quad \vec{v}\perp\vec{s} \qquad /\cdot\vec{s}$

$(1,0,2)\cdot\vec{s} + \lambda\cdot|\vec{s}|^2 + \underbrace{\langle\vec{s},\vec{v}\rangle}_{\overset{\shortparallel}{0}} = 0$

$4+0+10+\lambda\cdot 50 = 0 \quad \rightarrow \lambda = -\frac{14}{50}$

$\vec{v} = -(1,0,2) - \lambda\cdot\vec{s} = -(1,0,2) + \frac{14}{50}(4,3,5) =$

$= \frac{1}{50}(-50+56,\; 42,\; -100+70) =$

$= \frac{1}{50}(-6,\; 42,\; -30) = \frac{6}{50}(-1,7,-5)$

$\rightsquigarrow$ velikost v

(b) Σ: vsebuje $(0,0,0)$ in p: → iskanje prav. proj.

$\vec{s}$, $(1,0,2)-(0,0,0)=(1,0,2)$

$$\begin{array}{ccc} i & j & k \\ 4 & 3 & 5 \\ 1 & 0 & 2 \end{array} = (6,-3,-3) \simeq (2,-1,-1) = \vec{n}$$

Σ: $2x-y-z=0$

$\vec{s} \perp \vec{v}$

$\vec{n} \perp \vec{s}, \vec{v} \Rightarrow \vec{v} = \lambda \cdot \vec{n} \times \vec{s}$

$$\begin{vmatrix} i & j & k \\ 4 & 3 & 5 \\ 2 & -1 & -1 \end{vmatrix} = (2,14,-10) \simeq (1,7,-5)$$

$$(1,0,2)+\lambda\cdot(4,3,5) = (0,0,0)+\underbrace{\mu\cdot(1,7,-5)}$$

$$\left.\begin{array}{lr} 1+4\lambda=\mu & /\cdot 4 \\ 0+3\lambda=7\mu & /\cdot 3 \\ 2+5\lambda=-5\mu & /-5 \end{array}\right\} +$$

$$4\cdot 1-2\cdot 5=\mu\cdot(4+21+25)=50\mu$$

$$\mu=\frac{-6}{50}$$

$$\left|-\frac{6}{50}(1,7,-5)\right| = \frac{6}{50}\sqrt{1+49+25} =$$

$$= \frac{6}{50}\sqrt{3\cdot 5^2} = \frac{6\cdot 5\cdot\sqrt{3}}{50} =$$

$$= \frac{6\sqrt{3}}{10} = \frac{3\sqrt{3}}{5}$$

(c) Še analitični

$$d(A, p) := \min \{d(A,T),\ T \in p\}$$

$T: \quad (1,0,2) + \lambda \cdot (4,3,5)$

$A: \quad (0,0,0)$

$d(T,A) = |(1+4\lambda,\ 3\lambda,\ 2+5\lambda)|$: iščemo min glede na λ

Ta je zavzet pri istem λ kot $\min \{d^2(T,A),\ T \in p\}$.

$$f(\lambda) = (1+4\lambda)^2 + 9\lambda^2 + (2+5\lambda)^2$$

$$f'(\lambda) = 2(1+4\lambda)\cdot 4 + 18\lambda + 2\cdot(2+5\lambda)\cdot 5 = 0$$

$$\lambda \cdot (32 + 18 + 50) + 8 + 20 = 0$$

$$\lambda = -\frac{28}{100} = -\frac{14}{50}$$

Vektorski prostori

Def.: Množica V je vektorski prostor nad $\mathbb{R}$, če velja naslednje.

(a) Na V je definirano seštevanje, ki je asociativno in komutativno, obstaja nevtralni element za seštevanje (vektor 0) in vsak vektor ima nasprotni element ($\forall\, u \in V \ \exists\, v \in V \ni: u+v=0$).

(b) Na V je definirano množenje s skalarjem, ki vektorju u in skalarju a priredi vektor $a \cdot u$. Ta operacija distributivna glede na seštevanje vektorjev ($a(u+v) = au + av$) in na seštevanje skalarjev ($(a+b)u = au + bu$), je asociativna za množenje s skalarji. Množenje z 1 je identična preslikava: $1 \cdot u = u$.

Podmnožica $U \subset V$ je vekt. podprostor, če je U v. p. za iste operacije. V tem primeru pišemo $U \leq V$.

V praksi: V: vemo, da je v. prostor. Naj bo $U \subset V$.
Da je $U \leq V$ je ekv. temu, da za $\forall\, u, v \in U$, $\alpha, \beta \in \mathbb{R}$ $\alpha u + \beta v \in U$.

$\mathbb{R}^n$ je v.p. za sešt. po komp. in množ. s skalarjem po komponentah.

Def. linearne neodvisnosti je enaka tudi za splošen V.

Ugotovi, kaj od naštetega je vekt. prostor (oz. podprostor).

$V=\mathbb{R}^4$; $U=\{(x, 2x, 1-x, z), \ x,z\in\mathbb{R}\}$ $U\le V$?

prvi test: ali $0\in U$: ne. U je ravnina v $\mathbb{R}^4$, ki ne gre skozi izhodišče.

$$(0,0,1,0)+x(1,2,-1,0)+z\cdot(0,0,0,1)$$

$V=\mathbb{R}^4$, $U=\{(2x, -3y+z, x+z, x+y+z), \ x,y,z\in\mathbb{R}\}$

$$\alpha(2x, -3y+z, x+z, x+y+z)+\beta\cdot(2x_0, -3y_0+z_0, x_0+z_0, x_0+y_0+z_0)$$

$$\overset{?}{=}(2x_1, -3y_1+z_1, x_1+z_1, x_1+y_1+z_1) \text{ za neke } x_1,y_1,z_1 \text{ ?}$$

$$\Big(2(\alpha x+\beta x_0), -3(\alpha y+\beta y_0)+\alpha z+\beta z_0, (\alpha x+\beta x_0)+(\alpha z+\beta z_0),$$

$$\underbrace{(\alpha x+\beta x_0)}_{\overset{\parallel}{x_1}}+\underbrace{(\alpha y+\beta y_0)}_{y_1}+\underbrace{(\alpha z+\beta z_0)}_{z_1}\Big)$$

$$T = x\cdot\underset{v_1}{(2,0,1,1)}+y\cdot\underset{v_2}{(0,-3,0,1)}+z\underset{v_3}{(0,1,1,1)}$$

Za DN: poišči $v\perp v_1, v_2, v_3$

Potem psl. $\pi: u\xrightarrow{\text{ek}}\langle u,v\rangle$, preslika ravno ves U v 0

$\pi^{-1}(0)=U$.

Skalarni produkt na $\mathbb{R}^n$: $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n)$

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i \cdot y_i$$

$\underset{v_1}{(2,0,1,1)}$, $\underset{v_2}{(0,-3,0,1)}$, $\underset{v_3}{(0,1,1,1)}$ $\qquad v = (\alpha, \beta, \gamma, \delta)$

$\langle v, v_i \rangle = 0$, $i = 1,2,3$

$$\begin{aligned} 2\alpha + 0\beta + \gamma + \delta &= 0 \\ 0\alpha - 3\beta + 0\gamma + \delta &= 0 \\ 0\alpha + \beta + \gamma + \delta &= 0 \end{aligned} \qquad \left[\begin{array}{cccc|c} 2 & 0 & 1 & 1 & 0 \\ 0 & -3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \sim$$

$$\begin{bmatrix} 2 & 0 & 1 & 1 \\ 0 & -3 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 1 & 1 \\ 0 & 1 & \textcircled{1} & 1 \\ 0 & 0 & 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & -1/3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 4/3 \end{bmatrix} \qquad \begin{aligned} \alpha &= 1/6\,\delta \\ \beta &= 1/3\,\delta \\ \gamma &= -4/3\,\delta \end{aligned}$$

$$v = \left(1/6\,\delta, 1/3\,\delta, -4/3\,\delta, \delta\right) = \underline{\delta\left(1/6, 1/3, -4/3, 1\right)}$$

↑ 1 dim. v. podprostor

$U^{\perp} = \{ v \in \mathbb{R}^4, \langle v, u \rangle = 0 \ \forall u \in U \}$: ortogonalni komplement

$\dim U + \dim U^{\perp} = 4$ (v $\mathbb{R}^4$)

$U^{\perp} = \{ \delta(1/6, 1/3, -4/3, 1), \delta \in \mathbb{R} \}$ 1-dim

$\forall u \in U$: $u = x \cdot v_1 + y \cdot v_2 + z \cdot v_3$, $x, y, z \in \mathbb{R}^3$

$u \simeq (x, y, z)$ v bazi $\{v_1, v_2, v_3\}$.

Def: Vektorji $v_1, .., v_n \in V$ so lin. neodvisni če velja sklep:

$$\sum_{i=1}^{n} \alpha_i v_i = 0 \iff \alpha_i = 0 \quad \forall i = 1, .., n.$$

Mnozica $\{v_1, .., v_n\} \subset V$ je baza V, če lahko $\forall v \in V$ zapišemo kot

$$v = \sum_{i=1}^{n} \alpha_i v_i$$

na EN SAM način. Število n je dimenzija v.p. V.

Naj bo $\langle \, , \, \rangle$ dan skalarni produkt. Vektor v je normiran, če je $\langle v, v \rangle = 1$. Baza $\{v_1, .., v_n\}$ je ortonormirana, če $\langle v_i, v_j \rangle = \delta_{ij}$. (ONB : ortonorm. baza)

δ_{ij}: Kroneckerja δ; $\quad \delta_{ij} = \begin{cases} 1, & i=j \\ 0 & \text{sicer} \end{cases}$

vsi v_i so velikosti 1 in vsak je pravokoten na $\forall$ drugega.

Primer: $P_2 := \{\text{polinomi stopnje} \leq 2\}$

Pokaži, da je P_2 v.p. za običajno množenje in seštevanje. Poišči kako bazo. Naj bo s predpisom

$$\langle p, q \rangle := \int_{-1}^{1} p(x) q(x) \, dx$$

dan skalarni produkt.

Poišči ONB glede na ta skal. produkt.

$p(x) = ax^2 + bx + c$
$q(x) = a_1x^2 + b_1x + c_1$

A: asociativnost: ✓ komut ✓ ; imamo polinom 0: ničla za +.
nasspr. polinom: $-p(x) = -ax^2 - bx - c$
B: množ. s skal: ✓ distr., asoc ✓ ; $1 \cdot p = p$. ✓

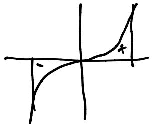
Baza: $x^2, x, 1$
$\quad v_1 \;\; v_2 \;\; v_3$

$$\langle v_1, v_1 \rangle = \int_{-1}^{1} x^2 \cdot x^2 dx = 2\int_0^1 x^4 dx = \frac{2}{5}$$

$$\langle v_3, v_3 \rangle = \int_{-1}^{1} 1 \cdot 1 = 2$$

$$\langle v_1, v_2 \rangle = \int_{-1}^{1} x^2 \cdot x\, dx = 0.$$

$$\langle v_1, v_3 \rangle = \int_{-1}^{1} x^2 \cdot 1\, dx = 2 \cdot \int_0^1 x^2 dx = \\ = \frac{2}{3}.$$

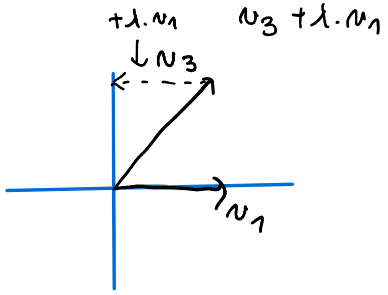


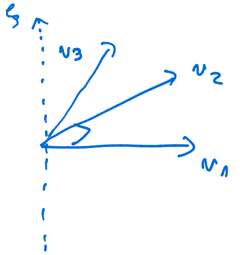
$$\langle v_2, v_3 \rangle = \int_{-1}^{1} x \cdot dx = 0$$

$$\langle v_2, v_2 \rangle = \int_{-1}^{1} x^2 = 2/3$$

Ali znamo bazo v_1, v_2, v_3 popraviti do ortonormirane?
- najprej do ortogonalne
- nazadnje normiramo.

$\langle v_1, v_2 \rangle = 0, \quad \langle v_1, v_3 \rangle \neq 0$
$\langle v_2, v_3 \rangle = 0$





v_3: leži v ravnini v_1, e_3.

Izbrati moramo λ, da bo $\underbrace{v_3 + \lambda \cdot v_1}_{\tilde{v}_3} \perp v_1$.

Toraj: $\langle v_3 + \lambda v_1, v_1 \rangle = 0$

$$\Updownarrow$$

$$\int_{-1}^{1} (1 + \lambda \cdot x^2) \cdot x^2 = 0$$

$$\Updownarrow$$

$$\int_{-1}^{1} \underbrace{x^2 \, dx}_{2/3} + \lambda \cdot \int_{-1}^{1} \underbrace{x^4 \, dx}_{2/5} = 0$$

$$\frac{2}{3} + \lambda \cdot \frac{2}{5} = 0$$

$$\lambda = -\frac{2}{3} \cdot \frac{5}{2} = -\frac{5}{3}$$

$$\tilde{v}_3 = 1 - \frac{5}{3} x^2$$

$\Rightarrow$ $v_1, v_2, \tilde{v}_3$: ortogonalna.

$\left\{ v_1 \cdot \sqrt{\frac{5}{2}},\ v_2 \cdot \sqrt{\frac{3}{2}},\ \tilde{v}_3 / \sqrt{\langle \tilde{v}_3, \tilde{v}_3 \rangle} . \right\}$ ONB

Oglejmo si še enkrat prvotno bazo, v_1, v_2, v_3 in napišimo v matriko skalarne produkte.

$A = \{ a_{\underset{\text{vrsta}}{i}\underset{\text{stolpec}}{j}} \}$

$a_{ij} = \langle v_i, v_j \rangle$: gramova matrika

$$A = \begin{bmatrix} \frac{2}{5} & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & 2 \end{bmatrix}$$

$\langle v_1, v_3 \rangle =$

Primer: Ugotovi, ali so vektorji

$v_1 = (1, 2, 1, 3)$

$v_2 = (0, 1, 1, 0)$

$v_3 = (1, 0, 1, 0)$

$v_4 = (1, 0, 0, 1)$

lin. odvisni / neodvisni: Ali tvorijo bazo?

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & \underline{1} & -1 \\ 0 & 2 & 1 & 2 \end{bmatrix} \begin{matrix} (-2) \\ \sim \\ (+1) \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$0 \quad 0 \quad -1 \quad 2$

$x = (x_1, \dots, x_4) := \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \alpha_4 v_4$

$$\begin{array}{c} \\ (-3)\ (-2) \end{array} \left[\begin{array}{cccc|c} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \\ 1 & 0 & 1 & 1 & x_1 \\ 2 & 1 & 0 & 0 & x_2 \\ 1 & 1 & 1 & 0 & x_3 \\ 3 & 0 & 0 & 1 & x_4 \end{array}\right] \sim$$

$$\sim (-1) \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & x_1 \\ 0 & 1 & -2 & -2 & x_2 - 2x_1 \\ 0 & 1 & 0 & -1 & x_3 - x_1 \\ 0 & 0 & -3 & -2 & x_4 - 3x_1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & x_1 \\ 0 & 1 & -2 & -2 & x_2 - 2x_1 \\ 0 & 0 & 2 & 1 & x_3 - x_1 - x_2 + 2x_1 \\ 0 & 0 & -3 & -2 & x_4 - 3x_1 \end{array}\right] \begin{matrix} (3/2) \\ \swarrow + \end{matrix}$$

zg. trikotna matrika

rang 4 : razš. matrika

$$\left[\begin{array}{cccc|l} 1 & 0 & 1 & 1 & x_1 \\ 0 & 1 & -2 & -2 & x_2 - 2x_1 \\ 0 & 0 & 2 & 1 & x_3 + x_1 - x_2 \\ 0 & 0 & 0 & -1/2 & x_4 - 3x_1 + \\ \end{array}\right]$$

$+3/2\,(x_1 + x_3 - x_2)$

osn. matrika. rang 4

Def: Naj bo A dana matrika dimenzije $n \times m$ (n vrst, m stolpcev)
Rang A je število lin. neodv. vrstic oz stolpcev.

Če je rang osn. matrike < rang razš. matrike, je sistem protisloven (to je na drug način povedano to, da imamo enačbo $0 = 1$).

Primer:

$$x + y + z = 1$$
$$3x + y + 2z = 4$$
$$x - y + 1 = 2 \quad +1$$

$$\begin{matrix}(-3)\\ \curvearrowright\\ \ \end{matrix}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & 1 & 2 & 4 \\ 1 & -1 & \underset{+1}{0} & \underset{3}{2} \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -1 & 1 \\ 0 & -2 & \underset{0}{-1} & 1_2 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -1 & 1 \\ 0 & 0 & \underset{+1}{0} & \underset{1}{0} \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right] \Leftarrow 0 = 1$$

r. osn: 2 ; rang razš: 2

r osn: 3 ; rang razš: 3

r osn: 2 ; rang razš: 3

Gaussova eliminacija ohranja rang.

Naloge iz v.p:

2N: Ali je katera od sp. množic v.p:

1. $V_1 = \{(x,y,z) \in \mathbb{R}^3, x \in \mathbb{Z}\}$; $0 \in V_1$, $(1,0,0) \in V_1$ $\frac{1}{2} \cdot (1,0,0) \notin V_1$ NI VP
2. $V_2 = \{(x,y,z) \in \mathbb{R}^3, y=0\}$; $0 \in V_2$, ravnina skozi izh: je v.p. JE VP
3. $V_3 = \{(x,y,z) \in \mathbb{R}^3, y \geq 0\}$; $0 \in V_3$; $(0,-1,0) \notin V_3$, $(0,1,0) \in V_3$ NI VP
4. $V_4 = \{(x,y,z) \in \mathbb{R}^3, x+z=0\}$; JE VP.
5. $V_5 = \{(x,y,z) \in \mathbb{R}^3, x+y=1\}$: NI VP
6. $V_6 = \{(x,y,z) \in \mathbb{R}^3, x=2y=3z\}$: JE VP

5N: (1) Ali je $(8,2,-14)$ lin. kombinacija $\underset{v_1}{(3,1,-5)}$ in $\underset{v_2}{(-2,0,4)}$?

$$(3,1,-5)\cdot\alpha + (-2,0,4)\cdot\beta = (8,2,-14)$$

$$\begin{matrix} \\ (5)\ (-3) \end{matrix}\left[\begin{array}{cc|c} 3 & -2 & 8 \\ 1 & 0 & 2 \\ -5 & 4 & -14 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & -2 & 2 \\ 0 & 4 & -4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & \underline{0} \end{array}\right]$$

$\alpha = 2$, $\beta = -1$

Množica $\{\alpha v_1 + \beta v_2, \alpha,\beta \in \mathbb{R}\}$ se imenuje linearna lupina ali ogrinjača, oznaka $\mathrm{Lin}(\{v_1,v_2\})$.
↳ njega

Ali je $\mathrm{Lin}(\{v_1,v_2\})$ vektorski prostor? DA. To je najmanjši v.p., ki vsebuje $\{v_1,v_2\}$.

Linearna ogrinjača baze prostora je kar cel prostor.