Naloga: Za matriko

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & -1 \\ 4 & 1 & 0 & 1 \\ 1 & 1 & 2 & -3 \end{bmatrix}$$

izračunajte LU razcep z delnim pivotiranjem.

Rešitev: Iščemo permut. matriko P, sp. trikotno matriko L z 1 na diag. in zg. trikotno matriko U, da je

$$PA = LU.$$

Matriko P lahko predstavimo z __vektorjem__

$p = [i_1, i_2, \dots, i_n]$, kjer je $i_1 i_2 \dots i_n$ permutacija števil $1, 2, \dots, n$.

Pri tem velja, da ima matrika P

v k-ti vrstici element 1 na

menjava i_k.

Začetek: $p = [1, 2, \ldots, n]$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad p = [3, 2, 1, 4]$

$$\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & -1 \\ \boxed{4} & 1 & 0 & 1 \\ 1 & 1 & 2 & -3 \end{array} \xrightarrow{\text{pivot.}} \begin{array}{cccc} 4 & 1 & 0 & 1 \\ 2 & 0 & 1 & -1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 2 & -3 \end{array}$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad p = [3, 4, 1, 2]$

$$\begin{array}{cccc} 4 & 1 & 0 & 1 \\ 1/2 & -1/2 & 1 & -3/2 \\ 1/4 & -1/4 & 0 & 3/4 \\ 1/4 & 3/4 & 2 & -13/4 \end{array} \xrightarrow{\text{pivot}} \begin{array}{cccc} 4 & 1 & 0 & 1 \\ 1/4 & 3/4 & 2 & -13/4 \\ 1/4 & -1/4 & 0 & 3/4 \\ 1/2 & -1/2 & 1 & -3/2 \end{array}$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad p = [3, 4, 2, 1]$

$$\begin{array}{cccc} 4 & 1 & 0 & 1 \\ 1/4 & 3/4 & 2 & -13/4 \\ 1/4 & -1/3 & 2/3 & -1/3 \\ 1/2 & -2/3 & 7/3 & -11/3 \end{array} \xrightarrow{\text{pivot.}} \begin{array}{cccc} 4 & 1 & 0 & 1 \\ 1/4 & 3/4 & 2 & -13/4 \\ 1/2 & -2/3 & 7/3 & -11/3 \\ 1/4 & -1/3 & 2/3 & -1/3 \end{array}$$

$$\begin{array}{cccc} 4 & 1 & 0 & 1 \\ 1/4 & 3/4 & 2 & -13/4 \\ 1/2 & -2/3 & 7/3 & -11/3 \\ 1/4 & -1/3 & 2/7 & 5/7 \end{array}$$

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 1 & 0 & 0 \\ 1/2 & -2/3 & 1 & 0 \\ 1/4 & -1/3 & 2/7 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 4 & 1 & 0 & 1 \\ 0 & 3/4 & 2 & -13/4 \\ 0 & 0 & 7/3 & -11/3 \\ 0 & 0 & 0 & 5/7 \end{bmatrix}$$