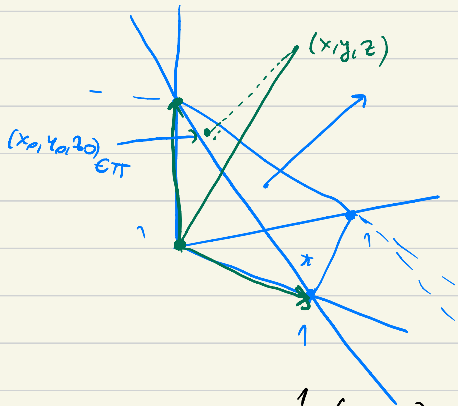
4. naloga

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$



lin. pol. preslika 0 v 0
slika lin. pol. je vekt. podprostor
Ta ravnina ni v. podprostor.

normala: $\frac{1}{\sqrt{3}}(1,1,1) = \vec{n}$

$(x, y, z) = (x_0, y_0, z_0) + \lambda \cdot \vec{n}$

$(x, y, z) - \lambda \cdot \vec{n} = (x_0, y_0, z_0) \in \Pi$

$(x - \frac{1}{\sqrt{3}}\lambda + y - \frac{1}{\sqrt{3}}\lambda + z - \frac{1}{\sqrt{3}}\lambda) - 1 = 0$

$(x + y + z) - \sqrt{3} \cdot \lambda - 1 = 0$

$\sqrt{3} \cdot \lambda = (x+y+z) - 1$

$\lambda = \frac{1}{\sqrt{3}}(x+y+z-1)$

$f: (x, y, z) \to (x_0, y_0, z_0)$

$f(x, y, z) = (x, y, z) - \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}(1,1,1) \cdot (x+y+z-1)$

$= (x, y, z) - \frac{1}{3}(1,1,1)(x+y+z-1)$

$= (x, y, z) - \frac{1}{3}(1,1,1)(x+y+z) + \frac{1}{3}(1,1,1)$

V matrični obliki: $\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\underbrace{m \times n \quad k \times l}_{n=k}$: $m \times l$

5. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ -1 & 5 & 0 \\ 0 & 1 & -2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 2 \\ 1 & 0 & 0 & 3 \end{bmatrix}$

$\cancel{A^3}$, ABA, BAB, $\cancel{B^2}$, če $\exists$.

$$AB = \begin{bmatrix} 3 & 1 & 0 & 3 \\ 2 & -1 & 1 & 8 \\ -2 & -6 & 5 & 10 \\ -2 & -1 & 1 & -4 \end{bmatrix}$$

ABA: DN

BAB

$(AB)\cdot A$:
$(4\times 4)\cdot(4\times 3) \to (4\times 3)$

$B\,(AB)$
$(3\times 4)\cdot(4\times 4) \to 3\times 4$

6: že naredili

7:

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & -2 \end{bmatrix}$$

: $\mathbb{R}^3 \to \mathbb{R}^2$

injektivna ni

surjektivna: je

ta 2 stolpca sta lin. neodvisna, $\det \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = 2 \neq 0$

1. geometrijsko:

$$\begin{array}{cc} & \begin{array}{ccc} i & j & k \end{array} \\ \begin{array}{c} v_1 = \\ v_2 = \end{array} & \begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & -2 \end{bmatrix} \end{array} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}^{v} = 0$$

$v = \lambda \cdot v_1 \times v_2$

$v = (-4,\ 4+4,\ 2) \simeq (-2, 4, 1)$

$\ker A = \mathbb{R} \cdot (-2, 4, 1)$

2. $\begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{array}{c} \begin{array}{ccc} & & z \end{array} \\ \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \end{bmatrix} \end{array}$

$x = -2z$

$y = +4z$

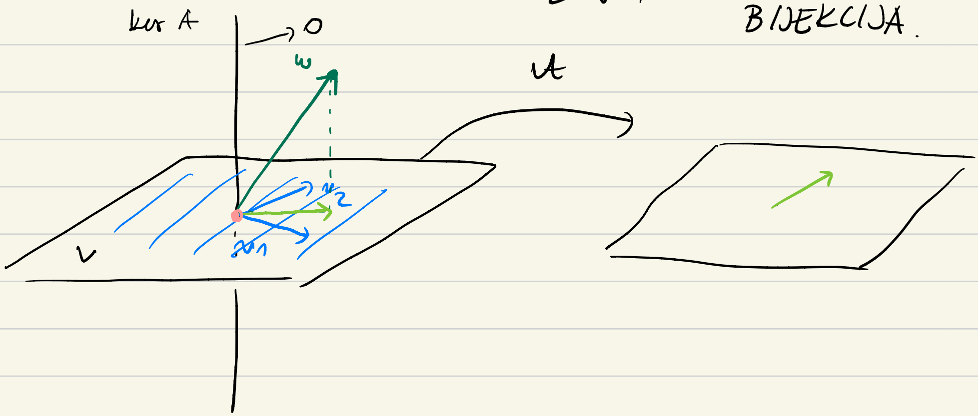
$z = z$

$(x, y, z) = z \cdot (-2, 4, 1)$

Naj bo $V = \ker A^{\perp}$. Baza $V = \{v_1, v_2\}$

$\mathbb{R}^3 = \ker A \oplus \ker A^{\perp}$ (velja: $W < V$: $V = W \oplus W^{\perp}$)

Preslikava $\mathcal{A}(x, y, z) := A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, zožena na V pa je BIJEKCIJA.



$w \in \mathbb{R}^3 \qquad w = \lambda \cdot v + \alpha v_1 + \beta v_2 \qquad \alpha v_1 + \beta v_2 \in V$

$$\underline{Aw} = \lambda \cdot \underbrace{Av}_{=0} + \alpha \cdot Av_1 + \beta \cdot Av_2 = \underbrace{\alpha A v_1 + \beta \cdot A v_2}_{\text{TO POMENI, DA JE } A|_V \text{ surjektivna}} ! \quad \text{surj. psl.}$$

$$Aw = A(\underbrace{\alpha v_1 + \beta v_2}_{= pr_V w})$$

Ali je $A|_V$ injektivna? To bo res, če ima trivialno jedro.

$\ker(A|_V) = (\ker A) \cap V \qquad A: V \to \mathbb{R}^2$ bijektivno

$$\begin{array}{ccccc} \mathbb{R}^3 & = & \ker A & \oplus & V \\ & & \downarrow & A & \downarrow \text{bij.} \\ & & 0 & \oplus & \mathbb{R}^2 \\ & & & & \vec{\mu} \end{array}$$

8. $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 0 & 2 \end{bmatrix} = A \qquad v = \begin{bmatrix} 4 \\ 3 \\ 5 \\ 4 \end{bmatrix}; \quad w = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Iščemo w: $A(w) = v$. Rešujemo 4×4 sistem.

(Denimo, da jedro ne bi bilo trivialno. Potem so rešitve $\vec{w} + \lambda \vec{k}$, $\vec{k} \in \ker A$).

9. $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $\qquad \mathcal{A}(x) = A \cdot x$

$\mathcal{A} = pr_{\mathcal{L}(e_1, e_2)}$

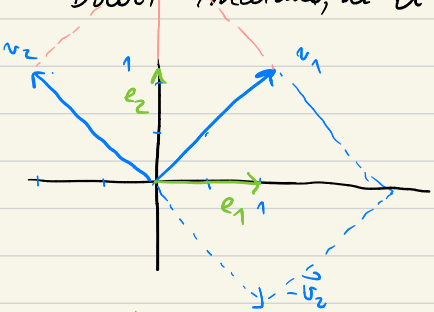
$B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

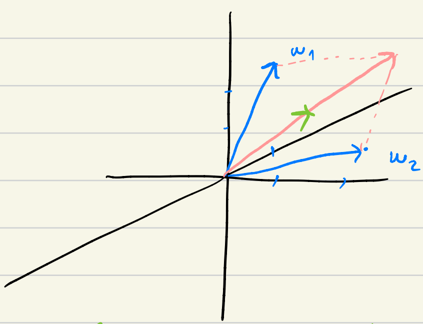
$C = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}$

$D = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

10. $\mathcal{A} : \mathbb{R}^2 \to \mathbb{R}^3$, $\quad \mathcal{A}(\underbrace{1,1}_{v_1}) = (\underbrace{0,1,2}_{w_1})$, $\mathcal{A}(\underbrace{-1,1}_{v_2}) = (\underbrace{2,1,0}_{w_2})$

Določi matriko, ki $\mathcal{A}$ pripada v std. bazah.





$e_1 = \lambda v_1 + \mu v_2 =$

$= \frac{1}{2}(v_1 - v_2) \Rightarrow \mathcal{A}(e_1) = \frac{1}{2}\left((0,1,2) - (2,1,0)\right) = \frac{1}{2}(-2,0,2) =$

$= (-1,0,1)$

$e_2 = \frac{1}{2}(v_1 + v_2)$, $\mathcal{A}(e_2) = \frac{1}{2}\big((0,1,2) + (2,1,0)\big) = (1,1,1)$

$$A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

11. $\mathcal{A}, \mathcal{B} : \mathbb{R}^2 \to \mathbb{R}^2$, $\mathcal{A}(x,y) = (y,x)$, $\mathcal{B}(x,y) = (x,-y)$
Izračunaj preslikave $\mathcal{A}+\mathcal{B}$, $\mathcal{A}\circ\mathcal{B}$, $\mathcal{B}\circ\mathcal{A}$ in zapiši pripadajoče matrike.

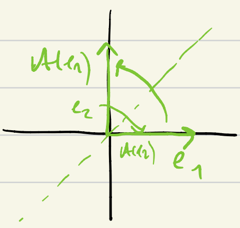
$$(\mathcal{A}+\mathcal{B})(x,y) = \mathcal{A}(x,y) + \mathcal{B}(x,y) = (y+x,\ x-y) = A\begin{bmatrix} x \\ y \end{bmatrix} + B\begin{bmatrix} x \\ y \end{bmatrix} = (A+B)\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathcal{A}(\mathcal{B}(x,y)) = \mathcal{A}(x,-y) = (-y,x) = A\cdot\left(B\cdot\begin{bmatrix} x \\ y \end{bmatrix}\right) = A\cdot B\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathcal{B}(\mathcal{A}(x,y)) = \mathcal{B}(y,x) = (y,-x)$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad A+B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathcal{A}\mathcal{B} \rightsquigarrow A\cdot B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \mathcal{B}\mathcal{A} \rightsquigarrow B\cdot A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



$\mathcal{A}$: zrcaljenje.

Naj bo $\mathcal{A}: \mathbb{R}^3 \to \mathbb{R}^3$ linearna bijekcija. Potem ima inverzno preslikavo $\mathcal{A}^{-1}: \mathbb{R}^3 \to \mathbb{R}^3$

$\mathcal{A}^{-1} \circ \mathcal{A} = id$

$\mathcal{A} \circ \mathcal{A}^{-1} = id$

Če $\mathcal{A}$ pripada A v std. bazah, potem $\mathcal{A}^{-1}$ pripada matrika B, tako da je

$B \cdot A = I$ in $A \cdot B = I$. Tako matriko označimo z A^{-1}.

$A^{-1} \exists \iff \det A \neq 0$

To velja tudi za $\mathbb{R}^n$.

$$[A \mid I] \overset{\text{gauss}}{\rightsquigarrow} [I \mid A^{-1}]$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \det A = ad - bc$$

$$A^{-1} = \frac{1}{\det A} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}; \quad A^{-1} \circ \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{\det A} \begin{bmatrix} ad-bc & 0 \\ 0 & -bc+ad \end{bmatrix} = I$$

11. Zapiši preslikavo, ki ji v std. bazi pripada matrika $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ v bazi $\{(1,1), (-1,1)\} = \{v_1, v_2\}$ z matriko B.

$\mathcal{A}(x, y) = (y, x) \qquad \mathcal{A}(x \cdot e_1 + y \cdot e_2) = y \cdot e_1 + x \cdot e_2$

$\mathcal{A}(\lambda \cdot v_1 + \mu \cdot v_2) = \alpha \cdot v_1 + \beta \cdot v_2$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \qquad \begin{aligned} \mathcal{A}(v_1) &= b_{11} v_1 + b_{21} v_2 \\ \mathcal{A}(v_2) &= b_{12} v_1 + b_{22} v_2 \end{aligned}$$

slika v_1 slika v_2

$\mathcal{A}(1,1) = (1,1) \Rightarrow b_{11} = 1, \quad b_{21} = 0$

$\mathcal{A}(-1,1) = (1,-1) \Rightarrow b_{12} = 0 \quad b_{22} = -1$

$\uparrow = -v_2$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Zapiši preslikavo P, ki preslika $e_1 \to v_1$, $e_2 \to v_2$

$$P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Kakšna je zveza med A, B, P?

Kolokvij: petek 18.12. ob 8^{00}.