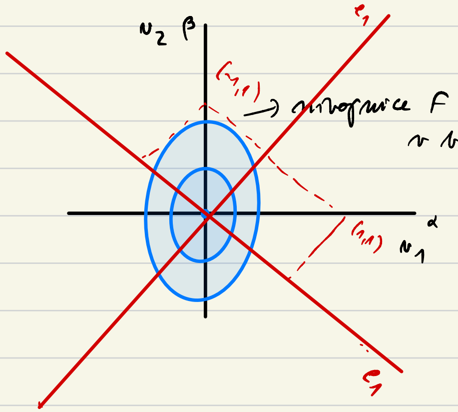
Primer: dana je forma $f(x,y) = 3x^2 + 2xy + 3y^2$. Pripadna je matrika

$$A = \begin{array}{c|cc} & x & y \\ \hline x & 3 & 1 \\ y & 1 & 3 \end{array} \quad ; \quad D = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$\{v_1, v_2\}$: $\rightsquigarrow 4\alpha^2 + 2\beta^2 = c$ : družina elips

$\left(\frac{\alpha}{a}\right)^2 + \left(\frac{\beta}{b}\right)^2 = 1$ : a, b: polosi

$$\left(\frac{\alpha}{1/2}\right)^2 + \left(\frac{\beta}{1/\sqrt{2}}\right)^2 = c$$




MIN v 0

graf f: eliptični paraboloid

Kolokvij: jedro, slika (baza, dimenzija), zapis matrike za dano lin. pres. inverzna matrika.

Izračunaj l. vrednosti in l. vektorje naslednjih matrik:

$$A_1 = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\det \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{vmatrix} \begin{matrix} \\ \cdot(-2) \\ \downarrow + \end{matrix} = \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 0 & \underbrace{4+2\lambda}_{2(2+\lambda)} & \underbrace{-2-\lambda}_{-(2+\lambda)} \end{vmatrix} =$$

$$= (2+\lambda) \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 0 & 2 & -1 \end{vmatrix} \begin{matrix} \\ \uparrow \\ \cdot 3 \end{matrix} =$$

$$= (2+\lambda) \begin{vmatrix} -2-\lambda & 2+\lambda & 0 \\ 3 & 1-\lambda & 0 \\ 0 & 2 & -1 \end{vmatrix} =$$

$$= (2+\lambda)^2 \begin{vmatrix} 1 & 1 & 0 \\ 3 & 1-\lambda & 0 \\ 0 & 2 & -1 \end{vmatrix} = (2+\lambda)^2 \left(-1+\lambda-3\right)$$

$\lambda_1 = -2$: dvojna $\qquad \lambda_2 = 4$

$$\lambda_1 = -2: \quad \begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix} \qquad v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

rang: 1 : dim jedra: 2

$\lambda_2 = 4$: $\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix}$ $\qquad v_3 = \begin{bmatrix} 1 \\ 1 \\ +2 \end{bmatrix}$

rang = 2

$$A_2 = \begin{bmatrix} 5 & 8 & 16 \\ 4 & +1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$$

$$\det(A_2 - \lambda I) = \begin{vmatrix} 5-\lambda & 8 & 16 \\ 4 & 1-\lambda & 8 \\ -4 & -4 & -11-\lambda \end{vmatrix} \begin{matrix} (-2) \\ \\ \\ \end{matrix} =$$

$+$

$\lambda - 3$

$$= \begin{vmatrix} 3-\lambda & 6+2\lambda & 0 \\ 4 & 1-\lambda & 8 \\ 0 & -3-\lambda & -3-\lambda \end{vmatrix} = (\lambda+3)(\lambda+3) \begin{vmatrix} -1 & +2 & 0 \\ 4 & 1-\lambda & 8 \\ 0 & -1 & -1 \end{vmatrix} \begin{matrix} \\ \\ (-8) \end{matrix}$$

$\lambda + 3$

$$= -(\lambda+3)(\lambda+3) \begin{vmatrix} 1 & -2 & 0 \\ 4 & 7-\lambda & 0 \\ 0 & -1 & -1 \end{vmatrix} =$$

$$= +(\lambda+3)(\lambda+3)\left(7-\lambda+8\right) = +(\lambda+3)(\lambda+3)(1-\lambda)$$

$\lambda = -3$: $\begin{bmatrix} 8 & 8 & 16 \\ 4 & 4 & 8 \\ -4 & -4 & -8 \end{bmatrix}$

rang 1; e. v. $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$$\lambda = 1: \begin{bmatrix} 4 & 8 & 16 \\ 4 & 0 & 8 \\ -4 & -4 & -12 \end{bmatrix} \sim \begin{bmatrix} \cancel{0} & \cancel{8} & \cancel{8} \\ 1 & 0 & 2 \\ 0 & -4 & -4 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 2-\lambda & -1 & 0 \\ 1 & -1-\lambda & 1 \\ -1 & -1 & 1-\lambda \end{bmatrix} +$$

$$\det(A_3 - \lambda I) = \begin{vmatrix} 2-\lambda & -1 & 0 \\ 1 & -1-\lambda & 1 \\ 0 & -2-\lambda & 2-\lambda \end{vmatrix} =$$

$$= (2-\lambda) \cdot \left( (\lambda+1)(\lambda-2) + 2+\lambda \right) + 1 \cdot (2-\lambda) =$$

$$= (2-\lambda)\left( \lambda^2 \cancel{-\lambda} \cancel{-2} \cancel{+2} \cancel{+\lambda} + 1 \right) = (2-\lambda)(\lambda^2+1)$$

$\lambda_1 = 2$: $\begin{bmatrix} 0 & +1 & 0 \\ 0 & -3 & 1 \\ -1 & -1 & -1 \end{bmatrix}$ $\quad u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_{2,3} = \pm i$ $\begin{bmatrix} 2\pm i & -1 & 0 \\ 1 & -1\pm i & 1 \\ -1 & -1 & 1\pm i \end{bmatrix} \sim \begin{bmatrix} 2\pm i & -1 & 0 \\ 1 & -1\pm i & 1 \\ 0 & -2\pm i & 2\pm i \end{bmatrix} \sim$

$\lambda = -i \quad \sim \begin{bmatrix} 2+i & -1 & 0 \\ 1 & -1+i & 1 \\ 0 & -2+i & 2+i \end{bmatrix} -(2+i) \sim$

$-(-1+i)(2+i) =$
$= -(-2+2i-i-1) =$
$= -(-3+i) = 3-i$

$\sim \begin{bmatrix} 0 & 2-i & -(2+i) \\ 1 & -1+i & 1 \\ 0 & -2+i & 2+i \end{bmatrix} +$

$\sim \begin{bmatrix} 0 & 2-i & -(2+i) \\ 1 & -1+i & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$v_2 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\qquad (2-i)b - (2+i)c = 0$

$v_3 = \overline{v_2}$ $\qquad c = 2-i$

↳ kor je A realna matrika $\qquad b = \dfrac{(2+i)\cdot c}{(2-i)}$

$a + (-1+i)\cdot(2+i) + (2-i) = 0 \qquad b = (2+i)$

$a = -2+i - (-3+2i-i-1) = -2+i+4-i = 2$

$$A_4 = \left[\begin{array}{cc|cc} 11-\lambda & -6 & -3 & 6 \\ 18 & -10-\lambda & -6 & 12 \\ \hline 0 & 0 & 14-\lambda & -30 \\ 0 & 0 & 6 & -13-\lambda \end{array}\right]$$

$$\det(A_4 - \lambda I) = (11-\lambda) \cdot \begin{vmatrix} -10-\lambda & -6 & 12 \\ 0 & 14-\lambda & -30 \\ 0 & 6 & -13-\lambda \end{vmatrix}$$

$$- 18 \cdot \begin{vmatrix} -6 & -3 & 6 \\ 0 & 14-\lambda & -30 \\ 0 & 6 & -13-\lambda \end{vmatrix} =$$

$$(11-\lambda)\cdot(-10-\lambda)\cdot \begin{vmatrix} 14-\lambda & -30 \\ 6 & -13-\lambda \end{vmatrix} - 18 \cdot (-6) \cdot \begin{vmatrix} 14-\lambda & -30 \\ 6 & -13-\lambda \end{vmatrix}$$

$$= \Big( (11-\lambda)(-10-\lambda) - 18 \cdot -6 \Big) \begin{vmatrix} 14-\lambda & -30 \\ 6 & -13-\lambda \end{vmatrix} =$$

$$\begin{vmatrix} 11-\lambda & -6 \\ 18 & -10-\lambda \end{vmatrix} \cdot \begin{vmatrix} 14-\lambda & -30 \\ 6 & -13-\lambda \end{vmatrix}$$

$$(\lambda^2 - \lambda - 110 + 108) \cdot (\lambda^2 - \lambda \underbrace{- 13 \cdot 14 + 6 \cdot 30}_{-2}) =$$

$$= (\lambda^2 - \lambda - 2)(\lambda^2 - \lambda - 2) = (\lambda - 2)^2 (\lambda + 1)^2$$

$$\lambda = 2: \quad \begin{bmatrix} 9 & -6 & -3 & 6 \\ 18 & -12 & -6 & 12 \\ 0 & 0 & 12 & -30 \\ 0 & 0 & 6 & -15 \end{bmatrix} \begin{matrix} \text{proporc.} \\ \\ \text{proporc.} \end{matrix}$$

$$\begin{bmatrix} 3 & -2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -5 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -1 \\ -2 \\ 5 \\ 2 \end{bmatrix}$$

$$\lambda = -1: \quad \begin{bmatrix} 12 & -6 & -3 & 6 \\ 18 & -9 & -6 & 12 \\ 0 & 0 & 15 & -30 \\ 0 & 0 & 6 & -12 \end{bmatrix} \begin{matrix} \text{prop.} \\ \\ \text{prop.} \end{matrix}$$

$$\sim \begin{bmatrix} 4 & -2 & -1 & 2 \\ 0 & 0 & 1 & -2 \\ & & 0 & \end{bmatrix} +$$

$$v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} \qquad v_4 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Izracunaj: $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}^{100}$

$A = PDP^{-1} \qquad A^{100} = PD^{100}P^{-1}$

$$\begin{vmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 = (\lambda+1)(\lambda+2)$$

$$D = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \qquad v_1: \begin{bmatrix} +1 & 1 \\ -2 & -2 \end{bmatrix} : \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$v_2: \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} : \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \qquad P^{-1} = \frac{1}{-1} \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

$$A^{100} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2^{100} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2^{100} & -2^{100} \end{bmatrix} = \begin{bmatrix} 2 - 2^{100} & 1 - 2^{100} \\ -2 + 2^{101} & -1 + 2^{101} \end{bmatrix}$$

# Kvadratne forme

Opiši družine krivulj: $2x^2 + 2xy + 2y^2 = c$ $= q(x,y)$

$x^2 + 4xy + y^2 = c$

$$q(x,y) = \overset{= v^T}{[x,y]} \cdot A \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$A$: simetrična

$\Rightarrow \exists\, Q : QQ^T = I$, $D$:

$A = QDQ^T$ $(P = Q)$

$$q = v^T A v =$$
$$= v^T Q D Q^T v =$$
$$= (Q^T v)^T D \underset{\substack{\parallel \\ w}}{(Q^T v)} = w^T D w = d_1 w_1^2 + d_2 w_2^2$$

$w = (w_1, w_2)$

$$A = \begin{array}{c|cc} & x & y \\ \hline x & \underline{2} & 1 \\ y & 1 & \underline{2} \end{array}$$

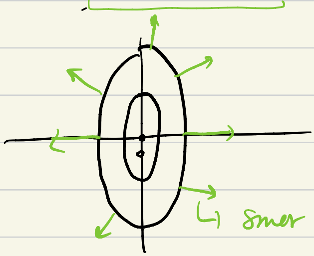
$\underline{\lambda_1 = 1}$: $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{2}}$

$\underline{\lambda_2 = +3}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{\sqrt{2}}$

$$Q = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}$$

$q(w_1, w_2) = \underline{w_1^2 + 3w_2^2} = c$ : elipse

$$\left(\frac{w}{1}\right)^2 + \left(\frac{w}{1/\sqrt{3}}\right)^2$$



↳ smer naraščanja: imam minimum

$$x^2 + 4xy + y^2 = c$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \qquad (1-\lambda)^2 - 4 = 0$$

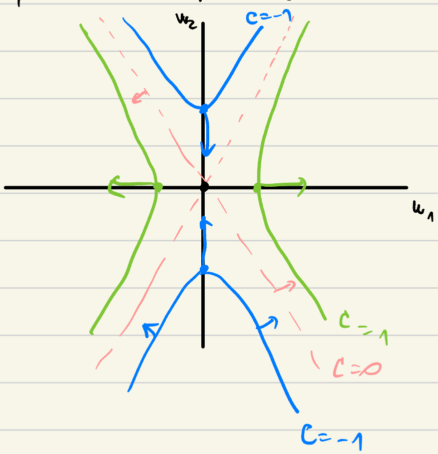
$$(\lambda-1)^2 = 4 \qquad \lambda - 1 = \pm 2$$

$$\lambda_1 = 1 + 2 = 3$$

$$\lambda_2 = 1 - 2 = -1$$

$$q(w_1, w_2) = 3 \cdot w_1^2 - w_2^2 = c$$

hiperbole




$c=0$: $3w_1^2 - w_2^2 = 0$

$$w_2 = \pm\sqrt{3} \cdot w_1$$

$c=1$ $\quad 3w_1^2 - w_2^2 = 1$

$$w_2^2 = 3w_1^2 - 1$$

$$w_2 = 0: \quad w_1 = \pm \frac{1}{\sqrt{3}}$$

$c=-1$: $3w_1^2 - w_2^2 = -1$

$$3w_1^2 + 1 = w_2^2$$

0 ni lokalni ekstrem, ampak sedlo.