$y = -z - w$

10. naloga: $U = \{(x,y,z,w) \in \mathbb{R}^4,\ y+z+w=0\}$

$V = \{(x,y,z,w) \in \mathbb{R}^4,\ x+y=0,\ z=2w\}$

Poišči kako bazo za $U, V, U \cap V, \mathrm{Lin}(U \cup V) = U \oplus V$.

baza za U: spl. vektor v U je

$$(x, -z-w, z, w) = 0$$

$$= (x,0,0,0) + (0,-z,z,0) + (0,-w,0,w)$$
$$= x(1,0,0,0) + z\cdot(0,-1,1,0) + w(0,-1,0,1) = 0$$

$$U = \mathrm{Lin}\{(1,0,0,0), (0,-1,1,0), (0,-1,0,1)\}$$

Tole ↗ ni baza, če so v. lin. odvisni, sicer pa je.

$y + z + w = 0$

V: spl. vektor je: $(x, -x, 2w, w) = 0 \iff x=0,\ w=0$

baza: $(1,-1,0,0), (0,0,2,1)$

$y = -z - w =$
$= -2w - w = -3w$

baza $U \cap V$: $(3w, -3w, 2w, w)$

$(x, -z-w, z, w)$; $(x, -x, 2w, w)$

$(z+w, -z-w, z, w) =$
$= (3w, -3w, 2w, w)$

$$\begin{aligned} x + y \quad\quad &= 0 \\ z - 2w &= 0 \\ y + z + w &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \sim \quad (-1)$$

$$\begin{array}{cccc} x & y & z & w \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix} (-1)$$

$$\begin{aligned} x &= +3w \\ y &= -3w \\ z &= 2w \\ w &= w \end{aligned}$$

$$(x, y, z, w) = w\,(3, -3, 2, 1)$$

baza $U \cup V$:

Vzamemo $B =$ baza $U \cup$ baza V in poiščemo največjo lin. neodv. podmnožico.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{: lin. neodv.} \Rightarrow U \oplus V = \mathbb{R}^4$$