4. DN - REŠITVE

①

a) $$\frac{4x^2+2x+6}{x^3-x^2-5x-3} = \frac{4x^2+2x+6}{(x+1)^2(x-3)} = \frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{x-3}$$

$$\int \frac{4x^2+2x+6}{x^3-x^2-5x-3}\,dx = \ln|x+1| + \frac{2}{x+1} + 3\ln|x-3| + C$$

b) $$\int \frac{x^3+1}{x^2-x}\,dx = \int\left(x+1+\frac{x+1}{x^2-x}\right)dx = \int\left((x+1)+\frac{2}{x-1}-\frac{1}{x}\right)dx =$$

$$= \frac{x^2}{2} + x + 2\ln|x-1| - \ln|x| + C$$

$$\frac{x+1}{x^2-x} = \frac{2}{x-1} - \frac{1}{x}$$

c) $t = \tan\frac{x}{2}$

$$\int \frac{1}{3-2\cos x}\,dx = \int \frac{1}{3-2\frac{1-t^2}{1+t^2}}\,\frac{2\,dt}{1+t^2} = 2\int\frac{dt}{3+3t^2-2+2t^2} = 2\int\frac{dt}{1+5t^2} =$$

$u = \sqrt{5}\,t$
$du = \sqrt{5}\,dt$

$$= \frac{2}{\sqrt5}\int\frac{du}{1+u^2} = \frac{2}{\sqrt5}\arctan\left(\sqrt5\tan\frac{x}{2}\right) + C$$

②

a) $t = \ln x$

$$I = \int_1^2 (t^2-2t)\,dt = -\frac{2}{3}$$

b) $u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

$$I = -xe^{-x}\Big|_0^2 + \int_0^2 e^{-x}dx = -2e^{-2} + e^{-x}\Big|_0^2 =$$

$$= -2e^{-2} - e^{-2} + 1 =$$

$$= 1 - 3e^{-2}$$

c) $$\int\frac{x}{x^2-x-6}\,dx = \int\left[\frac{3}{5}\,\frac{1}{x-3} + \frac{2}{5}\,\frac{1}{x+2}\right]dx = \frac{3}{5}\ln|x-3| + \frac{2}{5}\ln|x+2|$$

$$I = \left[\frac{3}{5}\ln|x-3| + \frac{2}{5}\ln|x+2|\right]_{-1}^{2} = \frac{2}{5}\ln4 - \frac{3}{5}\ln4 = -\frac{1}{5}\ln4$$

(3) a) $\lim\limits_{b\to\infty} \int_2^b \frac{1}{x^2-1}\,dx = \lim\limits_{b\to\infty}\left(\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Big|_2^b\right) = \lim\limits_{b\to\infty}\left(\frac{1}{2}\ln\left|\frac{b-1}{b+1}\right| - \frac{1}{2}\ln\frac{1}{3}\right)$

$= \frac{1}{2}\ln 3 = \ln\sqrt{3}$

$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$

b) $\int_{-1}^1 \frac{1}{x^2-1}\,dx = \int_{-1}^0 \frac{1}{x^2-1}\,dx + \int_0^1 \frac{1}{x^2-1}\,dx$

$\int_0^1 \frac{1}{x^2-1}\,dx = \lim\limits_{b\to 1}\int_0^b \frac{1}{x^2-1}\,dx = \lim\limits_{b\to 1}\left(\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\right)\Big|_0^b = \lim\limits_{b\to 1}\frac{1}{2}\ln\left|\frac{b-1}{b+1}\right|$

ne obstaja

c) $\int_0^{1/2}\frac{1}{\sqrt{1-2x}}\,dx = \lim\limits_{b\to 1/2}\int_0^b \frac{1}{\sqrt{1-2x}}\,dx = \lim\limits_{b\to 1/2}\left[-(1-2x)^{1/2}\right]\Big|_0^b =$

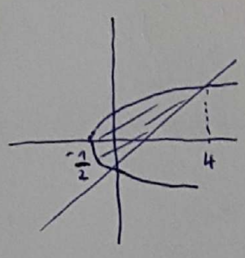
$= \lim\limits_{b\to 1/2}\left(1-(1-2b)^{1/2}\right) = 1$

(4) $f(c) = \frac{1}{2}\int_{-1}^1 f(x)\,dx$

$\frac{1}{1+c^2} = \frac{1}{2}\cdot\frac{2\pi}{4}$

$c = \pm\sqrt{\frac{4}{\pi}-1}$

(5)



$pl = \int_{-1/2}^0\left(\sqrt{2x+1} - (-\sqrt{2x+1})\right)dx + \int_0^4\left(\sqrt{2x+1} - (x-1)\right)dx = \frac{16}{3}$

(6) $V = \pi\int_0^8 4x\,dx = 128\pi$ $\qquad S = 2\pi\int_0^8 2\sqrt{x}\sqrt{1+\frac{1}{x}}\,dx = \frac{208\pi}{3}$

(7) $V = \pi \int_{-3\pi/2}^{\pi/2} (f(x)+1)^2 dx = \pi \int_{-3\pi/2}^{\pi/2} (\cos x + 2)^2 dx = \pi \left(\frac{9}{2}x + \frac{\sin 2x}{4} + 4\sin x \right)\Big|_{-3\pi/2}^{\pi/2} = 9\pi^2$

(8) $l = \int_{-\pi}^{\pi} \sqrt{\varphi^2 + 4\varphi^2}\, d\varphi = \int_{-\pi}^{\pi} \sqrt{5\varphi^2}\, d\varphi = 2\int_0^{\pi} \sqrt{5}\varphi\, d\varphi = 2\sqrt{5}\,\frac{\pi^2}{2} = \sqrt{5}\pi^2$

(9)

napaka $\leq \frac{1}{12n^2} \max_{x\in[0,1]} |g''(x)|$ za $g(x) = e^{x^2}$

$g'(x) = 2x e^{x^2}$

$g''(x) = 2e^{x^2} + 2x e^{x^2} \cdot 2x =$

$= 2e^{x^2}(1+2x^2) \leq 2e \cdot 3 = 6e$

za vse $x \in [0,1]$

$\frac{1}{12n^2} \cdot 6e \leq \frac{1}{100}$

$n^2 \geq \frac{100e}{2} \doteq 136$

$\Rightarrow n \geq 12$

(10) $f^{(n)}(x) = (-1)^n n!\, x^{-n-1}$

$f^{(n)}(1) = (-1)^n n!$

$T_n(f;1)(x) = \sum_{k=0}^{n} \frac{f^{(k)}(1)}{k!}(x-1)^k = \sum_{k=0}^{n} (-1)^k (x-1)^k$

(11) $f(x) = (1+x)\left(1 + x^2 + \frac{x^4}{2!} + \dots\right) =$

$= 1 + x + x^2 + x^3 + \frac{x^4}{2!} + \frac{x^5}{2!} + \dots$

(12) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$\cos\frac{\pi}{10} = 1 - \left(\frac{\pi}{10}\right)^2 \frac{1}{2!} + \left(\frac{\pi}{10}\right)^4 \cdot \frac{1}{4!} - \dots$

$|R_n| \leq \frac{|f^{(n+1)}(\xi)|}{(n+1)!} \cdot \left(\frac{\pi}{10}\right)^{n+1} \leq \left(\frac{\pi}{10}\right)^{n+1} \frac{1}{(n+1)!} \leq \left(\frac{4}{10}\right)^{n+1} \frac{1}{(n+1)!} \leq \frac{1}{100} \Rightarrow n = 3$

$\Rightarrow \cos\frac{\pi}{10} \doteq 1 - \frac{\pi^2}{200}$

(13) $\left|\frac{(x+1)^{2n+2}}{3^{n+1}((n+1)^2+1)} \cdot \frac{3^n(n^2+1)}{(x+1)^{2n}}\right| = \frac{(x+1)^2}{3} \frac{n^2+1}{n^2+2n+2} = D_n$

$$\lim_{n\to\infty} D_n = \frac{(x+1)^2}{3} < 1$$

$(x+1)^2 < 3$

$|x+1| < \sqrt{3}$

$x \in (-1-\sqrt{3}, -1+\sqrt{3})$

rob: $x = -1 \pm \sqrt{3}$

vrsta: $\sum_{n=1}^{\infty} \frac{3^n}{3^n(n^2+1)} = \sum_{n=1}^{\infty} \frac{1}{n^2+1} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$

$\Rightarrow$ območje konvergence: $x \in [-1-\sqrt{3}, -1+\sqrt{3}]$.