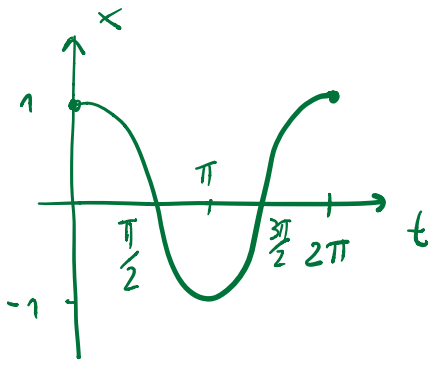
NALOGA: SKICIRAJ KRIVULJI, KI STA PODANI S PREDPISOMA:

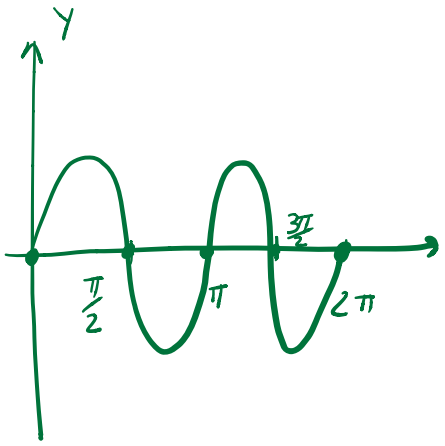
(a) $x(t) = \cos t$, $y(t) = \sin 2t$, $t \in [0, 2\pi]$.

(b) $x(t) = \cos^2 t$, $y(t) = \sin^2 t$, $t \in (0, \pi)$.

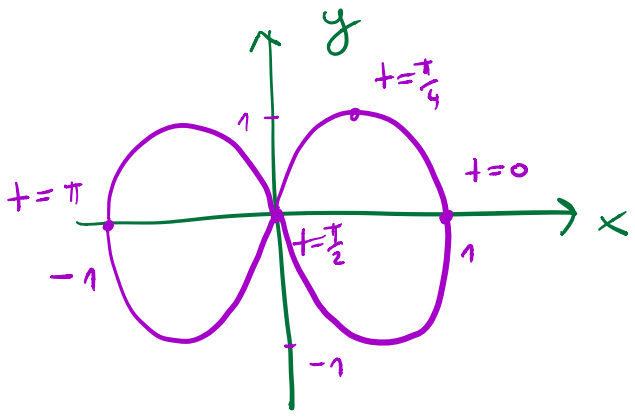
REŠITEV:

(a)





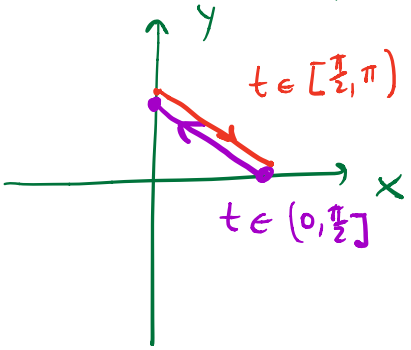
t	x	y
0	1	0
$\frac{\pi}{2}$	0	0
π	-1	0
$\frac{3\pi}{2}$	0	0
2π	1	0



(b) OPAZIMO: $x + y = \cos^2 t + \sin^2 t = 1$, $t \in (0, \pi)$.

t.j. KRIVULJA LEŽI NA PREMICI $y = 1 - x$.

t	x	y
0	1	0
$\frac{\pi}{2}$	0	1
π	1	0



KAJ SE ZGODI PRI $t = \frac{\pi}{2}$ oz. V $(x, y) = (0, 1)$?

$\dot{x} = -2 \cos t \sin t \,|_{t=\frac{\pi}{2}} = 0$

$\dot{y} = 2 \sin t \cos t \,|_{t=\frac{\pi}{2}} = 0$

} PARAMETRIZACIJA SE USTAVI.

NALOGA: DESCARTESOV LIST $x^3 - 3axy + y^3 = 0$, $a > 0$,

(a) Z NASTAVKOM $t = \frac{y}{x}$ PARAMETRIZIRAJ KRIVULJO.

(b) SKICIRAJ KRIVULJO (DOLOČI TUDI LINEARNE ASIMPTOTE).

REŠITEV: $y = tx$

$$x^3 - 3atx^2 + t^3x^3 = 0$$

ČE $x \neq 0$:

$$x(1+t^3) = 3at$$

$$\boxed{x = \frac{3at}{1+t^3}} \qquad t \in \mathbb{R} \setminus \{-1\}$$

$$\boxed{y = \frac{3at^2}{1+t^3}}$$

SKICIRAJNO $x(t)$:

NIČLA: $t = 0$

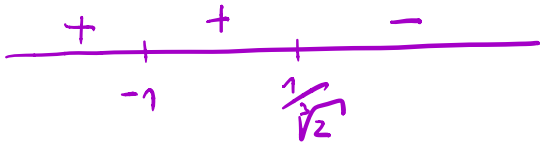
POL: $t = -1$

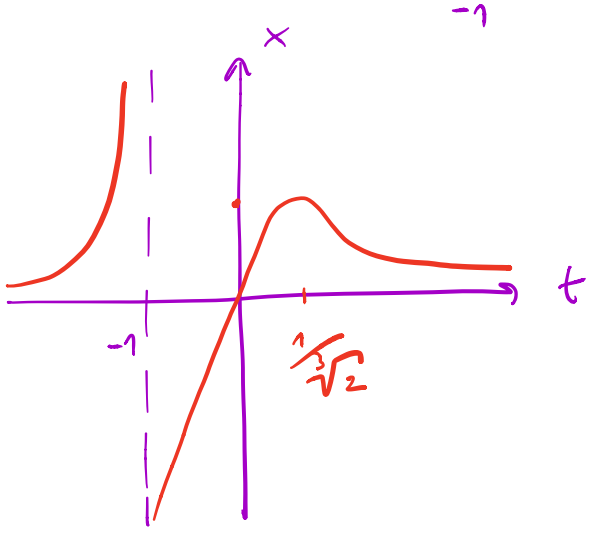
$$\lim_{t \to \pm\infty} \frac{3at}{1+t^3} = 0$$

$$\dot{x} = \frac{3a(1+t^3) - 3at \cdot 3t^2}{(1+t^3)^2} = \frac{3a(1-2t^3)}{(1+t^3)^2} = 0$$

STAC. TOČKA: $t = \frac{1}{\sqrt[3]{2}}$ $\quad x = \frac{3a\sqrt[3]{2}}{1+\frac{1}{2}} = 2a \cdot \frac{1}{\sqrt[3]{2}} = a\sqrt[3]{4}$

NAR. / PADANJE:





SKICIRAJMO $y(t)$:

~ DVOJNA NIČLA $t=0$.

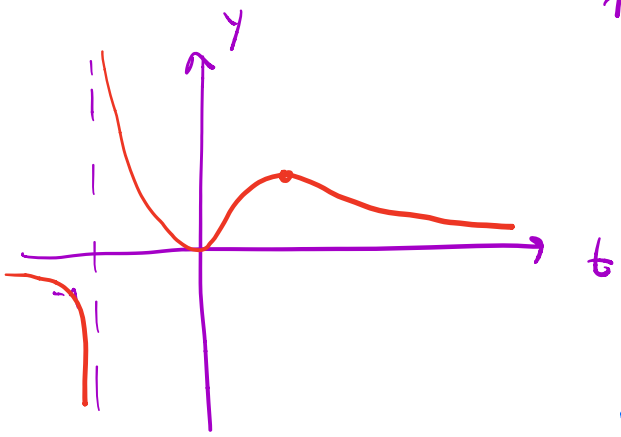
~ POL: $t=-1$

$$\lim_{t\to\pm\infty} \frac{3at^2}{1+t^3} = 0$$

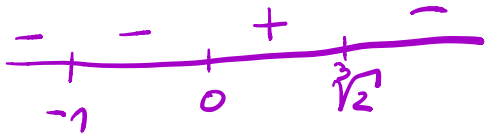
$$\dot{y} = \frac{3at+(2-t^3)}{(1+t^3)^2} = 0$$

STAC. TOČKI: $t=0$ $y=0$

$t=\sqrt[3]{2}$ $y = \frac{3a\sqrt[3]{4}}{1+2} = a\sqrt[3]{4}$



NAR./PADAJOČE:



PRVA SKICA:

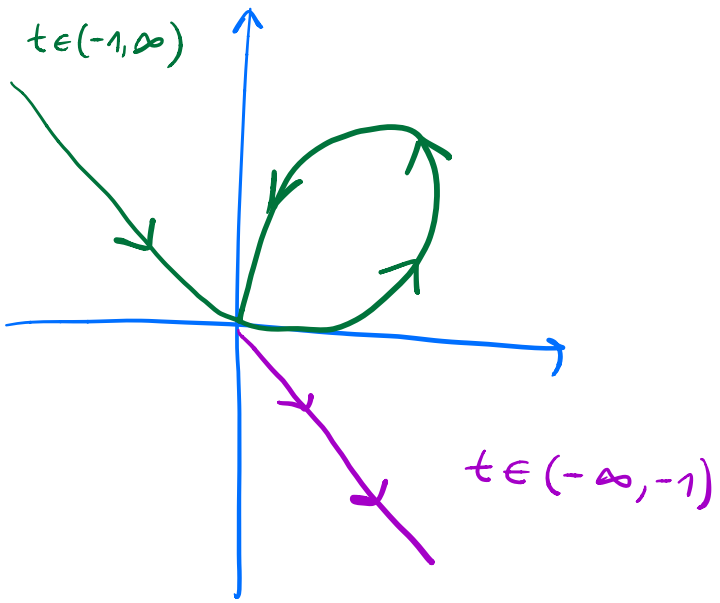


TABELA:

t	x	y
$\pm\infty$	0	0
-1^+	$-\infty$	∞
-1^-	∞	$-\infty$
0	0	0
$1/\sqrt[3]{2}$	$a\sqrt[3]{4}$	$a\sqrt[3]{2}$
$\sqrt[3]{2}$	$a\sqrt[3]{2}$	$a\sqrt[3]{4}$

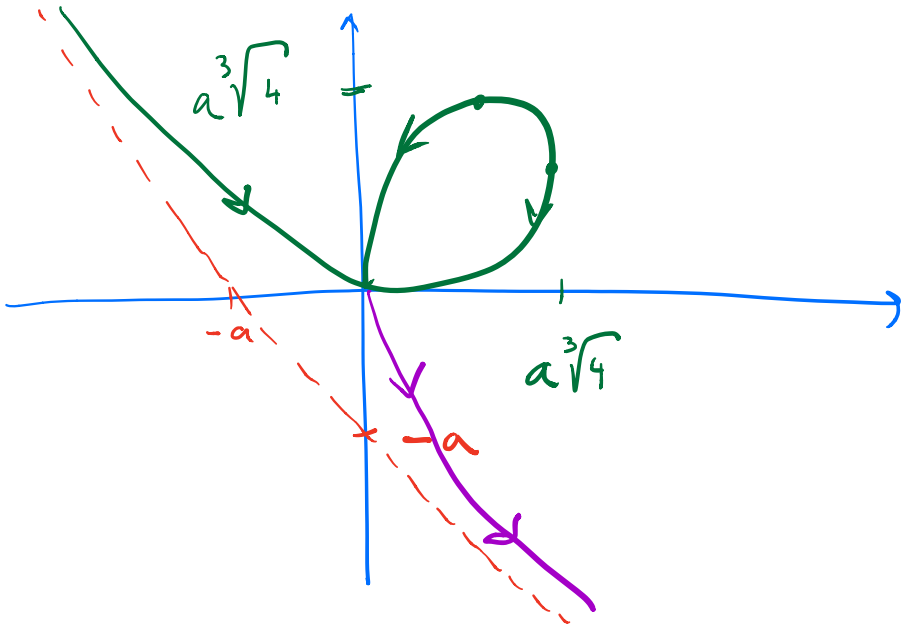
ŽELJA: BOLJ NATANČNO ANALIZIRATI OBNAŠANJE, KO $t\to-1$ T.J. SUMIMO, DA GRE ZA LINEARNO ASIMPTOTO.

$$k = \frac{\dot{y}}{\dot{x}} = \frac{t(2-t^3)}{1-2t^3} \xrightarrow{t\to-1} \frac{-3}{1+2} = -1$$

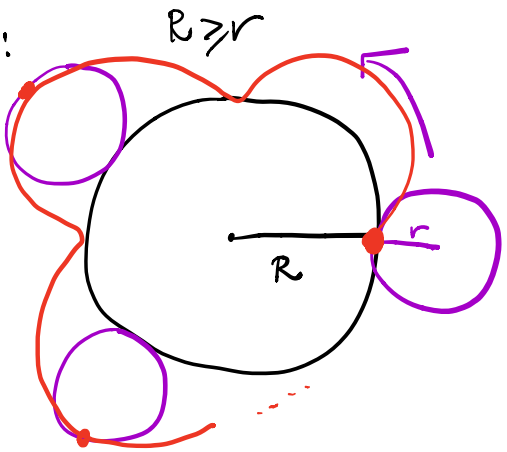
$$n = \lim_{t\to-1} y - kx = \lim_{t\to-1} (y+x) = \lim_{t\to-1} \left(\frac{3at^2}{1+t^3} + \frac{3at}{1+t^3}\right) =$$

$$= \lim_{t \to -1} \frac{3a + (t+1)}{(1+t)(1-t+t^2)} = \frac{-3a}{+3} = -a$$

BOLJ NATANČNA SKICA:

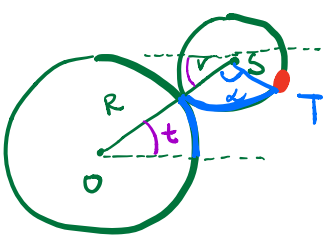


NALOGA:



PARAMETRIZIRAJ KRIVULJO, KI JO OPIŠE TOČKA, KI SE NAHAJA NA MANJŠEM KROGU, KI SE KOTALI PO VEČJEM. SKICIRAJ JO ZA $R = r$.

PARAMETRIZACIJA:



SREDIŠČE MANJŠEGA KROGA:

$\vec{OS} = ((R+r)\cos t, (R+r)\sin t)$

TOČKA T GLEDE NA S:

DOLŽINA MODRIH LOKOV JE ENAKA: $Rt = r\alpha$.

$\vec{ST} = (-r\cos(t+\alpha), -r\sin(t+\alpha))$

$\vec{ST} = \left(-r\cos\left(\left(1+\frac{R}{r}\right)t\right), -r\sin\left(\left(1+\frac{R}{r}\right)t\right)\right)$

SKUPNO:

$$\overrightarrow{OT} = \overrightarrow{OS} + \overrightarrow{ST} = \left((R+r)\cos t - r\cos\left((1+\tfrac{R}{r})t\right),\ (R+r)\sin t - r\left((1+\tfrac{R}{r})t\right)\right)$$

$$x(t) = (R+r)\cos t - r\cos\left((1+\tfrac{R}{r})t\right)$$

$$y(t) = (R+r)\sin t - r\sin\left((1+\tfrac{R}{r})t\right)$$

NAŠ PRIMER:

$R=r$: $x(t) = 2r\cos t - r\cos(2t)$

$y(t) = 2r\sin t - r\sin(2t)$

BREZ ŠKODE ZA SPLOŠNOST: $r=1$:

$$x = (2\cos t - \cos^2 t + \sin^2 t) =$$

$$= (2\cos t - \cos^2 t + 1 - \cos^2 t) =$$

$$= (1 + 2\cos t - 2\cos^2 t)$$

SKICIRAJMO FUNKCIJO BREZ ENICE IN JO NATO PRESTAVIMO:

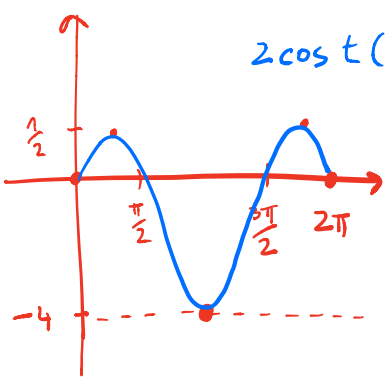
$$2\cos t - 2\cos^2 t = 2\cos t(1-\cos t) = 0$$

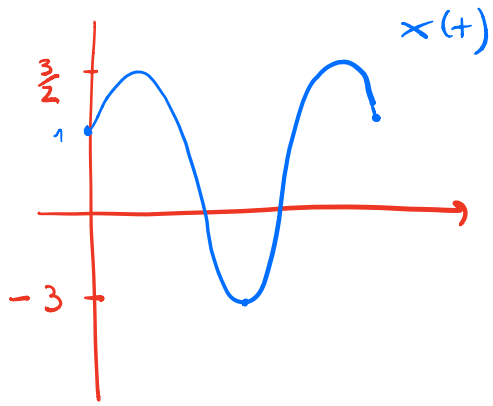
NIČLE: $\frac{\pi}{2}, \frac{3\pi}{2}, 0, 2\pi$

ODVOD: $-2\sin t + 4\cos t \sin t = 0$

S.T.: $\sin t = 0$: $t = 0, \pi, 2\pi$

$\cos t = \frac{1}{2}$: $t = \frac{\pi}{3}, \frac{5\pi}{3}$





$$y = 2\sin t - \sin 2t = 2\sin t - 2\sin t\cos t = 2\sin t(1-\cos t)$$

NIČLE: $t = 0, \pi, 2\pi$;

S.T.: $\dot{y} = 2\cos t(1-\cos t) + 2\sin t \cdot \sin t = 2(\cos t - 2\cos^2 t + 1)$

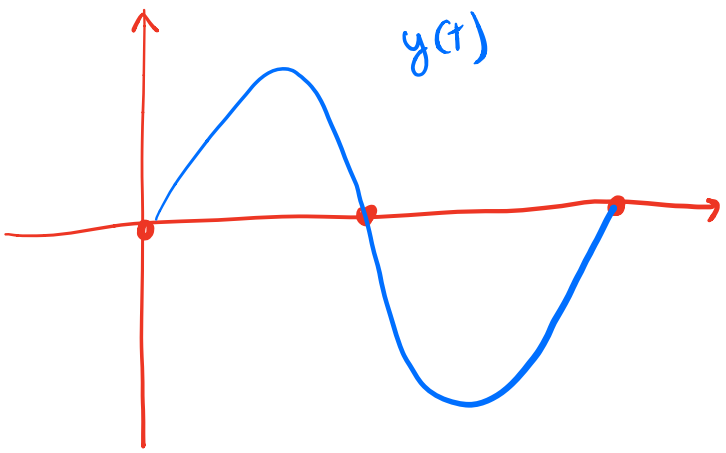
$$\cos t = \frac{-1 \pm \sqrt{1+8}}{-4} = \frac{-1\pm 3}{-4} = -\frac{1}{2} \text{ ALI } 1$$

$\cos t = 1 \Rightarrow t = 0$ ALI 2π

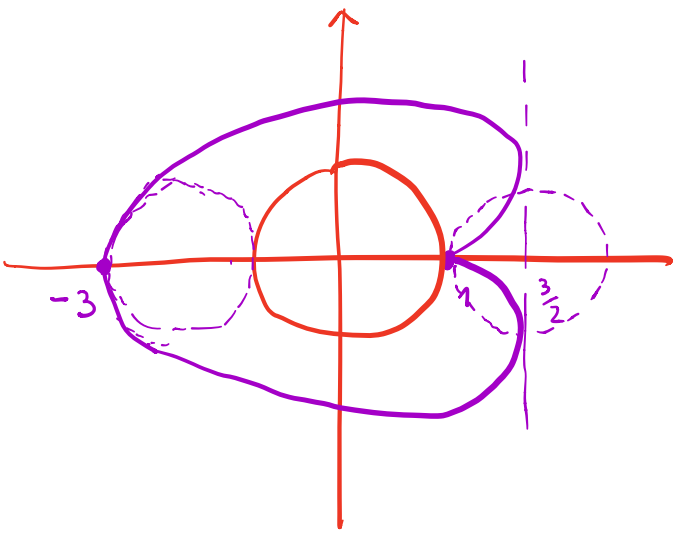
$\cos t = -\frac{1}{2} \Rightarrow t = \frac{2\pi}{3}$ ALI $\frac{4\pi}{3}$.

$y\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}\left(1+\frac{1}{2}\right) = \frac{3\sqrt{3}}{4}$

$y\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}\left(1+\frac{1}{2}\right) = -\frac{3\sqrt{3}}{4}$

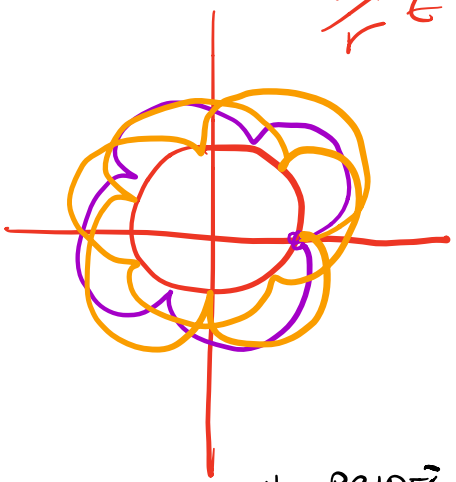


SKUPNA SLIKA:



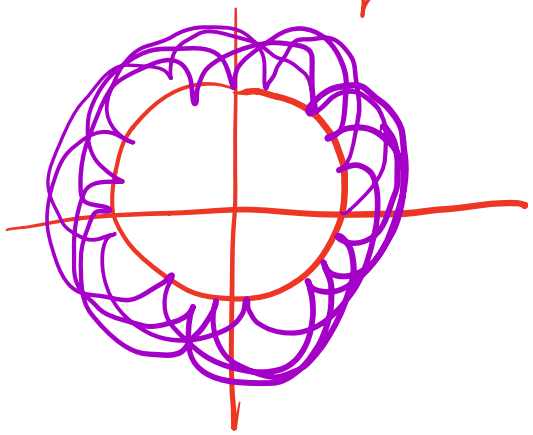
KAKŠNA JE SLIKA V SPLOŠNEM?

$\frac{R}{r} \in \mathbb{Q}$



PO NEKAJ KROGIH PRIDEŠ NA ZAČETEK

$\frac{R}{r} \in \mathbb{R} \setminus \mathbb{Q}$



NIKOLI VEČ NE PRIDEŠ NA ZAČETEK