DS2 VAJE 2

1. KATERA ZAP. SO GRAFOVSKA?

a) $(6,5,5,4,3,3,2,2,2) \iff$

SPOMNIMO SE:

$(d_1,\dots,d_n)$ GRAFOVSKO $\iff$

$(d_2-1,\dots,d_{d_1+1}-1, d_{d_1},\dots,d_n)$ GRAFOVSKO

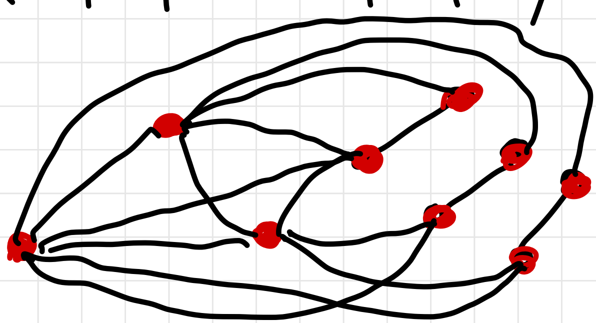
$\iff (4,4,3,2,2,1,2,2)$

$\iff (4,4,3,2,2,2,2,1)$

$\iff (3,2,1,1,2,2,1)$

$\iff (3,2,2,2,1,1,1)$

$\iff (1,1,1,1,1,1)$ JE GRAFOVSKO



b) $(5,5,4,3,3,3,2)$

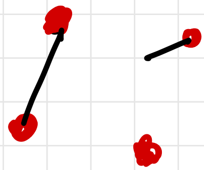
$\Leftrightarrow (4,3,2,2,2,2)$

$\Leftrightarrow (2,1,1,1,2)$

$\Leftrightarrow (2,2,1,1,1)$

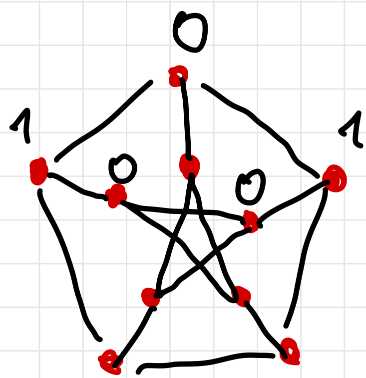
$\Leftrightarrow (1,0,1,1)$

NI GRAFOVSKO



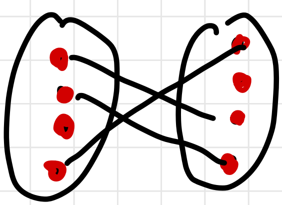
2. ZA KATERE $n, k \in \mathbb{N}$, KJER $k < \frac{n}{2}$ JE POSPLOŠENI PETERSENOV GRAF DVODELEN?

$P_{n,k}$ $\qquad$ $P_{5,2}$

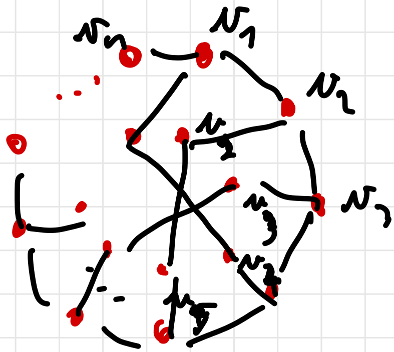


NI DVODELEN

DVODELNI:



- n MORA BITI SOD (ZUNANJI CIKEL)
- k MORA BITI LIH:
  - ČE k SOD:

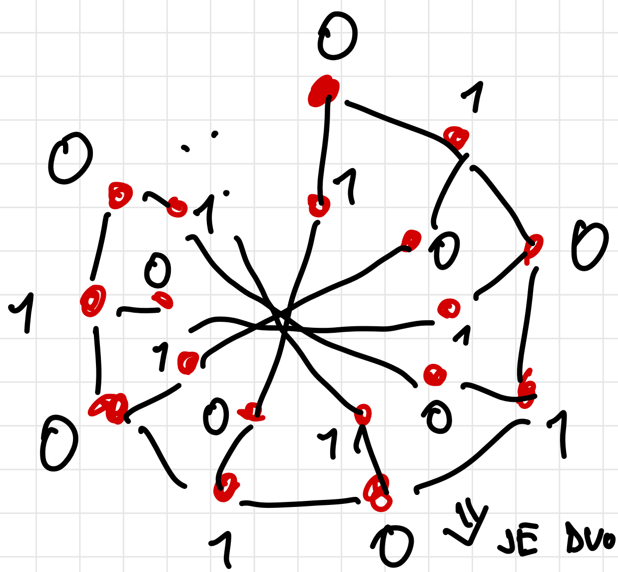


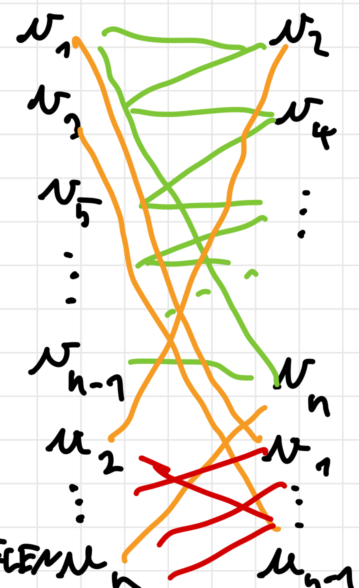
CIKEL $v_1\, u_1\, u_{k+1}\, v_{k+1}\, v_k\, v_{k-1} \ldots v_1$

DOLŽINA: $3 + k$ ← LIHO

⇒ NI DVODELEN

- ČE k LIH:





⇒ JE DVODELEN

3. GRAF HIPERKOCKE Q_n.

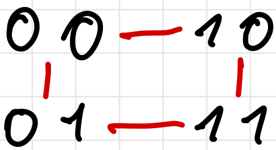
NARIŠI $Q_2, Q_3, Q_4, \dots$

$$V(Q_n) = \{0,1\}^n$$

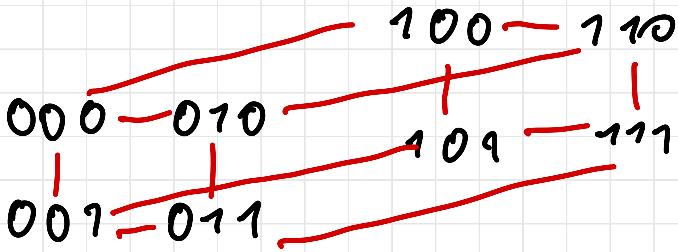
$$(x_1, x_2, \dots, x_n) \sim (y_1, \dots, y_n) \Leftrightarrow$$

$\exists i$ DA $x_i \neq y_i$ IN $x_j = y_j \ \forall j \neq i$

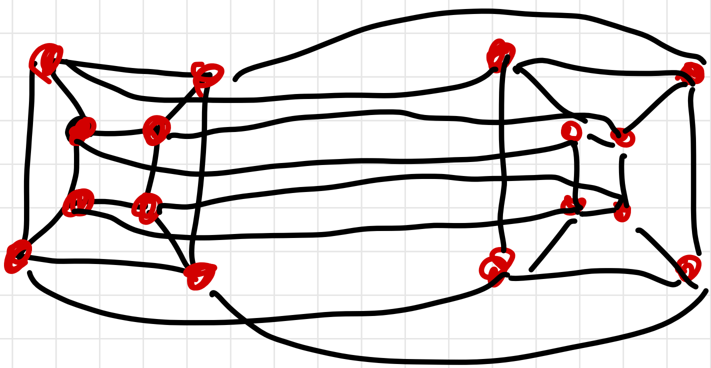
Q_2:



Q_3:



Q_4:



$|V(Q_n)| = ?$ $\quad |E(Q_n)| = ?$

$$|V(Q_n)| = 2^n$$

$$|E(Q_n)| \underset{\uparrow}{=} \frac{1}{2} \cdot \sum_{v \in V(Q_n)} \deg(v) = \frac{1}{2} \cdot n \cdot 2^n$$

LEMA O ROKOVANJU

$$= n \cdot 2^{n-1}$$

$$\operatorname{diam}(Q_n) = ?$$

$v = x_1 x_2 \ldots x_n$ JE NAJBOLJ ODDALJENO OD $u = \overline{x_1}\, \overline{x_2}\, \overline{x_3} \ldots \overline{x_n}$

$$(\overline{x_i} = 1 - x_i)$$

$$\Rightarrow \operatorname{diam}(Q_n) = n$$

ALI JE Q_n DVODELEN GRAF?

$A = \{ v \in V(Q_n) \mid v$ IMA SODO ENK $\}$

$$B = \{ v \in V(Q_n) \mid v \text{ IMA LIHO ENK} \}$$

$$V(Q_n) = A \cup B$$

↑ DISJUNKTNA

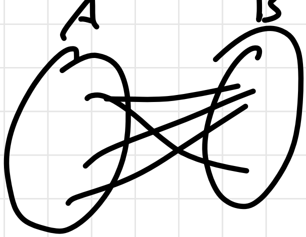
ČE $\{v, u\} \in E(Q_n) \Rightarrow$ SE RAZLIKUJETA V NATANKO 1 BITU $\Rightarrow$ EN IMA SODO ENK, EN LIHO ENK

$\Rightarrow \{v, u\}$ JE MED A in B

$\Rightarrow Q_n$ JE DVODELEN

4. NAJ BO G DVODELEN REGULAREN GRAF Z VSAJ ENO POVEZAVO. DOKAŽI, DA STA MNOŽICI DVODELNEGA RAZBITJA ENAKO MOČNI.

G



VELJA:

$$\sum_{v \in A} \deg(v) = |E(G)|$$

$$\sum_{v \in B} \deg(v) = |E(G)|$$

ČE G k-REGLAREN

$$\sum_{v \in A} \deg(v) = k \cdot |A|$$

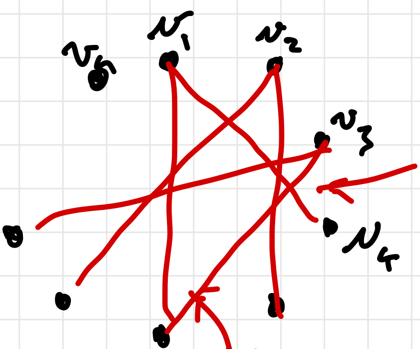
$$\|$$

$$\sum_{v \in B} \deg(v) = k \cdot |B|$$

$$\Rightarrow \quad k \cdot |A| = k\,|B|$$

$$|A| = |B|$$

5. $\mathrm{Circ}\left(n, \{s_1, \dots, s_k\}\right)$ – KROŽNI GRAF



ČE $s_i = 3$

ČE $s_j = 5$

● ALI JE GRAF ZA n LIH LAHKO

DVODELEN ?

$Circ(n, \{s_1, \dots, s_k\})$ JE VEDNO REGULAREN.

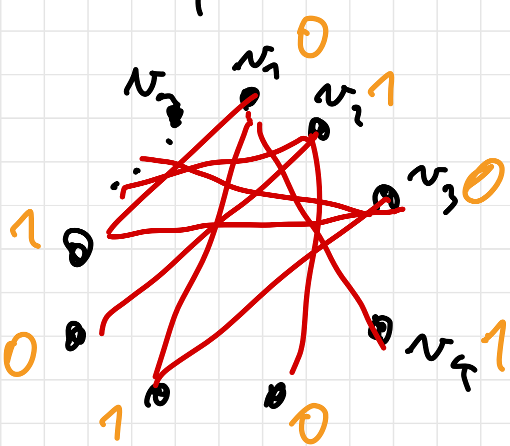
$\Rightarrow$ ČE JE DVODELEN, IMATA BIPART. ENAKO MOČ

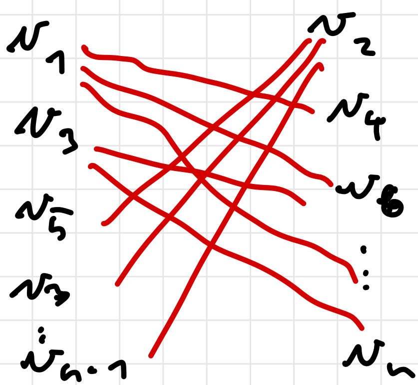
PO PREJŠNI NALOGI

$\Rightarrow$ KER n LIH TO NI MOGOČE

$\Rightarrow$ NI DVODELEN

- ČE n SOD IN VSI $s_1, \dots, s_k$ LIHI, ALI JE DVODELEN?





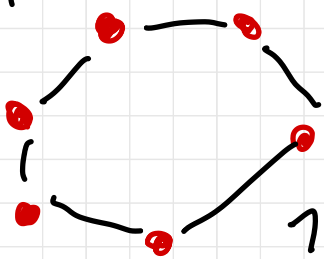
JE DVODELEN Z BIPARTICIJO

$A = \{v_1, v_3, \ldots, v_{n-1}\}$

$B = \{v_2, \ldots, v_n\}$

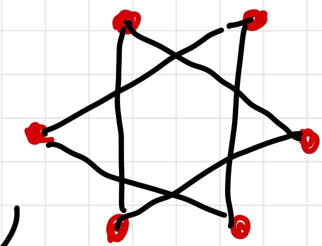
6. KOLIKO POVEZANIH KOMPONENT IMA $\mathrm{Circ}(n, \{s, n-s\})$?

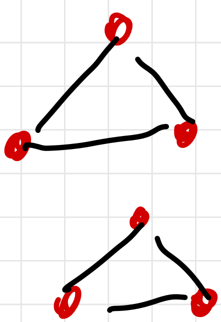
$n=6, s=1$



1 KOMPONENTA

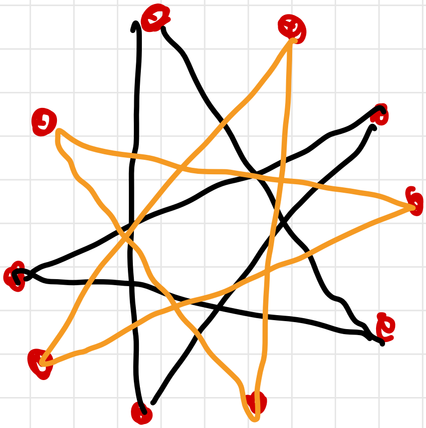
$n=6 \quad s=2$





2 KOMPONENTI

$n=10 \quad s=4$



2 KOMPONENTI

KDAJ STA v_0 IN v_1 V ISTI KOMPONENTI?

$$0+s+s+s+\dots+s = 1 \pmod{n}$$

$$\Leftrightarrow \exists k \quad k\cdot s = 1 \pmod{n}$$

$$\Leftrightarrow \exists k,l \quad k\cdot s = 1 + l\cdot n$$

$$\Leftrightarrow \exists k,l' \quad k\cdot s + l' n = 1$$

$$\Leftrightarrow \gcd(s,n) = 1$$

$\Rightarrow$ GRAF POVEZAN $\Leftrightarrow \gcd(s,n)=1$

IZ v_0 LAHKO PRIDEMO DO

v_k $\iff$

$$0 + s + s + \dots + s = k \pmod n$$

$$\Updownarrow$$

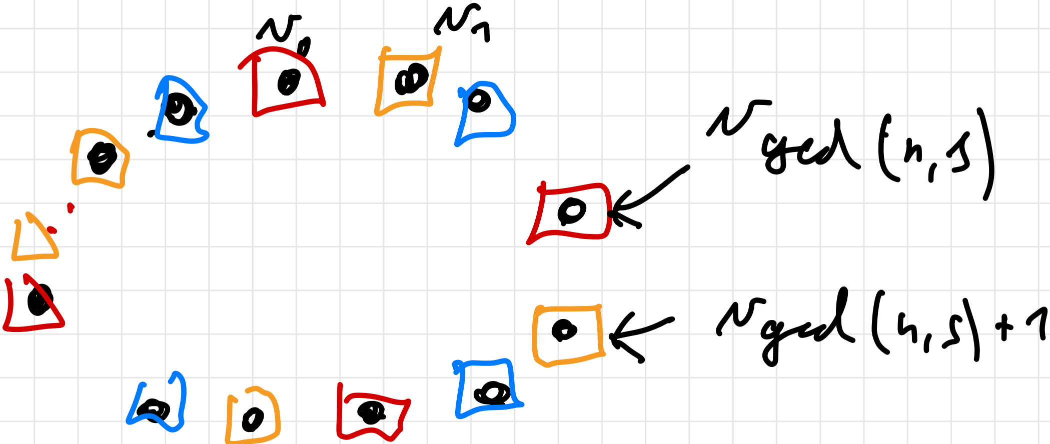
$$\exists\, l, l' \quad s \cdot l + n \cdot l' = k$$

$$\Updownarrow$$

$$\gcd(s, n) \mid k$$

$\Rightarrow$ v_0 V POVEZANI KOMPONENTI Z

$v_{\gcd(s,n)}$, $v_{2\cdot \gcd(s,n)}$, ...



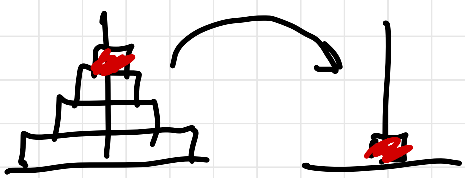
N_1 V KOM. s

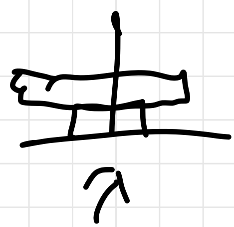
$N_{\gcd(s,n)+1}, N_{2\gcd(s,n)+1}, \ldots,$

$\vdots$

$\Rightarrow$ ŠTEVILO KOMPONENT JE $\gcd(s,n)$

7. GRAF HANOJSKEGA STOLPA H_n.





NI DOVOLJENO

GRAF: H_n – n DISKOV

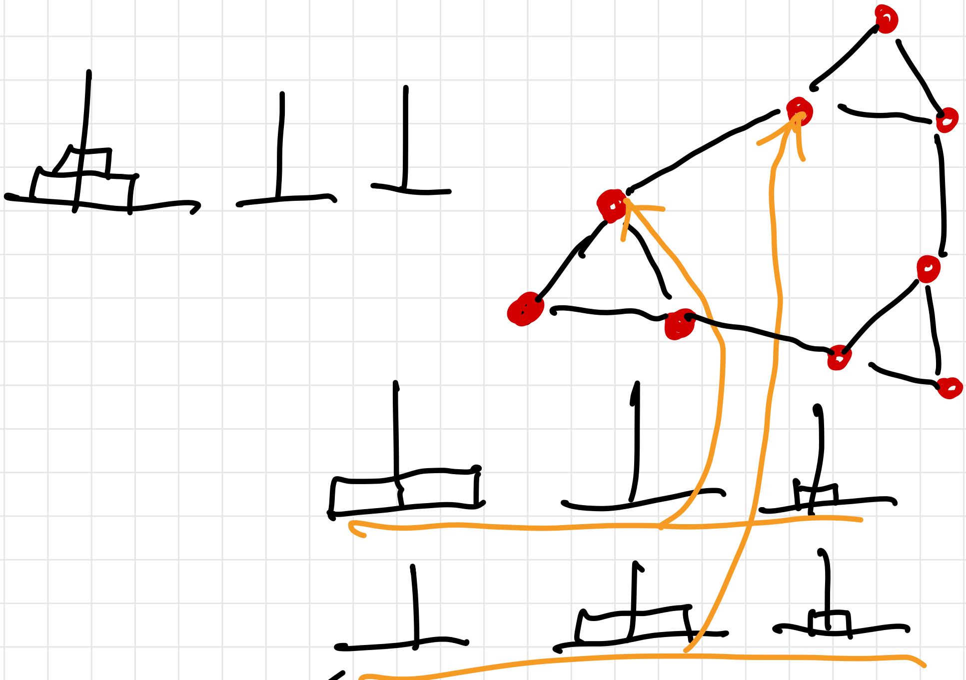
VOZLIŠČA: STANJA V IGRI

POVEZAVE: ČE LAHKO NAREDIMO POTEZO

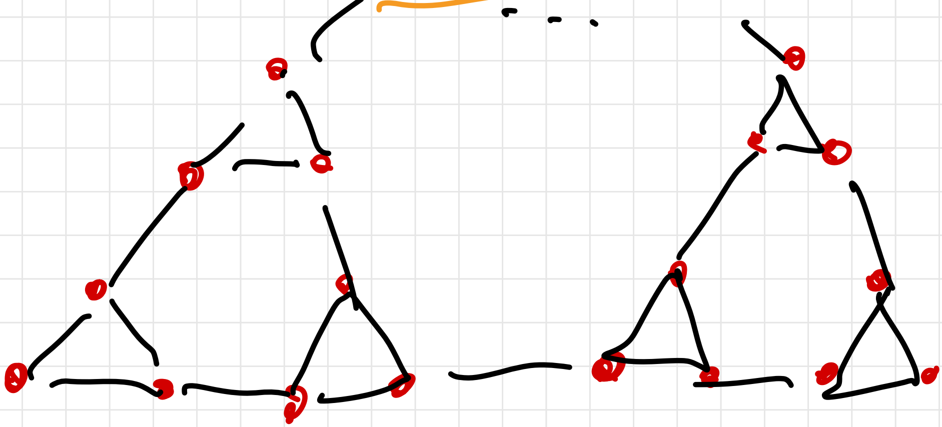
NARIŠI H_1, H_2

• H_1

• H_2



• H_3



$$|V(H_n)| = 3^n$$

$$3 \cdot 3 \cdot \ldots \cdot 3 = 3^n$$

ZA n-TI

ZA 2. NAJVEČJI

MOŽNOSTI ZA NAJVEČJI DISK

$$|E(H_n)| = ?$$

$$|E(H_n)| = 3 \cdot |E(H_{n-1})| + 3$$

$$|E(H_1)| = 3$$

$$|E(H_2)| = 3 \cdot 3 + 3 = 3^2 + 3$$

$$|E(H_3)| = 3 \cdot (3^2 + 3) + 3 = 3^3 + 3^2 + 3$$

$$\vdots$$

$$|E(H_n)| = \sum_{i=1}^{n} 3^i = \sum_{i=0}^{n} 3^i - 1 = *$$

$$(3^{n+1} - 1) = (3-1)(3^n + 3^{n-1} + \ldots + 1)$$

$$\ast = \frac{3^{n+1}-1}{2} - 1$$

$$\operatorname{diam}(H_n) = ?$$

DN