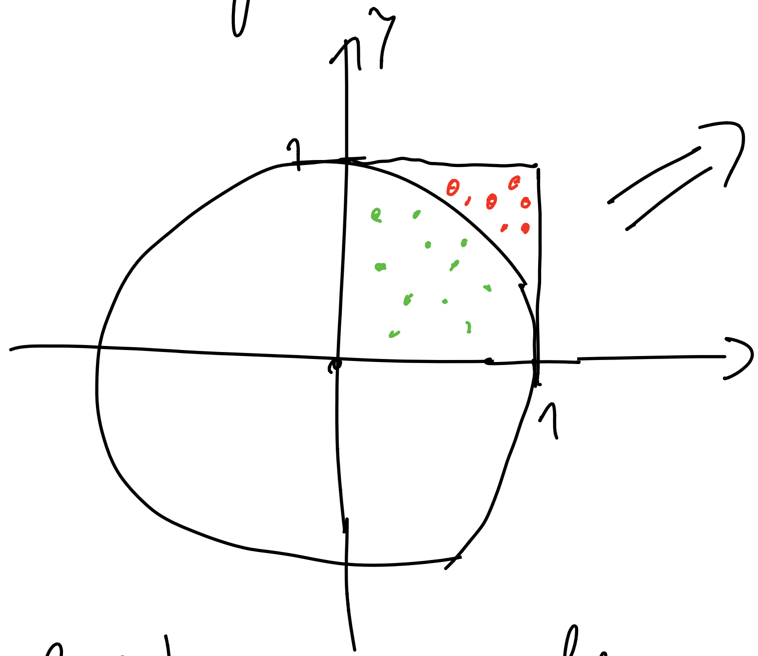
Računanje števila π

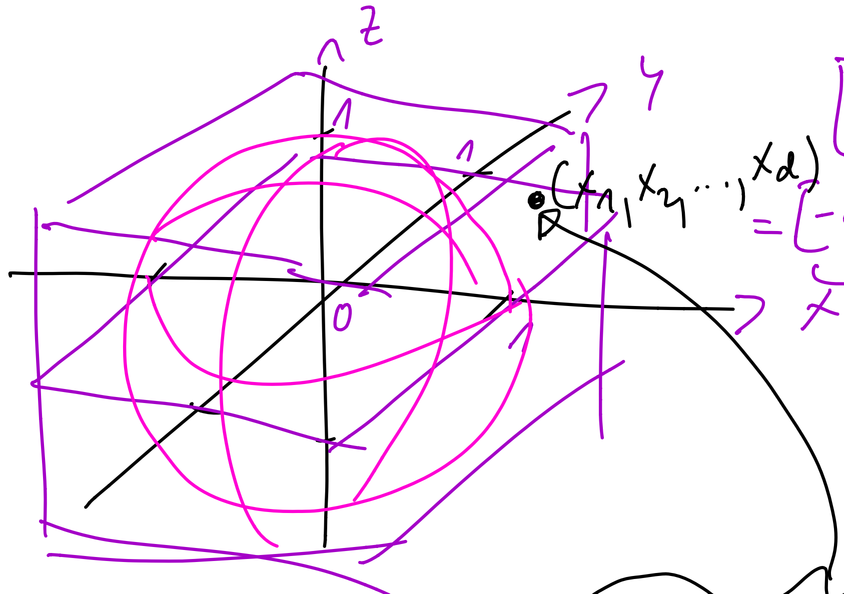


$$\Longrightarrow \quad \pi \approx \frac{4K}{N}$$

K: št. zelenih

N: št. vseh

Posplošitev na dim. d



$$[-1,1]^d = \underbrace{[-1,1] \times \dots \times [-1,1]}_{d\text{-krat}}$$

Naključno izbiramo $(x_1, x_2, \dots, x_d) \in [-1,1]^d$

Naj bo N št. vseh izbranih d-teric, K pa je število tistih, za katere je

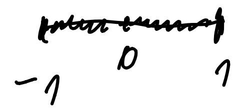
$$x_1^2 + x_2^2 + \dots + x_d^2 \le 1 \quad (\text{točka je v krogli})$$

Ideja: $\dfrac{V_{krogle}}{V_{kocke}} \approx \dfrac{K}{N}$

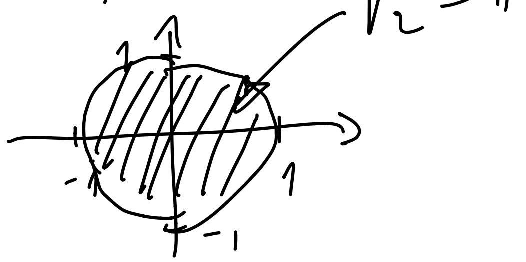
$V_{kocke} = 2^d$

$V_{krogle} = \,? =: V_d$

$d = 1: \quad V_1 = 2$



$d = 2: \quad V_2 = \pi$



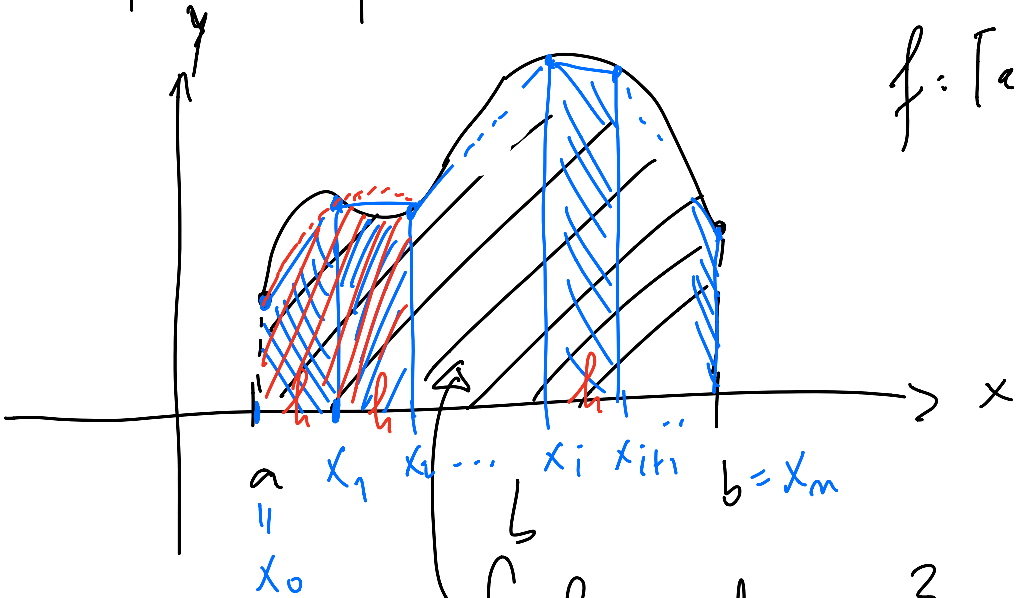
$d = 3: \quad V_3 = \dfrac{4\pi}{3}$

$d = 4: \quad ?$

Izkaže se: $V_d = \dfrac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}+1\right)} \Rightarrow$

$$\pi \approx \left(2^d \, \Gamma\left(\frac{d}{2}+1\right) \frac{K}{N}\right)^{\frac{2}{d}}$$

Trapezno pravilo



$f : [a,b] \to \mathbb{R}$

$$\int_a^b f(x)\,dx = ?$$

$$\int_a^b f(x)\,dx \approx \sum_{i=0}^{n-1} \underbrace{\text{ploščina } (i+1)\text{-ga trapeza}}_{S_{i+1}}$$

$$S_{i+1} = \frac{f(x_i) + f(x_{i+1})}{2} \cdot \underbrace{(x_{i+1} - x_i)}_{h}$$

$$= \frac{h}{2}\left(f(x_i) + f(x_{i+1})\right); \text{ Trapezna formula}$$

$$\int_a^b f(x)\,dx \approx \sum_{i=0}^{n-1} S_{i+1}$$

$f(x_i) =: f_i$

$$= \sum_{i=0}^{n-1} \frac{h}{2}\left(f(x_i) + f(x_{i+1})\right) =$$

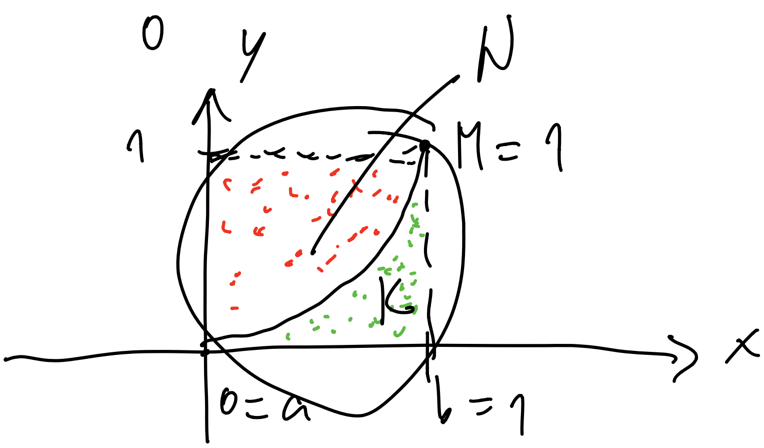
$$= \sum_{i=0}^{n-1} \frac{h}{2}\left(f_i + f_{i+1}\right)$$

$$= \underbrace{h\left(\frac{1}{2} f_0 + f_1 + f_2 + \ldots + f_{n-1} + \frac{1}{2} f_n\right)}_{\text{Trapezno pravilo!}}$$

<u>Primer</u>: Statistično bi radi izračunali

$$\int_0^1 x^2 \, dx .$$

Točno: $\int_0^1 x^2 \, dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$



$f(x) = x^2$

$M = 1$

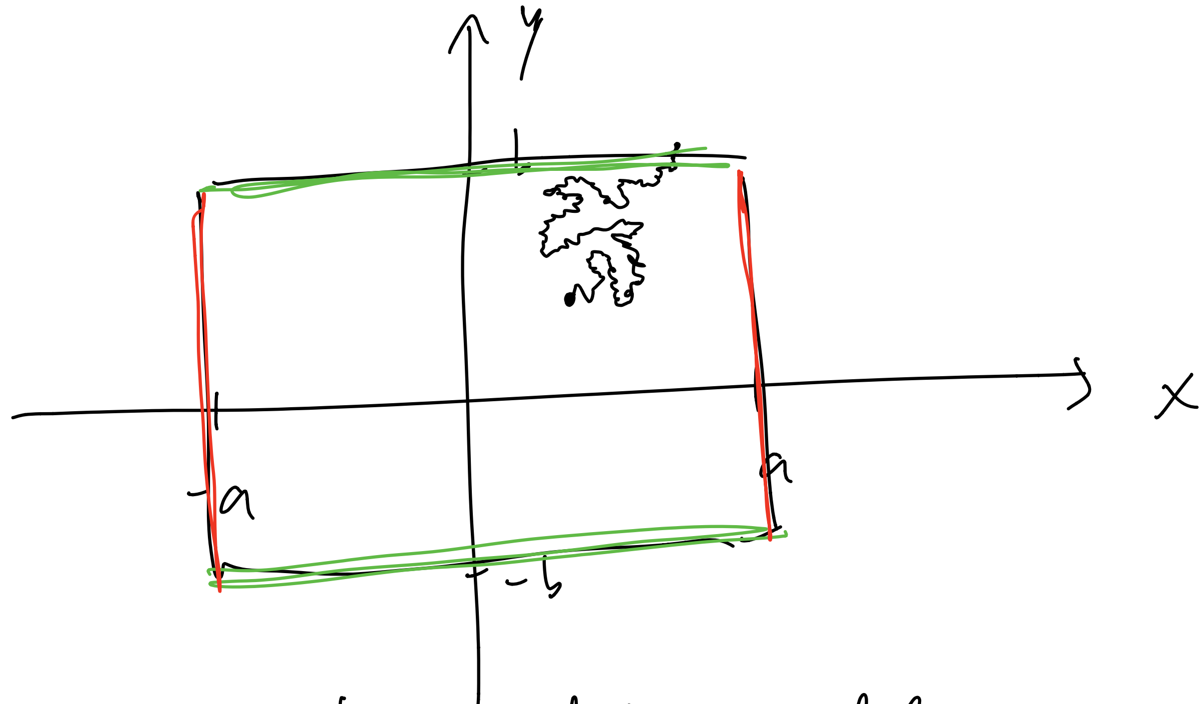
$$\int_a^b f(x)\,dx \approx \frac{K}{N}\cdot M(b-a)$$

K – število izžrebanih točk po grafom funk. f

N – število vseh točk

V našem primeru: $\int_0^1 x^2\,dx \approx \frac{K}{N}\cdot 1\cdot 1 = \frac{K}{N}$

Naključni sprehod delca



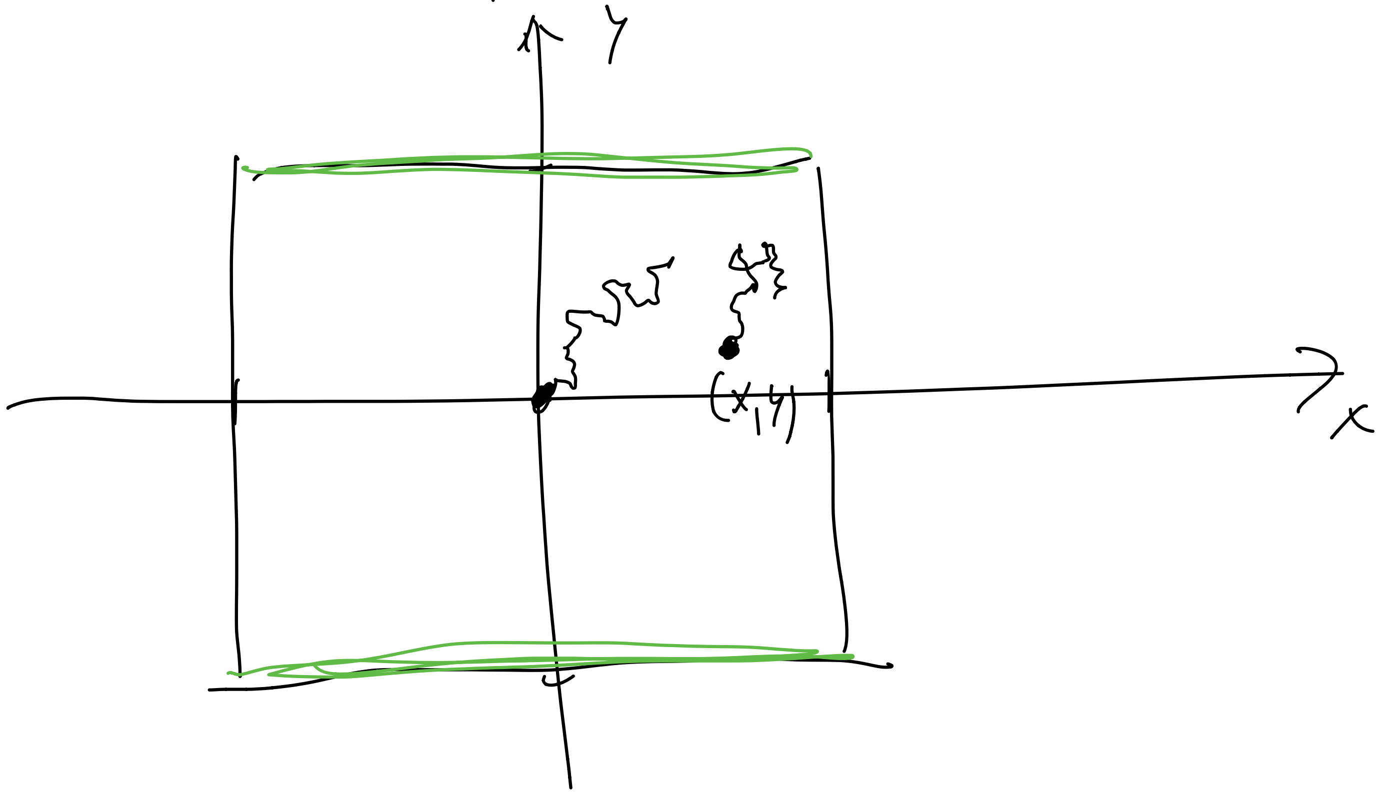
Brownovo gibanje delca: delec se v vsakem trenutku premakne infinitezimalno malo v poljubno (naključno) izbrano smer!

Kolikšna je verjetnost, da delec zapusti pravokotnik preko stranic:

$[-a, a] \times \{-b\}$ ali $[-a, a] \times \{b\}$

<u>Poseben primer</u>: $a = b = 1$ in

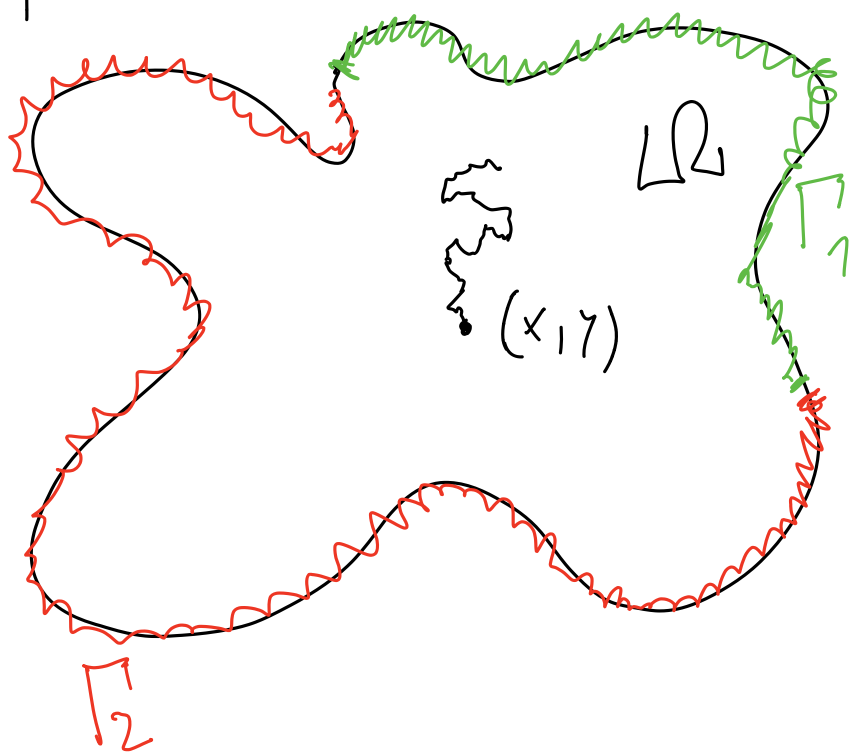
delec začne pot v $(0, 0)$:



Verjetnost je $1/2$!

Splošno:

$\Omega \subseteq \mathbb{R}^2$



Zanima nas verjetnost, da delec zapusti Ω skozi Γ_1.

Izkaže se, da je verjetnost enaka $u(x,y)$, kjer je $u : \Omega \to \mathbb{R}$ rešitev

$$\boxed{\begin{aligned} \Delta u &= 0 \\ u\big|_{\Gamma_1} &= 1 \\ u\big|_{\Gamma_2} &= 0 \end{aligned}}$$

Dirichletov problem (robni problem za PDE).