REŠITVE 3. DOMAČE NALOGE

1. a) $f'(x) = \frac{1-x^2}{(1+x^2)^2}$

   b) $f'(x) = \frac{2}{1-x^2}$

   c) $f'(x) = \frac{-1}{3(\sqrt[3]{x^4}+\sqrt[3]{x^2})}$

   d) $f'(x) = 2\ln\frac{x}{1-x} + \frac{2x-1}{x(1-x)}$

   e) $f'(x) = \frac{-x}{\sqrt{x^2-x^4}}$

   f) $f(x) = \sqrt[x]{x} = e^{\frac{\ln x}{x}}$

   $f'(x) = e^{\frac{\ln x}{x}} \cdot \frac{1-\ln x}{x^2} = \sqrt[x]{x}\,\frac{1-\ln x}{x^2}$

2. $e^{xy}(y+xy') + \frac{y'}{y} = 1$

   $x=0: \quad 1+\ln y = 0$

   $y = e^{-1}$

   $T(0, e^{-1})$

   $y'(1+xye^{xy}) = y(1-ye^{xy})$

   $y' = \frac{y(1-ye^{xy})}{1+xye^{xy}}$

   $y'(T) = e^{-1} - e^{-2} = k$

   tangenta : $y = kx + n$, $n = e^{-1}$

   $y = (e^{-1} - e^{-2})x + e^{-1}$

(3) Presečišči: $\frac{1}{1+x^2} = \frac{1}{2} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$f'(x) = \frac{-2x}{(1+x^2)^2}$

$x=1: \quad k = -\frac{1}{2}, \quad n = 1$ $\qquad$ $x=-1: \quad k = \frac{1}{2}, \quad n = 1$

$y = -\frac{1}{2}x + 1$ $\qquad$ $y = \frac{1}{2}x + 1$

presečišče tangent: $-\frac{1}{2}x + 1 = \frac{1}{2}x + 1 \Rightarrow x = 0 \Rightarrow T(0,1)$

kot: $\varphi = \arctan \frac{4}{3}$

(4) $k_{tangente} = 1$

$g'(x) = \ln x + 1 = 1 \Rightarrow x = 1$

$k_{normale} = -1$

normala: $y = -x + n$

$0 = -1 + n \Rightarrow n = 1$

$y = -x + 1$

(5) a) $f(x) = \cos x$ $\qquad$ $a = \frac{3\pi}{9} = \frac{\pi}{3}$

$f'(x) = -\sin x$ $\qquad$ $h = -\frac{\pi}{9}$

$f\left(\frac{2\pi}{9}\right) \doteq \cos\frac{\pi}{3} + \frac{\pi}{9}\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\pi}{9}\frac{\sqrt{3}}{2}$

b) $f(x) = \ln\left(\sqrt{\sin(-x) + 1 - x}\right)$ $\qquad$ $a = 0$

$f'(x) = \frac{-\cos x - 1}{2(-\sin x + 1 - x)}$ $\qquad$ $h = 0{,}05$

$f(0{,}05) \doteq f(0) + 0{,}05 \cdot f'(0)$

$= 0 + 0{,}05 \cdot (-1) = -0{,}05$

(6) $f'(x) = 2x \arctan x + \frac{x^2}{1+x^2} > 0$ za vse x

(7) v krajiščih: $f(-2) = -17$ $\boxed{MIN}$

$f(2) = -1$

stacionarne točke: $f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

$f(0) = 3$ $\boxed{MAX}$

(8) $f'(x) = e^{-x}(2x - x^2)$

$f''(x) = e^{-x}(x^2 - 4x + 2)$

stacionarni t.: $x = 0$ LOKALNI MINIMUM $\quad f''(0) > 0$

$x = 2$ LOKALNI MAKSIMUM $\quad f''(2) < 0$

(9) Točka na grafu: $T\left(x, \frac{1}{1+x^2}\right)$

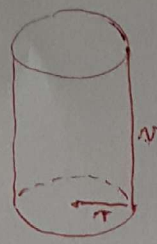
Razdalja do izhodišča: $d^2 = x^2 + \frac{1}{(1+x^2)^2} = \tilde{f}(x)$ iščemo minimum

$\tilde{f}'(x) = 2x - 4x(1+x^2)^{-3} = 0$

$\Rightarrow x = 0$ ali $x = \pm\sqrt{\sqrt[3]{2} - 1}$ $\leftarrow \boxed{MIN}$

$\tilde{f}(0) = 1 \qquad \tilde{f}\left(\pm\sqrt{\sqrt[3]{2}-1}\right) \doteq 0{,}89$

(10) $500 = \pi r^2 v \Rightarrow v = \frac{500}{\pi r^2}$



$$S = 2\pi r v + 2\pi r^2 = \frac{1000}{r} + 2\pi r^2$$

$$S'(r) = -\frac{1000}{r^2} + 4\pi r = 0$$

$$r^3 = \frac{250}{\pi} \Rightarrow r = \sqrt[3]{\frac{250}{\pi}}$$

(11) a)

$$\lim_{x \to \frac{\pi}{4}} \frac{\ln(\tan x)}{\sin x - \cos x} \overset{\frac{0}{0}}{=} \lim_{x \to \frac{\pi}{4}} \frac{1}{\sin x \cos x\,(\sin x + \cos x)} = \sqrt{2}$$

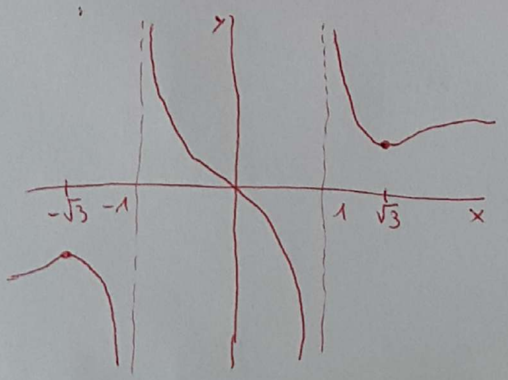
b)

$$\lim_{x \searrow 0} e^{\frac{1}{x^2}\ln\left(\frac{\sin x}{x}\right)} = e^{-\frac{1}{6}}$$

$$\lim_{x \searrow 0} \frac{\ln\left(\frac{\sin x}{x}\right)}{x^2} \overset{\frac{0}{0}}{=} \lim_{x \searrow 0} \frac{x\cos x - \sin x}{2x^2 \sin x} = \lim_{x \searrow 0} \frac{x\cos x - \sin x}{2x^3} \overset{\frac{0}{0}}{=} -\frac{1}{6}$$

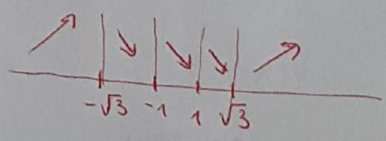
(12) $D_f = \mathbb{R} \setminus \{\pm 1\}$

$$f'(x) = \frac{x^2 - 3}{3\sqrt[3]{(x^2-1)^4}} = 0 \Rightarrow x = \pm\sqrt{3}$$



f je liha ; ničla : $x = 0$

naraščanje :



$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

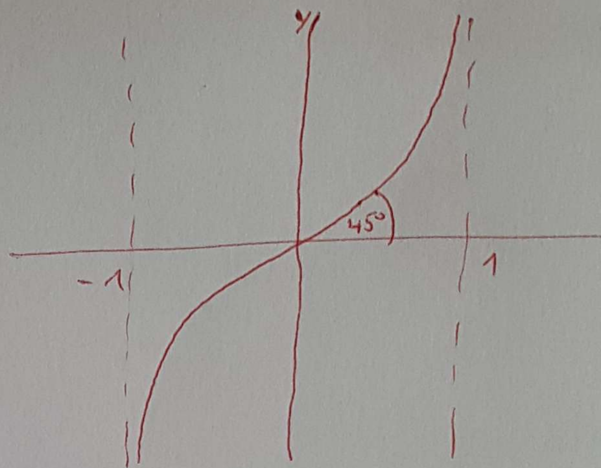
(13) $D_f = (-1, 1)$

nula: $x = 0$

$y' = \dfrac{1}{1-x^2} > 0$ na D_f

$y'' = \dfrac{2x}{(1-x^2)^2} \geq 0$ za $x \geq 0$

$\lim\limits_{x \to 1} f(x) = \infty \qquad \lim\limits_{x \to -1} f(x) = -\infty$



(14) a) $t = \arctan x$

$dt = \dfrac{dx}{1+x^2}$

$$\int \sqrt{t}\, dt = \frac{2\sqrt{(\arctan x)^3}}{3} + C$$

b) $u = \ln(1+x)$ $\qquad dv = x\,dx$

$du = \dfrac{dx}{1+x}$ $\qquad v = \dfrac{x^2}{2}$

$$\int x \ln(1+x)\, dx = \frac{x^2}{2}\ln(1+x) - \frac{1}{2}\int \frac{x^2}{x+1}\, dx =$$

$$= \frac{x^2}{2}\ln(1+x) - \frac{1}{2}\left(\frac{x^2}{2} - x + \ln|x+1|\right) + C$$