Naloga: Računamo zaporedje približkov

$$x_{r+1} = g(x_r), \quad r = 0, 1, \ldots,$$

kjer je $g(x) = \dfrac{x^2 + a}{2x}$, $a > 0$.

① Kam lahko konvergira $(x_r)_{r=0}^{\infty}$?

② Kakšen je red konvergence?

<u>Rešitev</u>: Konvergira lahko le k α:

$\alpha = g(\alpha)$, α je negibna točka.

Recimo, da je $\alpha = \lim_{r\to\infty} x_r$. Potem

je $\lim_{r\to\infty} x_{r+1} = \lim_{r\to\infty} g(x_r)$

$$\alpha = g(\lim_{r\to\infty} x_r) = g(\alpha)$$

$$= \frac{\alpha^2 + a}{2\alpha} \quad / \cdot 2\alpha$$

$$2\alpha^2 = \alpha^2 + a$$

$$\alpha^2 = a$$

$$\alpha = \pm\sqrt{a}$$

(2) Red konvergence:

a) $\underline{g(\sqrt{a}) = \sqrt{a}}$

$$g(\sqrt{a}) = \frac{(\sqrt{a})^2 + a}{2\sqrt{a}} = \frac{\cancel{2}a}{\cancel{2}\sqrt{a}} = \sqrt{a} \checkmark$$

b) $$g'(x) = \left(\frac{x^2+a}{2x}\right)' = \frac{2x \cdot 2x - (x^2+a)(2)}{4x^2}$$

$$= 1 - \frac{x^2+a}{2x^2}$$

$$g'(\sqrt{a}) = 1 - \frac{a+a}{2a} = 1 - 1 = \underline{\underline{0}}$$

$\Rightarrow$ red konvergence je ~~vsaj~~ $\underline{\underline{2}}$.

$$g''(x) = -\frac{1}{2}\left(\frac{2x\,x^2 - (x^2+a)x\cdot 2}{x^4}\right)$$

$$= +\frac{1}{\cancel{2}}\left(\frac{+\cancel{2}ax}{x^4}\right)$$

$$g''(\sqrt{a}) = \frac{\cancel{a}\sqrt{a}}{a^{\cancel{2}}} = \frac{\sqrt{a}}{a} \neq 0$$

$\Rightarrow$ red je točno $\underline{\underline{2}}$.

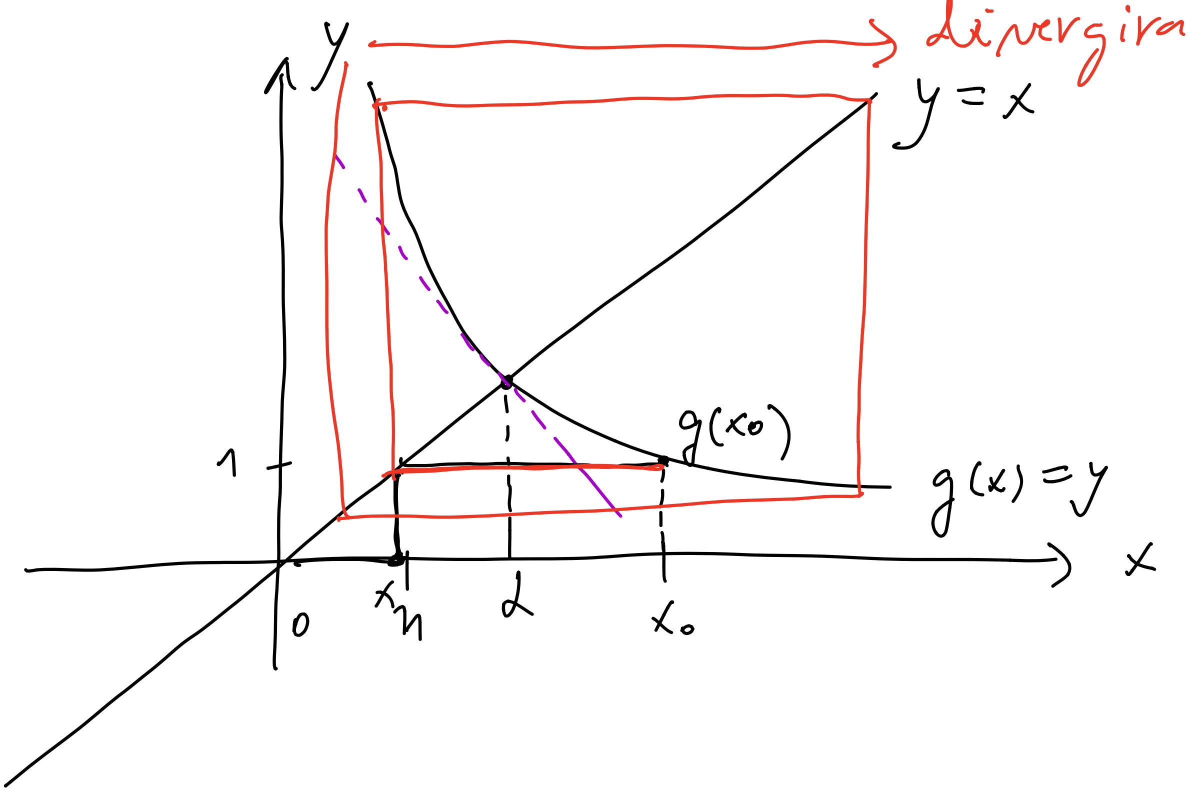
Posplošitev: $g(x) = \dfrac{(k-1)x_r^k + a}{k x_r^{k-1}}$; $a>0$, $k \in \mathbb{N}$, $k \geq 2$

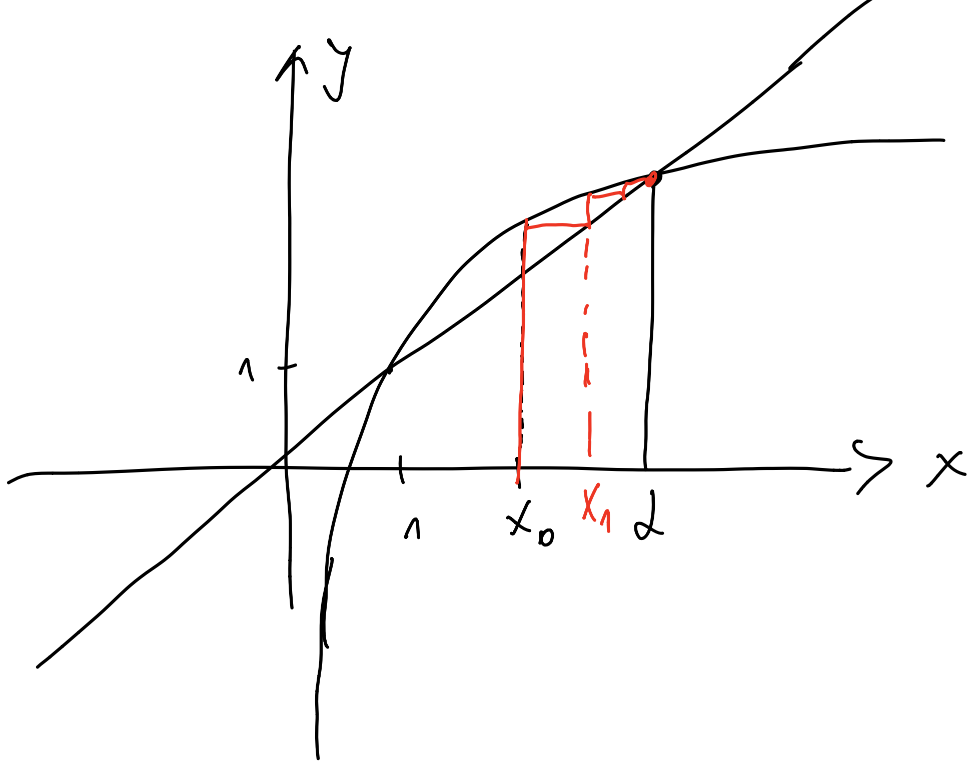
$\Rightarrow$ $(x_r)_{r=0}^{\infty}$, $x_{r+1} = g(x_r)$ konvergira k k-temu korenu iz a ($\sqrt[k]{a}$).

$k=3$: $g(x) = \dfrac{2x^3 + a}{3x^2}$

Red je 2 (doma).

"Grafična konvergenca $x_{r+1} = g(x_r)$"

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