① $13^{2n} + 6$ deljivo s 7 $\forall n \in \mathbb{N}$

$n = 1$: $13^{2\cdot 1} + 6 = 13^2 + 6 = 169 + 6 = 175 = 25 \cdot 7$

$\Rightarrow 13^{2\cdot 1} + 6$ je deljivo s 7.

$n \to n+1$: $7 \mid 13^{2n} + 6 \Rightarrow 7 \mid 13^{2(n+1)} + 6$

$$13^{2(n+1)} + 6 = 13^{2n+2} + 6 = 13^2 \cdot 13^{2n} + 6 =$$

$$= 169 \cdot 13^{2n} + 6 = (168+1) 13^{2n} + 6 =$$

$$= 168 \cdot 13^{2n} + 13^{2n} + 6 \stackrel{\text{I.P.}}{=}$$

I.P. $13^{2n} + 6 = 7k$

$$= 168 \cdot 13^{2n} + 7k$$

$$= 7 \cdot 24 \cdot 13^{2n} + 7k = 7 \cdot (\underbrace{24 \cdot 13^{2n} + k}_{k'}) = 7k'$$

2) $1+2x^2+\dots+nx^{2(n-1)}+(n+1)x^{2n}=\dfrac{1-(n+2)x^{2(n+1)}+(n+1)x^{2(n+2)}}{(x^2-1)^2}$

$\forall n\in\mathbb{N}$

$n=1:$ $L=1+2x^2$

$D=\dfrac{1-3x^4+2x^6}{(x^2-1)^2}=\dfrac{2x^6-3x^4+1}{x^4-2x^2+1}$

$2x^6-3x^4+1 : x^4-2x^2+1=2x^2+1$

$-2x^6+4x^4\mp 2x^2$

x^4-2x^2+1

$\Rightarrow D=2x^2+1=L$ ✓

$n\to n+1$

$\underbrace{1+2x^2+\dots+nx^{2(n-1)}+(n+1)x^{2n}}+(n+2)x^{2(n+1)}\overset{I.P.}{=}$

$\dfrac{1-(n+2)x^{2(n+1)}+(n+1)x^{2(n+2)}}{(x^2-1)^2}+(n+2)x^{2(n+1)}=$

$=\dfrac{1-(n+2)x^{2n+2}+(n+1)x^{2n+4}+(n+2)x^{2n+2}\cdot(x^4-2x^2+1)}{(x^2-1)^2}=$

$=\dfrac{1-\cancel{(n+2)x^{2n+2}}+(n+1)x^{2n+4}+(n+2)x^{2n+6}-2(n+2)x^{2n+4}+\cancel{(n+2)x^{2n+2}}}{(x^2-1)^2}=$

$=\dfrac{1+(n+1-2n-4)x^{2n+4}+(n+2)x^{2n+6}}{(x^2-1)^2}=\dfrac{1-(n+3)x^{2n+4}+(n+2)x^{2n+6}}{(x^2-1)^2}$

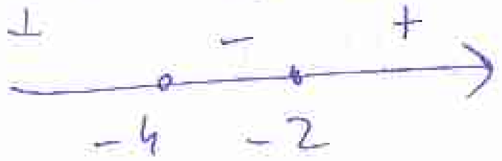
$=\dfrac{1-((n+1)+2)x^{2((n+1)+1)}+((n+1)+1)x^{2\cdot((n+1)+2)}}{(x^2-1)^2}$

3) a) $\frac{x}{x+2} > 2$

$2 - \frac{x}{x+2} < 0 \Rightarrow \frac{2x+4-x}{x+2} < 0$

$\Rightarrow \frac{x+4}{x+2} < 0$

predznaki ulomka



$R = (-4, -2)$

ali $\frac{x}{x+2} > 2 \quad / \cdot (x+2)^2$

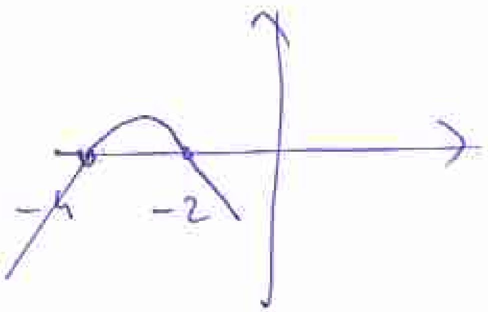
$x(x+2) > 2(x+2)^2$

$x(x+2) - 2(x+2)^2 > 0$

$(x+2) \cdot (x - 2(x+2)) > 0$

$(x+2)(-x-4) > 0$

$x_1 = -2 \quad x_2 = -4$



$\Rightarrow R = (-4, -2)$

3b) $\dfrac{x^2-9}{x^2-x-2} < 1$

Prvi način:

$$1 - \frac{x^2-9}{x^2-x-2} = \frac{x^2-x-2-x^2+9}{x^2-x-2} > 0$$

$$\frac{7-x}{x^2-x-2} > 0 \qquad \frac{7-x}{(x-2)(x+1)} > 0$$

$R = (-\infty,-1) \cup (2,7)$

Drugi način: $\dfrac{x^2-9}{x^2-x-2} < 1 \quad / \cdot (x^2-x-2)^2$

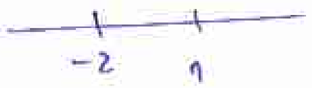
$$(x^2-9)(x^2-x-2) - (x^2-x-2)^2 < 0$$
$$(x^2-x-2)\cdot\left((x^2-9)-(x^2-x-2)\right) < 0$$
$$(x-2)(x+1)\cdot(x-7) < 0$$

$(-\infty,-1) \cup (2,7)$

4) $|x^2+|x-2|| > |x-1| + 1$

I: $x \geq 2 \Rightarrow |x^2 + x - 2| > |x-1| + 1$

$|(x+2)(x-1)| > |x-1| + 1$

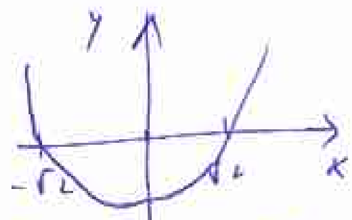


a) $x \geq 1$: $x^2 + x - 2 > x - 1 + 1$

$x^2 + x - 2 > x$

$x^2 > 2$

$x_{1,2} = \pm\sqrt{2} \Rightarrow$



$R_1 = \left((-\infty, -\sqrt{2}] \cup (\sqrt{2}, \infty)\right) \cap [2, \infty) = [2, \infty)$

b)* $x < 1$ nima smisla, ker $x \geq 2$

II. $x < 2$ $|x^2 + 2 - x| > |x-1| + 1$

nima realnih ničel

$x^2 + 2 - x = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1-8}}{2}$

$\Rightarrow x^2 + 2 - x > 0$

$\Rightarrow x^2 - x + 2 > |x-1| + 1$

a) $x \geq 1$: $x^2 - x + 2 > x - 1 + 1$

$x^2 - 2x + 2 > 0$ ✓

$(x-1)^2 + 1 > 0$ ✓

$R_2 = [1, \infty) \cap (-\infty, 2) = [1, 2)$

b) $x < 1$: $x^2 + 2 - x > 1 - x + 1$

$x^2 > 0 \Rightarrow R_3 = (\mathbb{R}\setminus\{0\}) \cap (-\infty, 1)$

$= (-\infty, 1) \setminus \{0\}$

$\Rightarrow R = \mathbb{R} \setminus \{0\}$

5) a) $\cos^2 x - \sin^2 x = \cos 2x$

$\cos^2 x + \sin^2 x = 1$

$\Rightarrow \quad 2\cos^2 x = 1 + \cos 2x$

$2\sin^2 x = 1 - \cos 2x$

$\Rightarrow \quad \cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$

$\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$

b) $\cos \frac{\pi}{8} = \pm \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

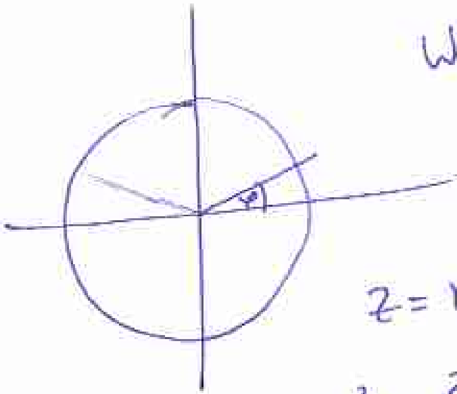
$\sin \frac{\pi}{8} = \pm \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

c) $w = -\frac{\sqrt{2+\sqrt{2}}}{5} + i\,\frac{\sqrt{2-\sqrt{2}}}{5}$

$r = \sqrt{\frac{2+\sqrt{2}}{25} + \frac{2-\sqrt{2}}{25}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$

$w = \frac{2}{5} \cdot \left(-\frac{\sqrt{2+\sqrt{2}}}{2} + i\,\frac{\sqrt{2-\sqrt{2}}}{2} \right)$

$w = \frac{2}{5} \cdot \left(\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8} \right)$



$z^3 = w$

$z = re^{i\varphi} \Rightarrow r^3 e^{i3\varphi} = w$

$\Rightarrow \quad r^3 = \frac{2}{5}\ ; \quad 3\varphi = \frac{7\pi}{8} + 2k\pi \quad k \in \mathbb{Z}$

$r = \sqrt[3]{\frac{2}{5}}\,, \qquad \varphi = \frac{7\pi}{8} + \frac{2k\pi}{3}\ ; \quad k = 0, 1, 2$

$z_k = \sqrt[3]{\frac{2}{5}} \left(\cos\left(\frac{7\pi}{8} + \frac{2k\pi}{3}\right) + i \sin\left(\frac{7\pi}{8} + \frac{2k\pi}{3}\right) \right)\ ; \quad \underline{k = 0, 1, 2}$

6) a) $z^4 = \bar{z} \Rightarrow z^4 - z = 0 \Rightarrow z(z^3 - 1) = 0$

$z_1 = 0$, $z_2 = 1$, $z_3 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, $z_4 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

b) $z^3 = (\bar{z})^3 \Rightarrow z = re^{i\varphi}$

$\Rightarrow r^3 e^{i3\varphi} = r^3 e^{-i3\varphi} \Rightarrow$

ó $r = 0 \Rightarrow z = 0$

sino $e^{i3\varphi} = e^{-i3\varphi} \Rightarrow e^{i6\varphi} = 1 = e^{i0}$

$\Rightarrow 6\varphi = 2k\pi$; $k \in \mathbb{Z}$

$\varphi = \frac{k\pi}{3}$; $k \in \mathbb{Z}$

$z_0 = r$, $z_1 = r(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = r(\frac{1}{2} + i\frac{\sqrt{3}}{2})$

$z_2 = r(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}) = r(-\frac{1}{2} + i\frac{\sqrt{3}}{2})$

$z_3 = r(\cos\pi + i\sin\pi) = -r$

$z_4 = r(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}) = r(-\frac{1}{2} - \frac{i\sqrt{3}}{2})$

$z_5 = r(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}) = r(\frac{1}{2} - \frac{i\sqrt{3}}{2})$

$z_7 = 0$

7a) $p(x) = x^4 + x^3 - x^2 + x - 2$

$p(1) = 0$; $1 + 1 - 1 + 1 - 2 = 0$ ✓

	1	1	-1	1	-2
1		1	2	1	2
	1	2	1	2	0

$p(x) = (x-1)(x^3 + 2x^2 + x + 2)$

$= (x-1)(x^2 \cdot (x+2) + (x+2))$

$= (x-1)(x+2)(x^2+1)$

b) $p(x) = (x-1)(x+2)(x-i)(x+i)$

8) $p(1-i)=0 \Rightarrow p(1+i)=0$

a) . . . . .

b) $q(x)=(x-1+i)(x-1-i)=(x-1)^2-i^2=x^2-2x+2$

q deli p

$$
\begin{array}{l}
x^4-6x^3+15x^2-18x+10 : x^2-2x+2 = x^2-4x+5 \\
\underline{-x^4+2x^3-2x^2} \\
\quad -4x^3+13x^2 \\
\quad \underline{4x^3-8x^2+8x} \\
\quad\quad 5x^2-10x+10
\end{array}
$$

kvadraten nerazcepen nad $\mathbb{R}$.

$p(x)=(x^2-2x+2)(x^2-4x+5)$

c) $p(x)=(x-1+i)(x-1-i)(x-2-i)(x-2+i)$

$x^2-4x+5=0$

$x_{3,4}=\frac{4\pm\sqrt{16-20}}{2}=\frac{4\pm 2i}{2}=2\pm i$

9) $f:\mathbb{R}^2\to\mathbb{R}^2$ rotacija v poz. smeri za $\frac{\pi}{4}$.

$g:\mathbb{R}^2\to\mathbb{R}^2$ —//— $-\frac{\pi}{4}$

Če $f(T)=f(T') \Rightarrow (g\circ f)(T)=(g\circ f)(T')$

$\qquad\qquad\qquad\qquad \| \qquad\qquad\qquad \|$

$\qquad\qquad\qquad\qquad T \qquad\qquad\qquad T'$

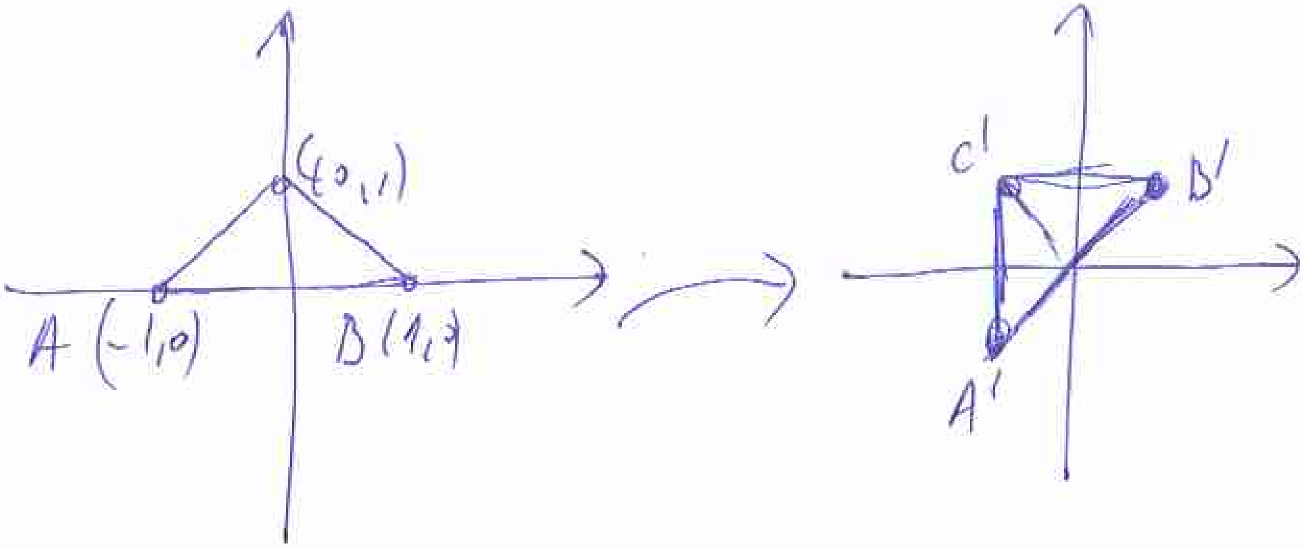
$\Rightarrow f$ inj.

f surj Kaj se slika v T?

v T se slika $g(T)$. $\Rightarrow f$ surj ✓

$\Rightarrow f$ bijekcija.

9 b)



$A' \ldots \quad x = \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ $\qquad A'\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\qquad y = \sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

$B' \ldots \quad x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\qquad B'\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\qquad y = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$C' \ldots \quad x = \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\qquad C'\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\qquad y = \sin\frac{2\pi}{4} = \frac{\sqrt{2}}{2}$

10) a) $\frac{2n^2+1}{n^2+2}$

$$\frac{2(n+1)^2+1}{(n+1)^2+2} - \frac{2n^2+1}{n^2+2} = \frac{(2(n+1)^2+1)(n^2+2) - (2n^2+1)((n+1)^2+2)}{((n+1)^2+2)(n^2+2)} =$$

$$= \frac{(2n^2+4n+3)(n^2+2) - (2n^2+1)(n^2+2n+3)}{((n+1)^2+2)(n^2+2)} =$$

$$= \frac{3+6n}{(2+n^2)((n+1)^2+2)} > 0$$

$\Rightarrow$ zap. strogo raste.

10b) $a_n \le 2$

$$2 - \frac{2n^2+1}{n^2+2} = \frac{2n^2+4-2n^2-1}{n^2+2} = \frac{3}{4n^2+2}$$ ✓

$a_n \geq 0$, DA saj je $a_n \geq 0$ kot kvocient dveh pozitivnih števil.

10c) Ker zap. raste $\exists \min\limits_{n\in \mathbb{N}} a_n = a_1 = \frac{3}{3} = 1$

$$\min_{n\in \mathbb{N}} a_n = \inf_{n\in \mathbb{N}} a_n = 1$$

Ker $(a_n)_{n\in \mathbb{N}}$ strogo raste $\Rightarrow$ $\max\limits_{n\in \mathbb{N}} a_n$ ne obstaja

$$\sup_{n\in \mathbb{N}} a_n = 2$$

2 je zgornja meja. (videli v (b))

2 je natančna zg. meja

$\varepsilon > 0$: $\dfrac{2n^2+1}{n^2+2} > 2-\varepsilon \Rightarrow 2n^2+1 > 2n^2+4-n^2\varepsilon-2\varepsilon$

$n^2\varepsilon > 3-2\varepsilon$

$\Rightarrow n^2 > \dfrac{3-2\varepsilon}{\varepsilon}$; če $2\varepsilon > 3$ $\Rightarrow$ vsak n ustreza.

če $3-2\varepsilon \geq 0 \Rightarrow n > \sqrt{\dfrac{3-2\varepsilon}{\varepsilon}}$ ustreza neki naravni.

Torej v I. primeru, če $2\varepsilon > 3$ je tu $a_n > 2-\varepsilon$

v II. če $2\varepsilon \leq 3$, $\forall n: n > \sqrt{\frac{3-2\varepsilon}{\varepsilon}}$ velja $a_n > 2-\varepsilon$. $\Rightarrow \sup\limits_{n\in \mathbb{N}} a_n = 2$