$$P = [\underbrace{v_1 \ldots v_m}_{\text{baza za } \ker(A-\lambda I)} \; v_{m+1} \ldots v_n]$$

$$AP = [\underset{\lambda v_1}{\underset{\parallel}{Av_1}} \ldots \underset{\lambda v_m}{\underset{\parallel}{Av_m}} \; \underset{\text{razvoj po } v_1, \ldots, v_n}{\underset{\parallel}{Av_{m+1}}} \ldots \underset{\text{razvoj po } v_1 \ldots v_n}{\underset{\parallel}{Av_n}}] = ?$$

$$v_1 \in \ker(A - \lambda I) \Rightarrow (A - \lambda I)v_1 = 0 \Rightarrow$$

$$Av_1 - \lambda \underbrace{I v_1}_{v_1} = 0 \Rightarrow Av_1 = \lambda v_1$$

razvoj Av_{m+1} razvoj od Av_n

$$? = \underbrace{[v_1 \ldots v_m \; v_{m+1} \ldots v_n]}_{P} \underbrace{\left[\begin{array}{ccccc} \overbrace{\lambda \quad \quad 0}^{m} & * & \cdots & * & \\ \vdots \quad \quad \vdots & \vdots & & & \} B \\ 0 \quad \quad \lambda & * & \cdots & * & \\ 0 \quad \quad 0 & * & \cdots & * & \\ \vdots \quad \quad \vdots & \vdots & & & \} C \\ 0 \quad \quad 0 & * & \cdots & * & \end{array}\right]}_{\left[\begin{array}{cc} \lambda I_m & B \\ 0 & C \end{array}\right]}$$

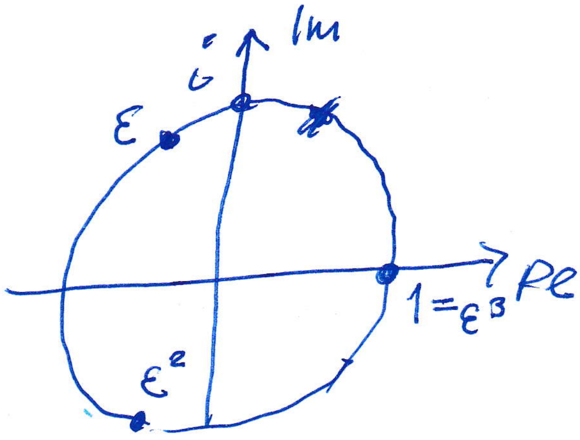
Dokazali smo, da je

$$P^{-1}AP = \left[\begin{array}{cc} \lambda I_m & B \\ 0 & C \end{array}\right]$$

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$e^{2\pi i} = 0$$

$$e^{2\pi i/n} = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$$



$$n=3 \Rightarrow e^{2\pi i/n} = e^{2\pi i/3} = \underbrace{\cos\frac{2\pi}{3}}_{} + i\underbrace{\sin\frac{2\pi}{3}}_{} = \varepsilon$$

$$= -\frac{1}{2} \qquad \frac{\sqrt{3}}{2}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cancel{0} & \cancel{1} & \cancel{0} & \cancel{0} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\underbrace{c_0 \underset{\underset{I}{\parallel}}{A^0} + c_1 \underset{\underset{A}{\parallel}}{A^1} + c_2 A^2 + c_3 A^3}_{} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix}$$

$= p(A)$, kjer je $p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$

Lema (v dokazih Trditev 3 in 4)

Če velja $n \leq m_1 + \dots + m_k$ in $n = n_1 + \dots + m_k$ in $m_1 \leq n_1, \dots, m_k \leq n_k$, potem velja $m_1 = n_1, \dots, m_k = n_k$

Dokaz: Ker je $m_i \leq n_i$, obstajajo taki $r_i \geq 0$, da je

$$n_1 = m_1 + r_1$$
$$\vdots$$
$$n_k = m_k + r_k$$

Če te enačbe seštejemo, dobimo

$$\underbrace{n_1 + \dots + n_k}_{n} = m_1 + \dots + m_k + r_1 + \dots + r_k$$

predpostavka

Odtod sledi $r_1 + \dots + r_k = n - (m_1 + \dots + m_k) \leq 0$

Torej je $r_1 = \quad = r_k = 0$, ker so $r_i \geq 0$

Lema: Za vsako permutacijo π je diagonalna matrika $D = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix}$ podobna diagonalni matriki $\begin{bmatrix} d_{\pi(1)} & & \\ & \ddots & \\ & & d_{\pi(n)} \end{bmatrix}$

Primer: Matrika $\begin{bmatrix} 1 & & & \\ & 2 & & \\ & & 1 & \\ & & & 2 \end{bmatrix}$ je podobna matriki $\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 2 & \\ & & & 2 \end{bmatrix}$

ker je

$$\begin{bmatrix} 1&0&0&0 \\ 0&0&1&0 \\ 0&1&0&0 \\ 0&0&0&1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 2 & & \\ & & 1 & \\ & & & 2 \end{bmatrix} \underset{\begin{matrix} \parallel \\ \begin{bmatrix} 1&0&0&0 \\ 0&0&1&0 \\ 0&1&0&0 \\ 0&0&0&1 \end{bmatrix}^{-1} \end{matrix}}{\begin{bmatrix} 1&0&0&0 \\ 0&0&1&0 \\ 0&1&0&0 \\ 0&0&0&1 \end{bmatrix}} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 2 & \\ & & & 2 \end{bmatrix}$$

Dokaz: $D[e_{\pi(1)}, \dots, e_{\pi(n)}] =$

$= [De_{\pi(1)} \dots De_{\pi(n)}]$

$= [d_{\pi(1)}e_{\pi(1)} \dots d_{\pi(n)}e_{\pi(n)}]$

$= [e_{\pi(1)}, \dots e_{\pi(n)}] \begin{bmatrix} d_{\pi(1)} & \ddots & d_{\pi(n)} \end{bmatrix}$

Opomba! Torej v diagonalizaciji matrike lahko predpost. da enaki diagonalni elementi ležijo skupaj.