Jordanova kanonična forma

A; λ: večkratna lastna vrednost, recimo k-kratna.
Lahko se zgodi, da je premalo lastnih vektorjev, torej da je jedro $\ker A-\lambda I$ dimenzije manj kot k.

Ideja: pogledamo $\ker (A-\lambda I)^2$.

l.v.

$$v_1 \xleftarrow{A-\lambda I} k_{11} \xleftarrow{A-\lambda I} k_{12} \qquad \text{veriga}$$

$$v_2 \xleftarrow{A-\lambda I} k_{21}$$

$$\vdots$$

$$\begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix} \quad a: \text{l.vr.} \quad A-aI = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{l.v.:} \overset{v_1}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} \longleftarrow \overset{k_1}{\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cancel{+ c \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}\right)} \longleftarrow \overset{k_2}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

$\in \ker A-aI$ $\qquad$ $\in \ker (A-aI)^2$ $\qquad$ $\in \ker (A-aI)^3$

$$\ker (A-aI)^2 = \ker \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \ker = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

$$(A-aI)k_1 = v_1$$
$$Ak_1 = v_1 + a\cdot k_1$$
$$(A-aI)k_2 = k_1$$
$$Ak_2 = k_1 + a\cdot k_2$$

$B = \{e_1, e_2, e_3\}$; $P = I$

$$Av_1 = av_1$$
$$Ak_1 = v_1 + a\cdot k_1$$

$$J = \begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$$

$$\begin{bmatrix} a & 1 & & & 0 \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ 0 & & & \ddots & 1 \\ & & & & a \end{bmatrix}$$

Jord. kletka, ki pripada eni verigi

$$\begin{bmatrix} a & 1 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

a: 3x l. vr.

$$A - aI = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v_1 \leftarrow k_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$v_2 \leftarrow$ // ne gre

$$\{v_1, k_1\}: \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} \qquad [a]$$

$$\left[\begin{array}{cc|c} a & 1 & 0 \\ 0 & a & 0 \\ \hline 0 & 0 & a \end{array}\right]$$

Zapiši Jordanko za

$$A = \left[\begin{array}{c|cc} 2-\lambda & 0 & 0 \\ \hline 0 & 2-\lambda & 1 \\ 1 & 0 & 2-\lambda \end{array}\right] \qquad \det(A-\lambda I) = (2-\lambda)^3$$

korenski v.

$$A - 2I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \quad k_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad k_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\tilde{k}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}; \quad \tilde{k}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A = PJP^{-1} \qquad P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \tilde{P} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A = \tilde{P} J \tilde{P}^{-1}$$

$$P: e_1 \to e_2 \to e_3$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$PJP^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

Zapiši Jordanko za

$$B = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 0 \\ -2 & 1 & 3 \end{bmatrix} \qquad \begin{matrix} \lambda_1 = 2 \; ; \\ \lambda_2 = 3 \end{matrix}$$

$$\lambda_1 = 2: \quad \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \qquad v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 3: \quad \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ -2 & 1 & 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad k_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

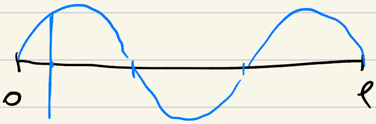
$$J = \left[\begin{array}{c|cc} 2 & 0 & 0 \\ \hline 0 & 3 & 1 \\ 0 & 0 & 3 \end{array}\right] \; ; \qquad P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$J = \left[\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 3 & 0 \\ \hline 0 & 0 & 2 \end{array}\right] \; ; \qquad P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$v_2 \quad k_1 \quad v_1$$

$$C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & -2 & 3 & 1 \\ 1 & -1 & 0 & 3 \end{bmatrix}$$

$$u_{tt} = c^2 (\overbrace{u_{xx} + u_{yy}}^{\Delta u}) \qquad u = X(x)\,Y(y)\,T(t)$$



$u_{tt} = c^2 u_{xx}$ val. enačba

$u(x,0) = f(x)$, $u_t(x,0) = g(x)$,

$u(0,t) = 0$, $u(l,t) = 0$: vpeta struna

$u(x,t) = X(x) \cdot T(t)$

amplituda

$u_{tt}(x,t) = X \cdot T''$

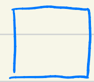
$u_{xx} = X'' \cdot T$

$X \cdot T'' = c^2 \cdot X'' T \qquad /: XTc^2$

$$\frac{T''}{c^2 T} = \frac{X''}{X} = \lambda$$

funkcija t — funkcija x

$$\frac{X''}{X} + \frac{Y''}{Y} = \lambda$$



za pravokotne opne

$D: X \to X''$: lin. psl. $\qquad X'' = \lambda X \iff DX = \lambda X$

<u>Problem</u>: reši $X'' = \lambda X$, na $[0,l]$, $X(0) = 0$, $X(l) = 0$.

(a) $\lambda = \mu^2 > 0 \qquad X'' = \mu^2 \cdot X \qquad$ vemo: rešitve $e^{\mu x}, e^{-\mu x}$

Določiti je treba μ, da je izpolnjen r-pogoj:

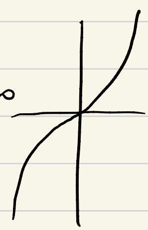
$X(x) = a \cdot e^{\mu x} + b \cdot e^{-\mu x}$

$X(0) = a + b = 0$

$X(l) = a \cdot e^{\mu l} - a \cdot e^{-\mu l} = 0$

$$\frac{e^{\mu l} - e^{-\mu l}}{2} = sh(\mu l)$$

$$2a \cdot \underbrace{\overbrace{sh(\mu l)}^{\neq 0}}_{\neq 0} = 0 \Rightarrow a = 0 \Rightarrow b = 0$$



(b) $\lambda = 0$: $\qquad X'' = 0$: $\quad X(x) = a + bx$

$X(0) = 0 \Rightarrow a = 0$

$X(l) = 0 \Rightarrow b = 0$

(c) $\underbrace{\lambda = -\mu^2} = (i\mu)^2 \rightsquigarrow e^{i\mu x}, e^{-i\mu x}$ $\qquad \frac{X''}{X} = -\mu^2$

$e^{i\mu x} = \cos\mu x + i\sin\mu x, \quad e^{-i\mu x} = \cos\mu x - i\sin\mu x$

$$J = \begin{bmatrix} -\left(\frac{\pi}{l}\right)^2 & & \\ & -\left(\frac{2\pi}{l}\right)^2 & \\ & & \ddots \end{bmatrix}$$

$$X(x) = \cancel{a\cos\mu x} + b\sin\mu x$$

$$X(0) = a = 0$$

$$X(l) = b\sin\mu\cdot l = 0 \qquad (b \neq 0!)$$

$$\sin\mu l = 0 \Rightarrow \mu l = k\pi \Rightarrow \underbrace{\mu_k = \frac{k\pi}{l}}$$

$$X_k(x) = \sin\frac{k\pi}{l}\cdot x \qquad \lambda_k = -\mu_k^2 = -\left(\frac{k\pi}{l}\right)^2$$

$$\frac{T_k''}{c^2 T_k} = -\left(\frac{k\pi}{l}\right)^2 \qquad \frac{T_k''}{T_k} = -\left(\frac{k\pi c}{l}\right)^2$$

$$T_k(t) = a_k\cdot\cos\left(\frac{k\pi c}{l}t\right) + b_k\sin\left(\frac{k\pi c}{l}\cdot t\right)$$

z konst!

lastna nihanja: $u_k(x,t) = X_k(x)\,T_k(x)$

$\forall$ rešitev lahko zapišemo kot $u(x,t) = \sum_0^\infty T_k(x)\cdot X_k(x)$

$$u(x,0) = \sum_0^\infty \sin\frac{k\pi}{l}x \cdot a_k = f(x) \qquad \text{Fourierova sinusna vrsta}$$

$$u_t(x,0) = \sum_0^\infty \underbrace{\sin\frac{k\pi}{l}\cdot x}\cdot b_k\cdot\frac{k\pi c}{l} = g(x)$$

↳ koef. razvoja g v Four. vrsto

Skal. prod: $\int_0^\pi f(x)g(x)\,dx$: skal. produkt

$$\int_0^\pi \underset{v_k}{\sin\frac{k\pi}{l}x}\,\sin\frac{m\pi}{l}x = 0 \text{ za } k\neq m$$

$$f = \sum v_k\cdot a_k$$

$$\langle f, v_n\rangle = \sum a_k\langle v_k, v_n\rangle = a_n\langle v_n, v_n\rangle \Rightarrow a_n$$