$$\int \frac{x^2}{(x-1)^{100}}\,dx$$ SUBSTITUCIJA

uvedemo novo spremenljivko $\quad t = x-1$
$\qquad\qquad\qquad\qquad\qquad\quad dt = dx$

$$\int \frac{x^2}{(x-1)^{100}}\,dx = \int \frac{(t+1)^2}{t^{100}}\,dt = \int \frac{t^2+2t+1}{t^{100}}\,dt =$$

$$= \int \frac{dt}{t^{98}} + 2\int \frac{dt}{t^{99}} + \int \frac{dt}{t^{100}} =$$

$$= \frac{t^{-97}}{-97} + 2\,\frac{t^{-98}}{-98} + \frac{t^{-99}}{-99} + C =$$

$$= \frac{(x-1)^{-97}}{-97} + 2\,\frac{(x-1)^{-98}}{-98} + \frac{(x-1)^{-99}}{-99} + C$$

$$\int \frac{dx}{\sqrt{x}\,(1+\sqrt[3]{x})}$$

uvedemo $\quad t := \sqrt[6]{x}$

$$dt = \frac{1}{6}\,x^{-\frac{5}{6}}\,dx$$

$$dx = 6\,x^{\frac{5}{6}}\,dt = 6\,t^5\,dt$$

$$\int \frac{dx}{\sqrt{x}\,(1+\sqrt[3]{x})} = \int \frac{6t^5}{t^3(1+t^2)}\,dt =$$

$$= 6\int \frac{t^2}{1+t^2}\,dt =$$

$$= 6\left(\int \frac{t^2+1}{t^2+1}\,dt - \int \frac{dt}{t^2+1}\right) =$$

$$= 6\left(t - \operatorname{arctg}(t)\right) + C =$$

$$= 6\sqrt[6]{x} - \operatorname{arctg}\left(\sqrt[6]{x}\right) + C$$

$$\int x e^{x^2} dx$$

$t = x^2$

$dt = 2x\,dx$

$$\int x e^{x^2} dx = \frac{1}{2}\int e^t dt = \frac{1}{2} e^t + C = \frac{1}{2} e^{x^2} + C$$

PER PARTES

$$\int x e^x dx$$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$$\int x e^x dx = x e^x - \int e^x dx =$$

$$= x e^x - e^x + C$$

$$\int e^x \cos x \, dx$$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \qquad v = e^x$

$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx = \cancel{*}$$

$$\int e^x \sin x = e^x \sin x - \int e^x \cos x \, dx$$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x \qquad v = e^x$

$$* = e^x (\sin x - \cos x) + \int e^x \cos x \, dx$$

$$\Rightarrow \int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x)$$

$$\int x^a \ln x \, dx$$

- $a \neq -1$

$u = \ln x \qquad dv = x^a dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^{a+1}}{a+1}$

$$\int x^a \ln x \, dx = \ln x \frac{x^{a+1}}{a+1} - \int \frac{x^a}{a+1} dx =$$

$$= \ln x \frac{x^{a+1}}{a+1} - \frac{x^{a+1}}{(a+1)^2} + C$$

$\cdot\ a = -1$

$$\int \frac{\ln x}{x}\,dx = \int t\,dt = \frac{(\ln x)^2}{2} + C$$

$t = \ln x$

$dt = \frac{dx}{x}$

$$\int x^3 e^{\frac{x}{2}}\,dx = \frac{1}{2^4}\int t^3 e^t\,dt = \star$$

$t = \frac{x}{2}$

$dt = \frac{dx}{2}$

$$\int t^3 e^t\,dt = t^3 e^t - 3\int t^2 e^t =$$

$u = t^3$ $\quad dv = e^t\,dt$

$du = 3t^2\,dt$ $\quad v = e^t$

$u = t^2$ $\quad dv = e^t\,dt$

$du = 2t\,dt$ $\quad v = e^t$

$$= t^3 e^t - 3\left(t^2 e^t - 2\int t e^t\,dt\right) =$$

$$= t^3 e^t - 3t^2 e^t + 6\left(t e^t - e^t\right) + C$$

glej zg. primer

$$* = \frac{1}{16} e^{\frac{x}{2}} \left(\left(\frac{x}{2}\right)^3 - 3\left(\frac{x}{2}\right)^2 + 6\left(\frac{x}{2}\right) - 1 \right) + C$$

$$\int x^5 \sqrt{x^3+1}\, dx = *$$

$$u = x^3 \qquad dv = x^2 \sqrt{x^3+1}\, dx$$

$$du = 3x^2 dx \qquad v = \frac{2}{9} (x^3+1)^{\frac{3}{2}}$$

$$\int x^2 \sqrt{x^3+1}\, dx = \frac{1}{3} \int \sqrt{t}\, dt = \frac{1}{3} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$t = x^3 + 1$$

$$dt = 3x^2 dx$$

$$= \frac{2}{9} (x^3+1)^{\frac{3}{2}} + C$$

$$* = \frac{2}{9} x^3 (x^3+1)^{\frac{3}{2}} - \frac{2}{3} \int x^2 (x^3+1)^{\frac{3}{2}} dx = \ast\ast$$

$$\int x^2 (x^3+1)^{3/2} dx = \frac{1}{3} \int t^{\frac{3}{2}} dt = \frac{1}{3} \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$t = x^3 + 1$$

$$dt = 3x^2 dx$$

$$= \frac{2}{15} (x^3+1)^{\frac{5}{2}} + C$$

$$\cancel{\oplus}\ \cancel{\oplus} = \frac{2}{9} x^3 (x^3+1)^{3/2} - \frac{4}{45} (x^3+1)^{5/2} + C$$

RAZCEP NA PARCIALNE ULOMKE

$$\int \frac{x-2}{(x+1)(x-1)^2}\,dx$$

$$\frac{x-2}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} =$$

$$= \frac{A\overbrace{(x-1)^2}^{x^2-2x+1} + B\overbrace{(x-1)(x+1)}^{x^2-1} + C(x+1)}{(x+1)(x-1)^2}$$

$$= \frac{x^2(A+B) + x(-2A+C) + (A-B+C)}{(x+1)(x-1)^2}$$

$$A + B = 0 \quad \Rightarrow \quad B = -A$$

$$-2A + C = 1 \quad \Rightarrow \quad C = 1 + 2A$$

$$A - B + C = 2 \quad \Rightarrow \quad 2A + 1 + 2A = 2$$

$$4A = 1$$

$$A = \frac{1}{4} \quad \Rightarrow \quad B = -\frac{1}{4}$$

$$C = \frac{3}{2}$$

$$\int \frac{x-2}{(x+1)(x-1)^2}\,dx = \frac{1}{4}\int \frac{dx}{x+1} - \frac{1}{4}\int \frac{dx}{x-1} + \frac{3}{2}\int \frac{dx}{(x-1)^2}$$

$$= \dots$$

$$\int \frac{x^3+3}{x^2-2x-3}\,dx$$

$(x^3+3):(x^2-2x-3) = x+2$, ostanek $= 7x+9$

$$\begin{array}{l} -(x^3-2x^2-3x) \\ \hline 2x^2+3x+3 \\ -(2x^2-4x-6) \\ \hline 7x+9 \end{array}$$

$$\int \frac{x^3+3}{x^2-2x-3}\,dx = \int (x+2)\,dx + \int \frac{7x+9}{x^2-2x-3}\,dx =$$

$$\frac{7x+9}{x^2-2x-3} = \frac{A}{x-3} + \frac{B}{x+1} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

$$= \frac{x(A+B)+(A-3B)}{(x-3)(x+1)}$$

$$x^2-2x-3 = (x-3)(x+1)$$

$A+B=7 \qquad B=7-A$

$A-3B=9$

$$A - 3(7 - A) = 9$$

$$-2A - 21 = 9$$

$$-2A = 30$$

$$\underline{\underline{A = -15}} \qquad B = 22$$

$$= \int x+2 \, dx - 15 \int \frac{dx}{x-3} + 22 \int \frac{dx}{x+1} = \ldots$$

$$\int \frac{e^x - 1}{e^x + 1} \, dx = \int \frac{t-1}{t(t+1)} \, dt = \underline{\underline{-\int \frac{dt}{t} + 2 \int \frac{dt}{t+1}}}$$

$$t = e^x$$

$$dt = e^x \, dx$$

$$dx = \frac{dt}{t}$$

$$\frac{t-1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} = \frac{A(t+1) + Bt}{t(t+1)}$$

$$= \frac{t(A+B) + A}{t(t+1)}$$

$$A = -1$$

$$A + B = 1 \Rightarrow B = 2$$

o..a o

$$\int \frac{dx}{\left(\sqrt[3]{x} - \sqrt[4]{x}\right)^2} =$$

$$t = \sqrt[12]{x}, \quad x = t^{12}$$

$$dx = 12t^{11}\,dt$$

$$\int \frac{12t^{11}}{\left(t^4 - t^3\right)^2}\,dt = \int \frac{12t^{11}}{t^6\,(t-1)^2} =$$

$$= \int \frac{12t^5}{(t-1)^2} =$$

$$= 12\int \frac{(u+1)^5}{u^2}\,du =$$

$$u = t-1$$

$$du = dt$$