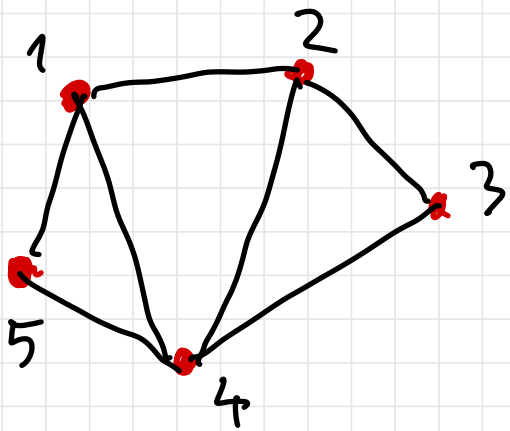
DS2 VAJE 1

1. DAN JE GRAF $G = (V, E)$

$V = \{1, 2, 3, 4, 5\}$

$E = \{ \{1,2\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{3,4\}, \{4,5\} \}$

a) GRAF G ČIM LEPŠE NARIŠITE.



b) POIŠČI STOPNJE VOZLIŠČ

$deg(1) = 3$ $deg(2) = 3$ $deg(3) = 2$

$deg(4) = 4$ $deg(5) = 2$

$\delta(G) = 2$ — min. st. v G

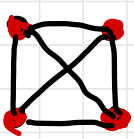
$\Delta(G) = 4$ — maks. st. v G

2. GRAF ŠAHOVSKEGA KRALJA H_n

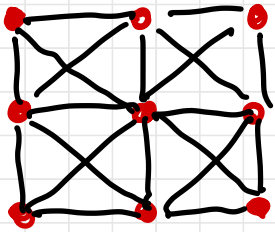
VOZLIŠČA: POLJA $n \times n$ ŠAHOVNICE

POVEZAVE: DVE POLJI POVEZAM, ČE SE LAHKO ŠAHOVSKI KRALJ PREMAKNE IZ ENEGA V DRUGEGA

a) NARIŠI H_2, H_3



H_2



H_3

b) KOLIKO VOZLIŠČ IN KOLIKO POVEZAV IMA H_n?

$|V(H_n)| = n^2$

$$|E(H_n)| = \underbrace{n\cdot(n-1)}_{\text{"VODORAVNE"}} + \underbrace{n\cdot(n-1)}_{\text{"NAPRIČ"}} + \underbrace{(n-1)\cdot 2}_{\text{"DIAGONALNE"}}$$

$$\sum_{v\in V} \deg(v) = 2\cdot|E| \quad \Leftarrow \text{LEMA O ROKOVANJU}$$

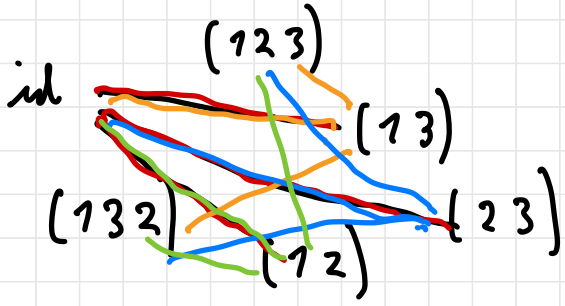
$$2\cdot|E| = 3\cdot 4 + 5\cdot\left(4\cdot(n-2)\right) + 8\cdot(n-2)^2$$

3. n - PERMUTACIJSKI GRAF

VOZLIŠČA : VSE PERMUTACIJE S_n

POVEZAVE : π_1 in π_2 POVEZANI, ČE OBSTAJA TRANSPOZICIJA τ, DA VELJA $\pi_1 = \tau\cdot\pi_2$

a) NARIŠI 3-PERM. GRAF.



1) $\pi_2 = id$

$\pi_1 = \tau\cdot id$

2) $\pi_2 = (13)$

$\pi_1 = \tau\cdot(13)$

2) SOSEDI (77)

$\pi_1 = (72)(73) = (123)$

$\pi_2 = (23)(13) = (213) = (132)$

$\pi = (13) \cdot (13) = id$

3) SOSEDI (23)

$\dots$ (123)

$\vdots$ (132)

(id)

b) ALI JE n-PERM. GRAF REGULAREN?

π : $deg(\pi)$

SOSEDI: $\pi_1 = \tau_1 \cdot \pi$

$\pi_2 = \tau_2 \cdot \pi$

$\pi_3 = \tau_3 \cdot \pi$

$\vdots$

} TOLIKO KOT JE TRANSPOZICIJ

$\frac{n \cdot (n-1)}{2} = \binom{n}{2}$

$\Rightarrow$ DA GRAF JE $\frac{n \cdot (n-1)}{2}$ - REGULAREN

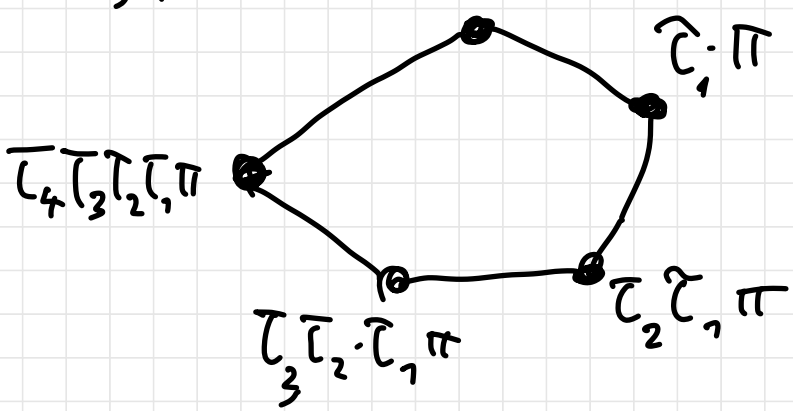
c) KOLIKO IMA VOZLIŠČ ($\sim$ POVEZAV)?

$$|V| = n!$$

$$|E| = \frac{\sum_{v \in V} \deg(v)}{2} = \frac{\frac{n \cdot (n-1)}{2} \cdot n!}{2}$$

d) ALI IMA n-PERM. GRAF 5-CIKEL KOT PODGRAF.

$$\tau_5 \tau_4 \tau_3 \tau_2 \tau_1 \pi = \pi$$



ČE OBSTAJA 5-CIKEL POTEM OBSTAJAJO $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$ DA VELJA:

$$\tau_5 \cdot \tau_4 \cdot \tau_3 \cdot \tau_2 \cdot \tau_1 = id$$

LICNA PERMUTACIJA

SODA PERMUTACIJA

4. 13 LJUDI, VSAK PRINESEL 3 DARILA, KI BI SI JIH IZMENJA S 3 DRUGIMI. ALI JE TO SPLOH MOGOCE ?

ISCEMO GRAF NA 13 VOZLISCIH, 3-REGULAREN.

$$2 \cdot |E| = \sum_{v \in V} deg(v)$$

$$= 13 \cdot 3$$

$$\Rightarrow \frac{39}{2} = |E| \quad \not\in$$

$\Rightarrow$ TAK GRAF NE OBSTAJA

5. $k < n$

KDAJ OBSTAJA K-REGULAREN GRAF NA n VOZLIŠČIH?

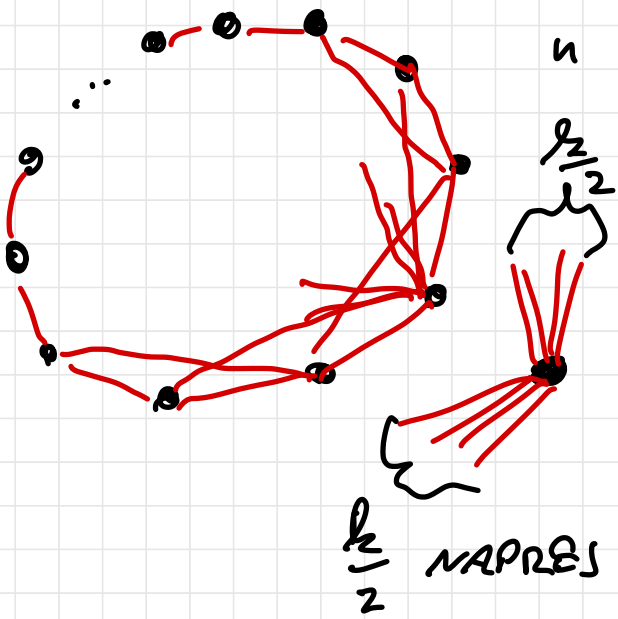
- ČE OBA LIHA

$$2 \cdot |E| = n \cdot k \quad \not\rightarrow\!\leftarrow$$

k LIHO

NE OBSTAJA

- k SOD:



n VOZLIŠČ

$\frac{k}{2}$ POVEZAV NAZAJ

$\frac{k}{2}$ NAPREJ

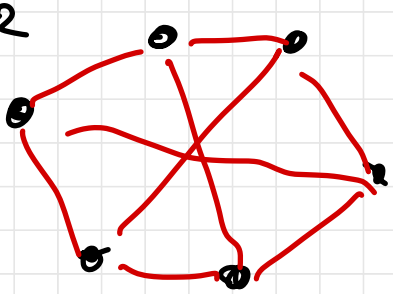
OBSTAJA

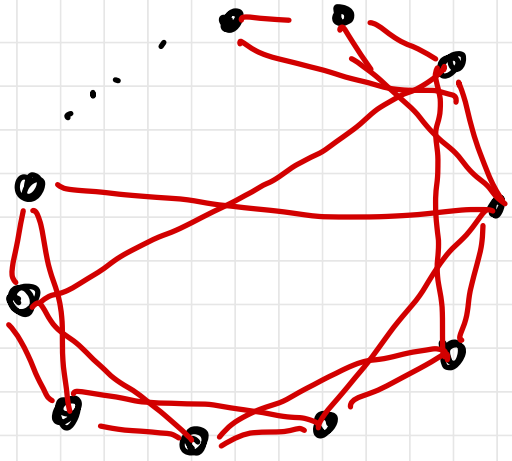
• k LIH, n SOD

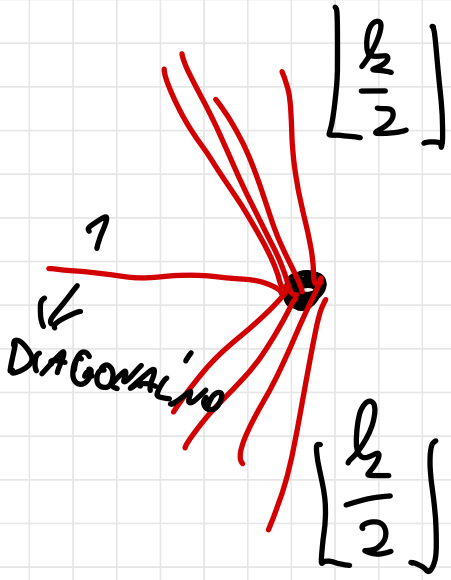
PRIMER

$k = 3$

$n = 6$







OBSTAJAJO.

6. G GRAF NA n VOZLIŠČIH IN m POVEZAVAH. KOLIKO IMA VPETIH PODGRAFOV IN KOLIKO INDUCIRANIH?

VPETI: 2^m

INDUCIRANIH: 2^n

KOLIKO JE VSEH PODGRAFOV K_n?

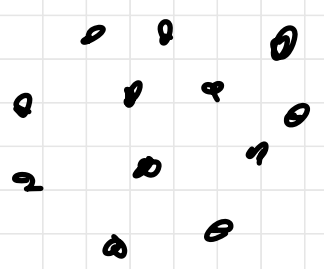
$$\sum_{k=0}^{n} \binom{n}{k} \cdot 2^{\binom{n-k}{2}}$$

↑ ODSTRANIMO k VOZLIŠČ

⇒ OSTANE $\frac{(n-k)\cdot(n-k-1)}{2} = \binom{n-k}{2}$ POVEZAV

ZA $n=10$

VPETIH $2^{\binom{10}{2}} = 2^{45}$



7. ZAPOREDJE $(d_1, d_2, \dots, d_n)$ JE GRAFOVSKO, ČE OBSTAJA GRAF, KI IMA STOPNJE $d_1, d_2, \dots, d_n$.

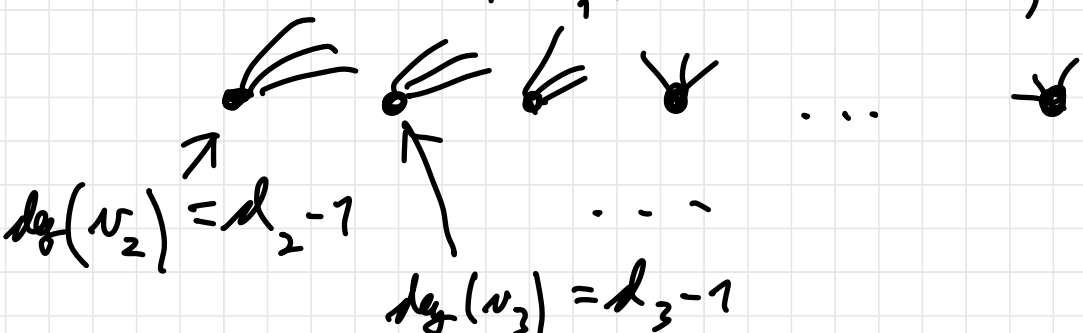
NAJ BO $n \geq 2$, $d_1 \geq 1$, $d_1 \geq d_2 \geq \dots \geq d_n$.

ZAP. $(d_1, \dots, d_n)$ JE GRAFOVSKO NATANKO TEDAJ, KO JE GRAFOVSKO ZAP.

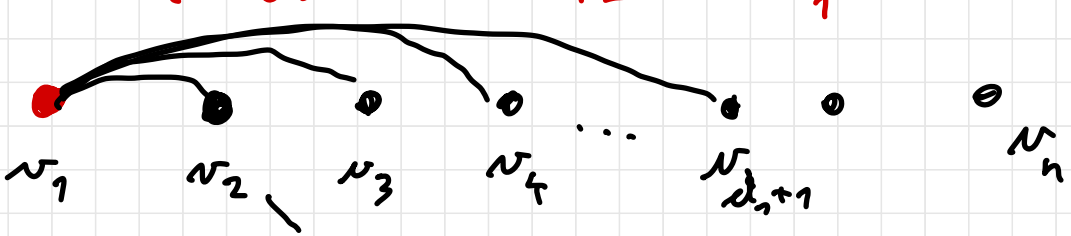
$$(d_2 - 1, d_3 - 1, \dots, d_{d_1+1} - 1, d_{d_1+2}, \dots, d_n)$$

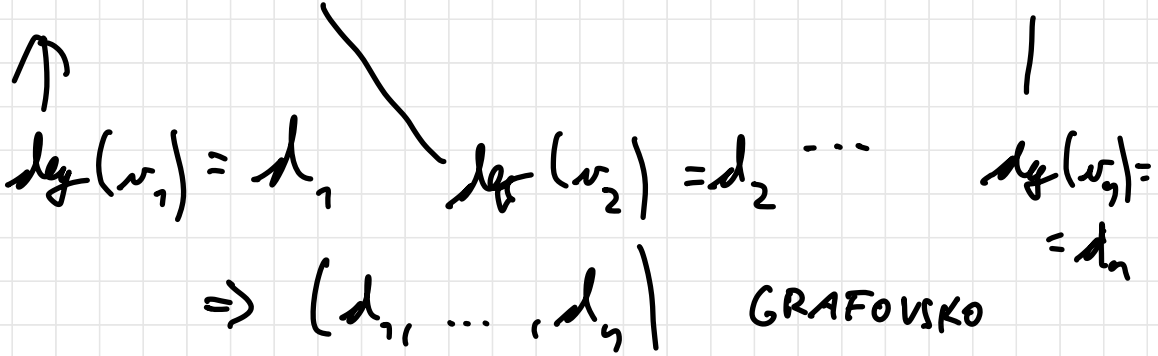
($\Leftarrow$) RECIMO, DA JE GRAFOVSKO

$$(d_2 - 1, \dots, d_{d_1+1} - 1, \dots, d_n)$$



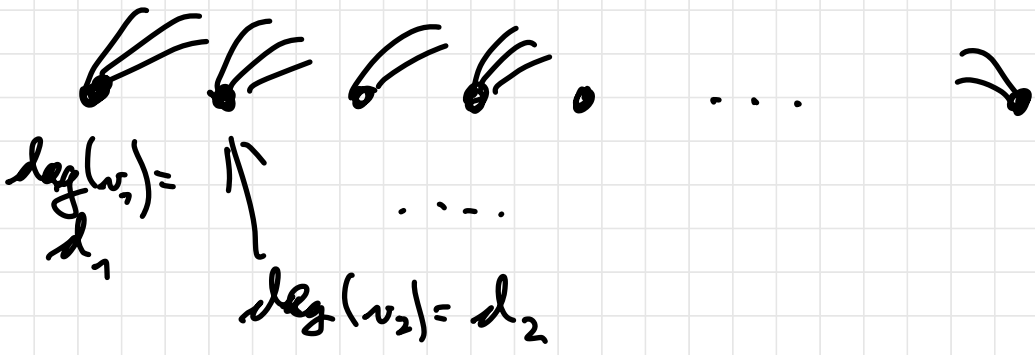
↓ DODAMO VOZLIŠČE v_1





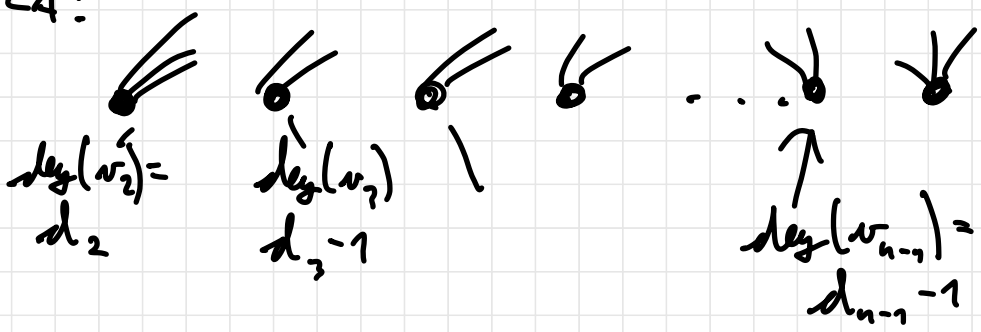
$\Rightarrow (d_1, \dots, d_n)$ GRAFOVSKO

($\Rightarrow$) RECIMO, DA JE $(d_1, \dots, d_n)$ GRAFOVSKO.



↓ ODSTRANIMO v_1

SKICA:



ČE OBSTAJA v_i $i \leq d_1 + 1$, DA

$deg(v_i) = d_i \Rightarrow$ OBSTAJA v_j

$j > d_1 + 1$, DA $deg(v_j) = d_j - 1$

KER JE $d_i > d_j - 1$, POMENI DA

IMA v_i SOSEDA, KI GA v_j NIMA

$\Rightarrow$ MENJAMO SOSEDA NA v_j

V NOVEM GRAFU $deg(v_i) = d_i - 1$

$deg(v_j) = d_j$

TO PONAVLJAMO, DOKLER NE DOBIMO PRAVEGA GRAFA.

□