Izpeljava formul (1) in (2) na 1. strani

$$A = \begin{bmatrix} \cancel{a_{11} \cdots a_{1n}} \\ a_{n1} \cdots a_{n,n} \end{bmatrix}$$

$$\vec{a}_1 \qquad \vec{a}_n$$

$$C = \begin{bmatrix} \cancel{c_{1,1} \cdots c_{1,n}} \\ \vdots \\ \cancel{c_{n,1} \cdots c_{n,n}} \end{bmatrix}$$

$$CA = \begin{bmatrix} c_{11}a_{11} + \cdots + c_{1n}a_{n1} & \cdots & c_{11}a_{1n} + \cdots + c_{1n}a_{nn} \\ \vdots & & \vdots \\ c_{n1}a_{11} + \cdots + c_{nn}a_{n1} & \cdots & c_{n1}a_{1n} + \cdots + c_{nn}a_{nn} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} c_{11} \cdots c_{1n} \\ \\ c_{n1} \cdots c_{nn} \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} a_{11} \\ \vdots \\ a_{n1} \end{bmatrix}}_{\vec{a}_1} \qquad \underbrace{\begin{bmatrix} c_{11} \cdots c_{1n} \\ \\ c_{n1} \cdots c_{nn} \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} a_{1n} \\ \vdots \\ a_{nn} \end{bmatrix}}_{\vec{a}_n}$$

$$= \begin{bmatrix} C\vec{a}_1 \cdots C\vec{a}_n \end{bmatrix}$$

$$A = \begin{bmatrix} \vec{a}_1 \cdots \vec{a}_n \end{bmatrix} \Rightarrow CA = \begin{bmatrix} C\vec{a}_1 \cdots C\vec{a}_n \end{bmatrix}$$

$$A\vec{x} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + \cdots + a_{1n}x_n \\ \vdots \\ a_{n1}x_1 + \cdots + a_{nn}x_n \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} a_{11}x_1 \\ \vdots \\ a_{n1}x_1 \end{bmatrix}}_{x_1\vec{a_1}} + \cdots + \underbrace{\begin{bmatrix} a_{1n}x_n \\ \vdots \\ a_{nn}x_n \end{bmatrix}}_{x_n\vec{a_n}}$$

$A = [\vec{a_1} \cdots \vec{a_n}]$ en $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

$$\Rightarrow A\vec{x} = x_1\vec{a_1} + \cdots + x_n\vec{a_n}$$

Opombe k trditvi 3)

- Reducirana vrstična stopničasta forma

$$A = \begin{bmatrix} 0 \cdots 0 & 1\ x \cdots x & 0\ x \cdots x & 0\ x \cdots x & 0\ x \cdots x \\ & & 1\ x \cdots x & 0\ x \cdots x & 0\ x \cdots x \\ & & & 1\ x \cdots x & 0\ x \cdots x \\ & 0 & & & 1\ x \cdots x \\ & & & & \end{bmatrix}$$

Poseben primer: Identična matrika

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Stolpci I so LN, ker iz $I\vec{x} = 0$ sledi $\vec{x} = 0$

Stolpci I so ogrodje, ker je $I\vec{x} = \vec{b}$ rešljiva pri vsakem $\vec{b}$

Še v drugo smer: Če je A kvadratna reducirana vrstična stopničasta forma in če so njeni stolpci LN, potem je $A = I$

S protislovjem: Če $A \neq I$, potem ima vsaj ena stopnica dolžino ≥ 2. Recimo

$$A = \begin{bmatrix} 1 & 0 & 0 & x_1 & \\ & 1 & 0 & x_2 & \\ & & 1 & x_3 & \\ & & & & \cdots \\ \uparrow & \uparrow & \uparrow & \uparrow & \\ \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \vec{a}_4 & \end{bmatrix}$$

$$\vec{a}_4 = x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3$$

Torej stolpci A niso LN kar je v nasprotju s predpostavko

Razvoj determinante po prvi/drugi/tretji vrstici

$$\det\begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\overset{\det}{=} a_{11}\det\left[\ \cancel{a_{11}}\ \right] - a_{12}\det\left[\ \cancel{a_{12}}\ \right] + a_{13}\det\left[\ \cancel{a_{13}}\ \right]$$

To je bil razvoj det A po prvi vrstici (To vzeli za definicijo determinante)

Determinanto lahko razvijemo tudi po drugi vrstici

$$\det\begin{bmatrix} {}^{+}a_{11} & {}^{-}a_{12} & {}^{+}a_{13} \\ {}^{-}\cancel{a_{21}} & {}^{+}\cancel{a_{22}} & {}^{-}\cancel{a_{23}} \\ {}^{+}a_{31} & {}^{-}a_{32} & {}^{+}a_{33} \end{bmatrix} = -a_{21}\det\left[\ \cancel{a_{21}}\ \right] +$$

$$+ \,a_{22}\det\left[\ \cancel{a_{22}}\ \right] - a_{23}\det\left[\ \cancel{a_{23}}\ \right]$$

Podobno bi lahko naredili tudi razvoj po tretji vrstici $\det A = a_{31}\det A_{31} - a_{32}\det A_{32} + a_{33}\det A_{33}$