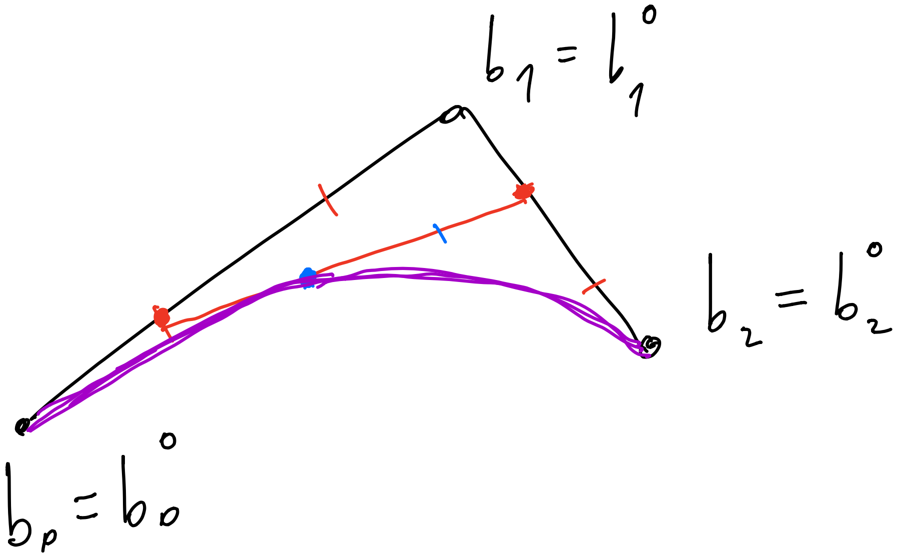
Bézierjeve krivulje

de Casteljauov algoritem

$b_0, b_1, b_2 \in \mathbb{R}^2, \quad t \in [0,1]$



$$b_0^1(t) = (1-t)\, b_0 + t\, b_1$$
$$b_1^1(t) = (1-t)\, b_1 + t\, b_2$$
$$b_0^2(t) = (1-t)\, b_0^1(t) + t\, b_1^1(t)$$

Račun: $b_0^2(t) = (1-t)\big((1-t)\, b_0 + t\, b_1\big)$

$$+ t\big((1-t)\, b_1 + t\, b_2\big)$$

$$= \underbrace{(1-t)^2 b_0 + 2t(1-t)\, b_1 + t^2 b_2}_{\text{parametrična krivulja}}$$

<u>Algoritem</u>: Podatki: $b_0, b_1, \dots, b_n \in \mathbb{R}^d$ (ponavadi $d = 2, 3$), $t \in [0,1]$

Definiramo: $b_j^0(t) = b_j$, $j = 0, 1, \dots, n$.

Ponavljamo:

$$b_j^k(t) = (1-t)\, b_j^{k-1}(t) + t\, b_{j+1}^{k-1}(t),$$

$$k = 1, 2, \dots, n; \quad j = 0, 1, \dots, n-k.$$

Izhod: $b^n(t) = b_0^n(t)$ je točka na Bézierjevi krivulji pri param. t.

$$\begin{array}{cccc}
b_0^0 \overset{(1-t)}{\searrow} & & & \\
b_1^0 \xrightarrow{t} & b_0^1 & & \\
b_2^0 & b_1^1 & \ddots & \\
\vdots & \vdots & \ddots & b_0^{n-1} \\
b_n^0 & b_{n-1}^1 & \cdots & b_1^{n-1} \quad b_0^n
\end{array}$$

Bernsteinova oblika Bézierjeve krivulje

Bernsteinov polinom:

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}, \quad i=0,1,\dots,n.$$

Lastnosti: $t \in [0,1]$

① $B_i^n(t) \geq 0$.

② Baza prostora pol. st. $\leq n$.

③ $B_i^n(t) = B_{n-i}^n(1-t)$

④ Particija enote:

$$\sum_{i=0}^{n} B_i^n(t) = 1, \quad \forall t$$

⑤ $B_i^n(t) = (1-t) B_i^{n-1}(t) + t B_{i+1}^{n-1}(t)$

⋮

glejte skripto

Izrek: Naj bodo $b_j \in \mathbb{R}^d$, $j = 0, 1, \dots, n$, kontrolne točke Bézierjeve krivulje b^n.

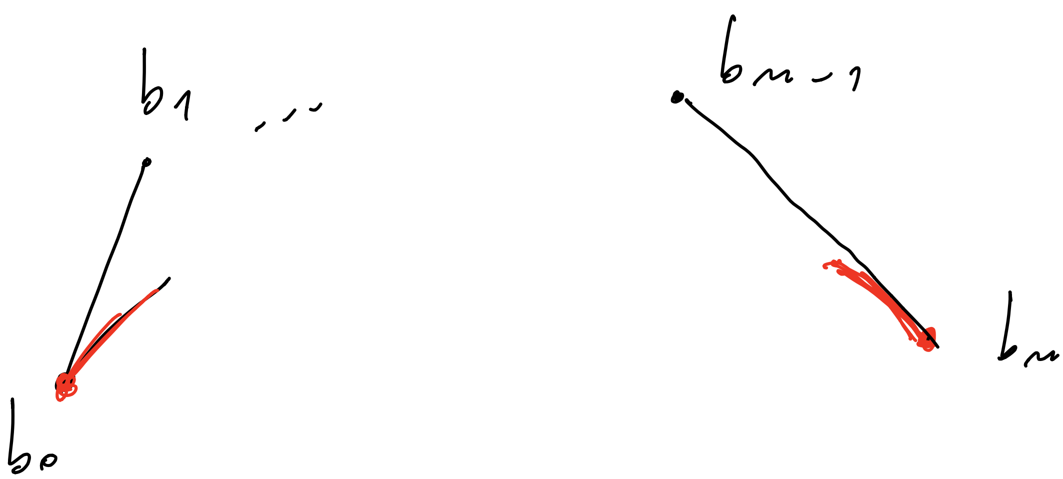
Potem je

$$b^n(t) = \sum_{j=0}^{n} B_j^n(t)\, b_j, \quad t \in [0,1].$$

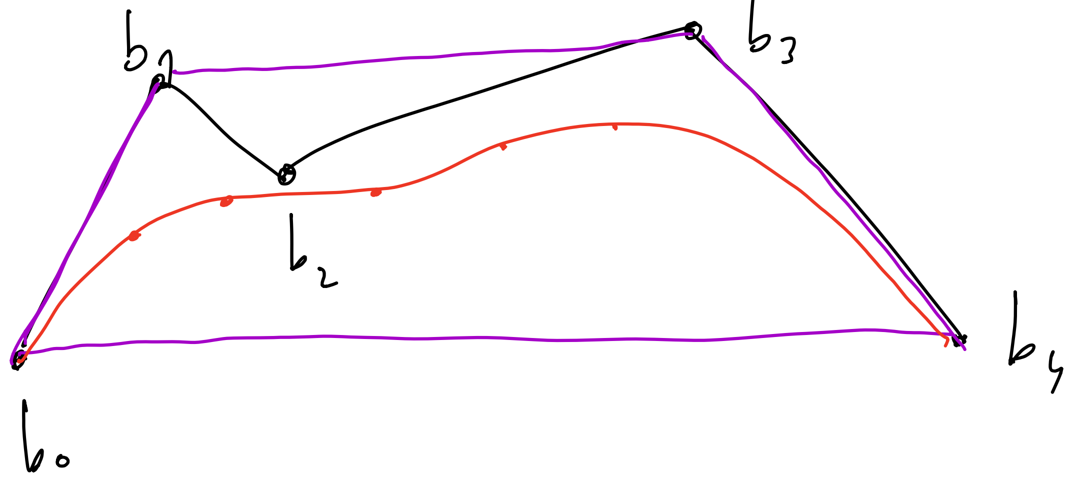
Lastnosti krivulje b^n:

(1) Krivulja je afino invariantna.

(2) $b^n(0) = b_0$ in $b^n(1) = b_n$



(3) Krivulja leži v konveksni ovojnici $b_0, b_1, \dots, b_n$.



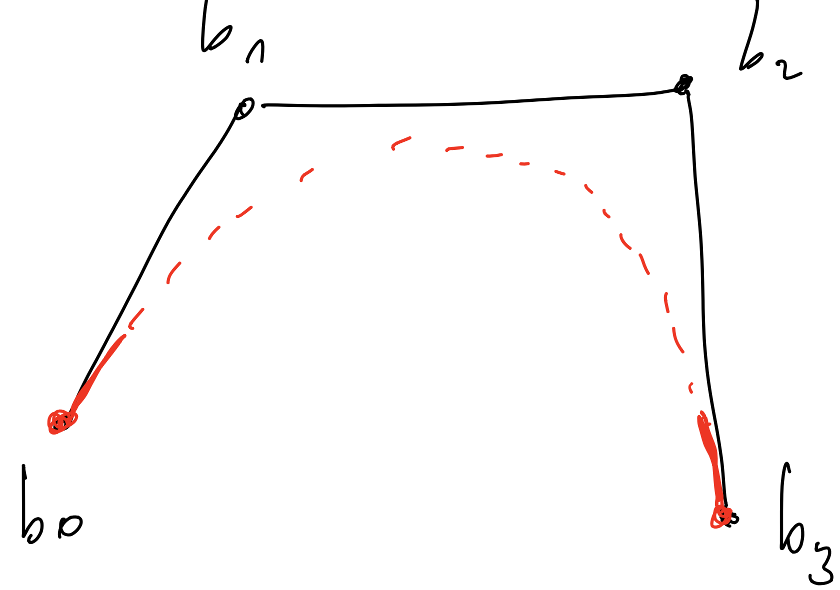
(4) $$\frac{d^r b^n}{dt^r}(t) = n(n-1)\dots(n-r+1)\cdot \sum_{j=0}^{n-r} B_j^{n-r}(t)\,\Delta^r b_j ,$$

$\Delta b_j = b_{j+1} - b_j$ și $\Delta^r b_j = \Delta(\Delta^{r-1}) b_j$

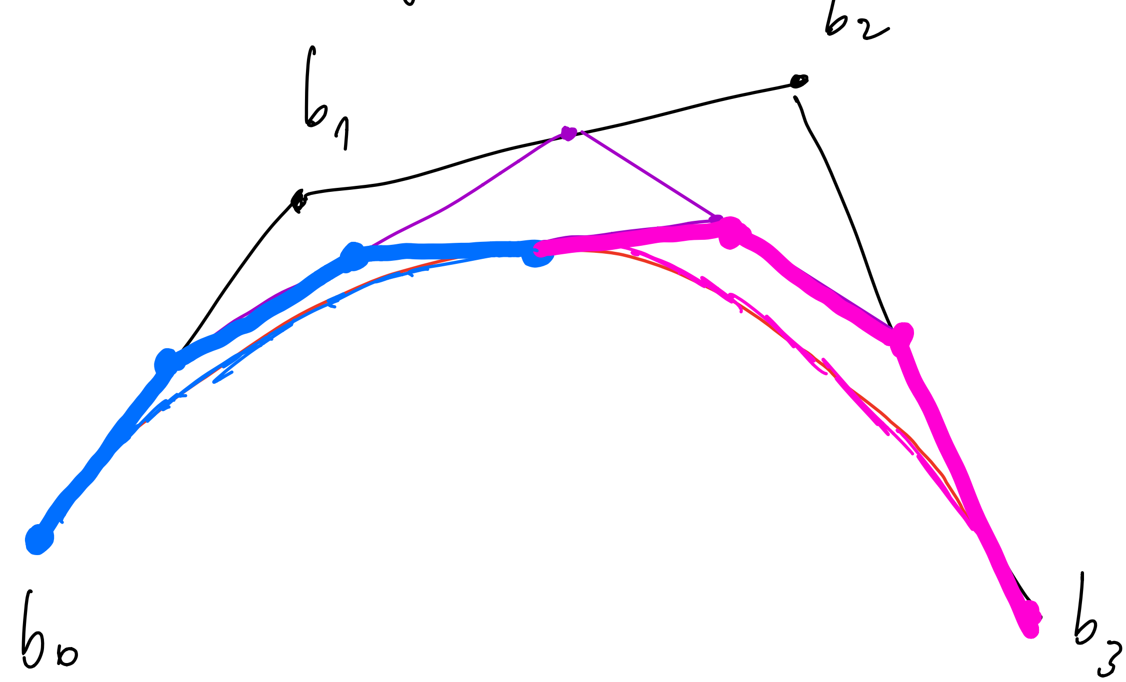
$r=1$. $$\frac{d b^n}{dt}(t) = n \sum_{j=0}^{n-1} B_j^{n-1}(t)\,(b_{j+1} - b_j)$$

$$\frac{d b^n}{dt}(0) = n\cdot (b_1 - b_0)$$

$$\frac{d b^n}{dt}(1) = n\cdot (b_n - b_{n-1})$$



Subdivizija :



$t = 1/2$ mi izvedemo de Castiljanov alg.