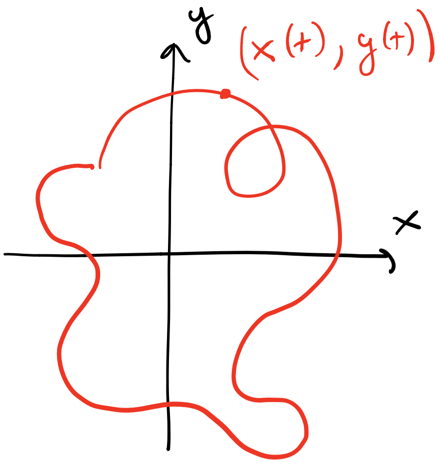
PARAMETRIČNE KRIVULJE V RAVNINI

$K = \{(x(t), y(t)),\ t \in I \subset \mathbb{R}\}$, I INTERVAL.



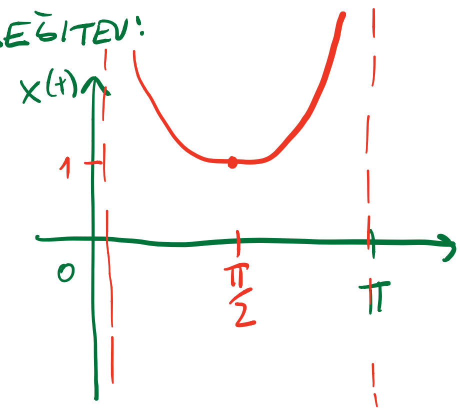
PARAMETRIČNA KRIVULJA JE ENODIMENZIONALEN OBJEKT V RAVNINI, PRI KATEREM STA KOORDINATI ODVISNI OD ISTEGA PARAMETRA $t \in I \subset \mathbb{R}$.

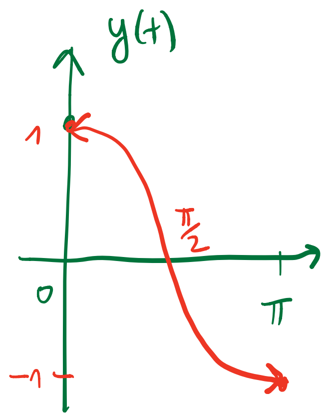
NALOGA: SKICIRAJ KRIVULJO, KI JE PODANA Z

$$x(t) = \frac{1}{\sin t}, \quad y(t) = \cos t, \quad t \in (0, \pi).$$

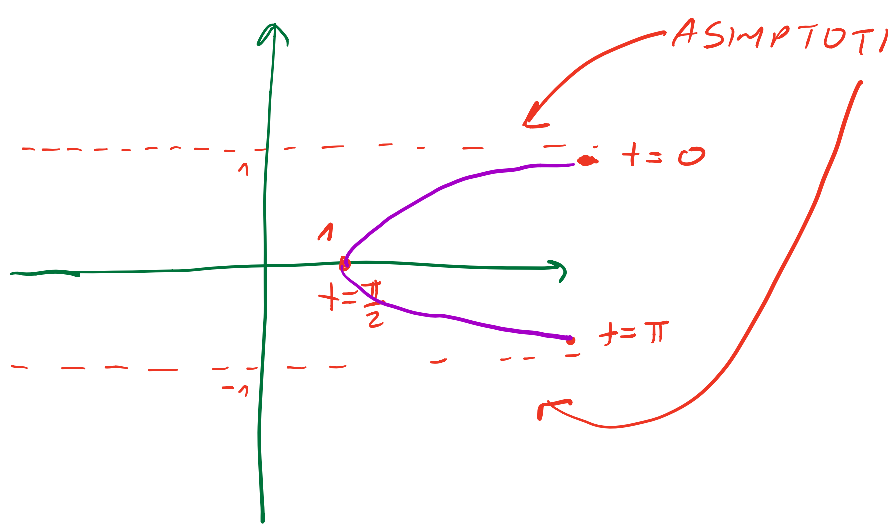
DOLOČI TANGENTO NANJO V TOČKAH $\left(\sqrt{2}, \frac{\sqrt{2}}{2}\right)$ IN $(1,0)$.

REŠITEV:





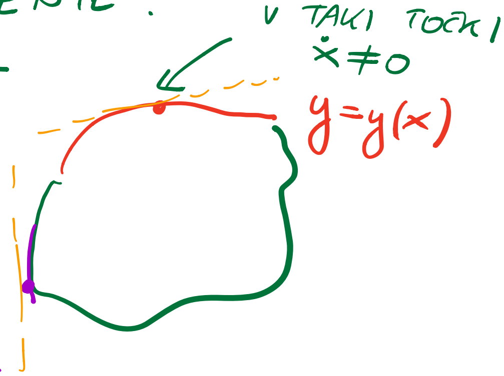
t	x	y
0	∞	1
$\frac{\pi}{2}$	1	0
π	∞	-1



KAKO IZRAČUNAMO KOEFICIENT TANGENTE?

$$k = y'(x) = \frac{dy}{dx} = \frac{dy}{dt}\,\frac{dt}{dx} = \frac{\dot{y}}{\dot{x}}$$

ČE SE DA Y IZRAZITI Z X



ČE $\dot{x} = 0$, JE TANGENTA NAVPIČNA

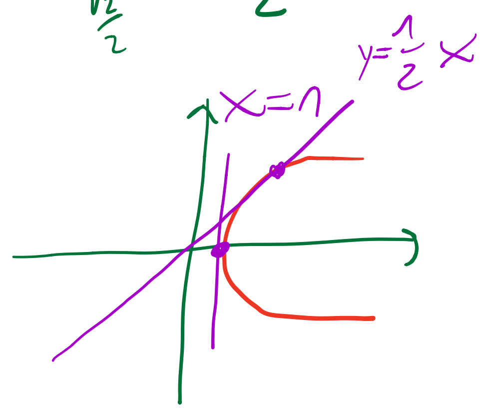
NAŠ PRIMER:

- $(x,y) = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right) \Rightarrow \left.\begin{array}{l} \frac{1}{\sin t} = \sqrt{2} \\ \cos t = \frac{\sqrt{2}}{2} \end{array}\right\} \boxed{t = \frac{\pi}{4}}$

$$k = \left.\frac{\dot{y}}{\dot{x}}\right|_{t=\frac{\pi}{4}} = \left.\frac{-\sin t}{-\frac{1}{\sin^2 t}\cos t}\right|_{t=\frac{\pi}{4}} = \frac{\left(\frac{\sqrt{2}}{2}\right)^3}{\frac{\sqrt{2}}{2}} = \frac{1}{2}$$

ENAČBA: $y - \frac{\sqrt{2}}{2} = \frac{1}{2}\left(x - \sqrt{2}\right)$

$$\boxed{y = \frac{1}{2}x}$$



- $(x,y) = (1,0) \Rightarrow t = \frac{\pi}{2} \Rightarrow \dot{x}\left(\frac{\pi}{2}\right) = \left.-\frac{\cos t}{\sin^2 t}\right|_{t=\frac{\pi}{2}} = 0$

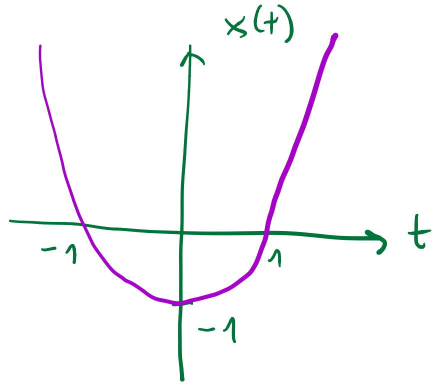
$$\boxed{x = 1}$$

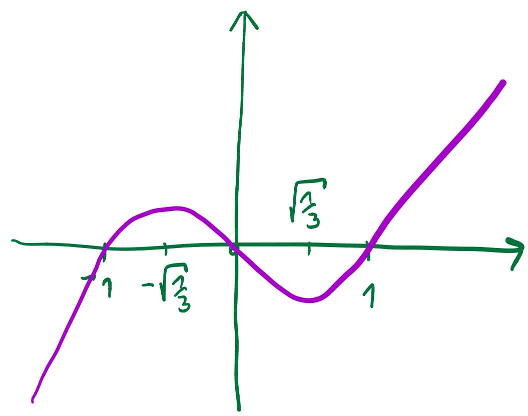
NALOGA: SKICIRAJ KRIVULJO, KI JE PODANA S PREDPISOMA

$$x(t) = t^2 - 1, \quad y(t) = t^3 - t, \quad t \in \mathbb{R}.$$

ALI SE ZA $t \to \pm\infty$ PRIBLIŽUJE KAKI LINEARNI ASIMPTOTI?

REŠITEV: POMAGAJMO SI S TABELO IN SKICO $x(t)$ IN $y(t)$.



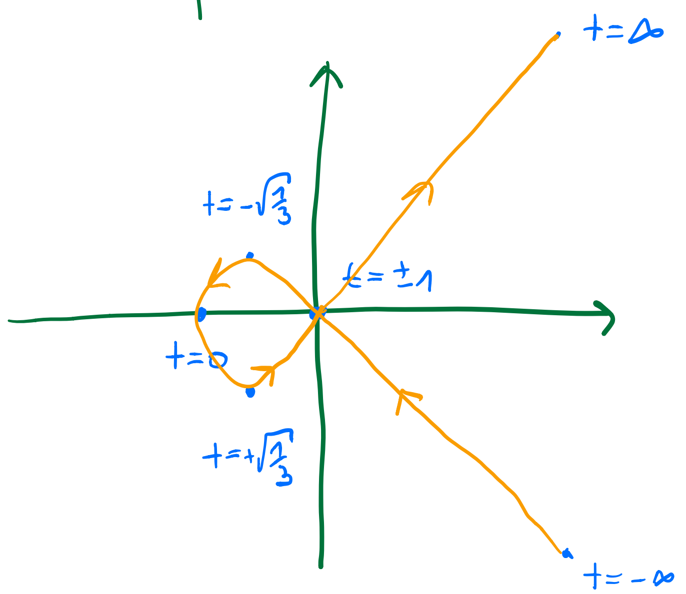


$$\dot{y} = 3t^2 - 1 = 0$$

$$t = \pm\sqrt{\frac{1}{3}}$$

$$y\left(\pm\sqrt{\tfrac{1}{3}}\right) = \mp\frac{2}{3}\sqrt{\frac{1}{3}}$$

t	x	y
$-\infty$	∞	$-\infty$
-1	0	0
$-\sqrt{\frac{1}{3}}$	$-\frac{2}{3}$	$\frac{2}{3}\sqrt{\frac{1}{3}}$
0	-1	0
$\sqrt{\frac{1}{3}}$	$-\frac{2}{3}$	$-\frac{2}{3}\sqrt{\frac{1}{3}}$
1	0	0
∞	∞	∞

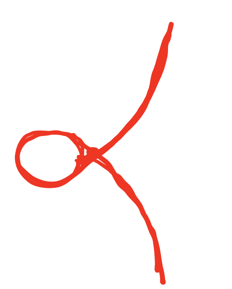


LINEARNA ASIMPTOTA?

ČE OBSTAJA, ZANJO VELJA

$$k = \lim_{t\to\pm\infty} \frac{\dot{y}}{\dot{x}} = \lim_{t\to\pm\infty} \frac{3t^2-1}{t} = \pm\infty.$$

BOLJ PRAVA SLIKA



ODGOVOR JE TOREJ 'NE'. TANGENTE SO VEDNO BOLJ 'STRME', x PA GRE ČEZ VSE MEJE (NI NAVPIČNIH ASIMPTOT).

NALOGA: PREVERI, DA JE S PREDPISOM

$$x(t)=2\cos t-\sin t,\quad y(t)=2\cos t+\sin t,\quad t\in[0,2\pi],$$

PODANA ELIPSA. POKAŽI, DA ZA $t=0$ LAHKO IZRAZIMO $y=y(x)$ IN IZRAČUNAJ y'' V TEJ TOČKI.

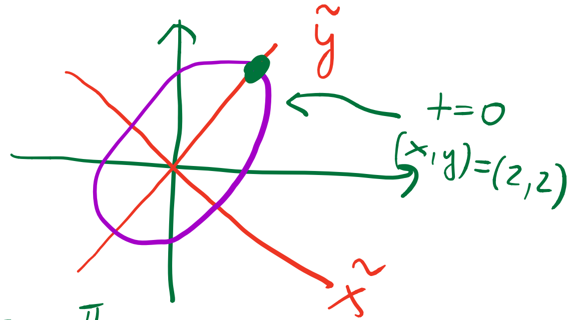
NAMIG: $\cos^2 t+\sin^2 t=1$

REŠITEV:

$$\left.\begin{aligned} x-y&=-2\sin t\\ x+y&=4\cos t \end{aligned}\right\}\quad \left(\frac{x-y}{2}\right)^2+\left(\frac{x+y}{4}\right)^2=1$$

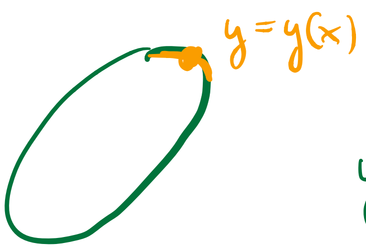
ČE ZAPIŠEMO $\tilde{x}=x-y$ IN $\tilde{y}=x+y$, JE TO ENAČBA ELIPSE:

$$\frac{\tilde{x}^2}{4}+\frac{\tilde{y}^2}{16}=1$$



t.j. IMAMO ELIPSO, KI JE ZAROTIRANA ZA $\frac{\pi}{4}$.

VIDIMO, DA $(2,2)$ NI 'SKRAJNA TOČKA', SAJ $\dot{x}(0)\neq 0$.
TOREJ SE DA IZRAZITI $y=y(x)$ VSAJ NA OKOLICI



ODVAJAMO Z VERIŽNIM PRAVILOM:

$$y''=\frac{d}{dx}y'=\frac{d}{dx}\frac{\dot{y}}{\dot{x}}=\frac{dt}{dx}\frac{d}{dt}\cdot\frac{\dot{y}}{\dot{x}}=$$

$$=\frac{1}{\dot{x}}\;\frac{\ddot{y}\dot{x}-\ddot{x}\dot{y}}{\dot{x}^2}$$

IZRAČUNAJ SAM:

$$\boxed{y''(2)=-4}$$

ALTERNATIVNA POT: $y(x)=\frac{3}{5}x+\frac{4}{5}\sqrt{5-x^2}$
POIŠČI EKSPLICITEN ZAPIS