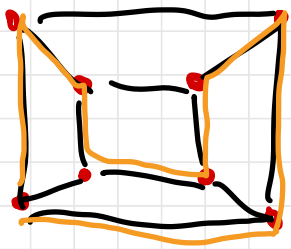
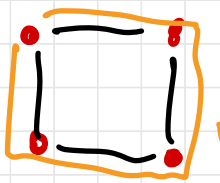
DS 2 VAJE 6

1. ZA KATERE $n > 0$ SO HIPERKOCKE Q_n EULERJEVI GRAFI? ZA KATERE $n, m > 0$ SO $K_{n,m}$ EULERJEVI?



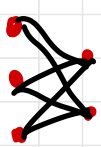
MMA EULER. OBHODA



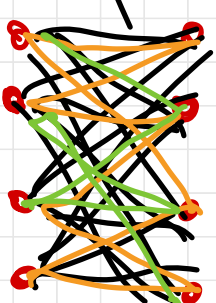
EULERJEV

EULERJEV OBHOD $\Leftrightarrow$ VSA VOZLIŠČA SODE STOPNJE

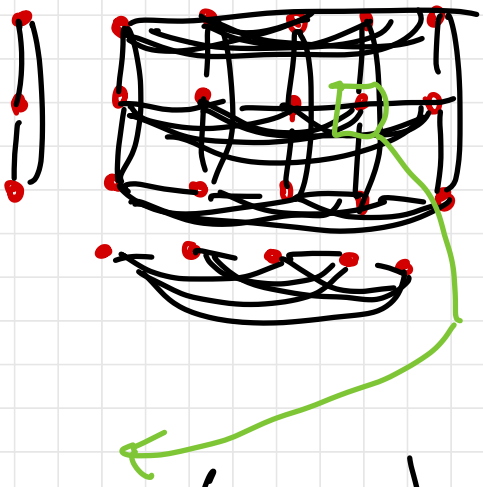
Q_n : STOPNJE n $\Rightarrow$ (EULERJEV $\Leftrightarrow$ n SOD)

$K_{n,m}$:  STOPNJE m, n

$\Rightarrow$ (EULERJEV $\Leftrightarrow$ n, m SODA)



2. DOKAŽI, DA JE KARTEZIČNI PRODUKT DVEH EULERJEVIH GRAFOV TUDI EULERJEV.

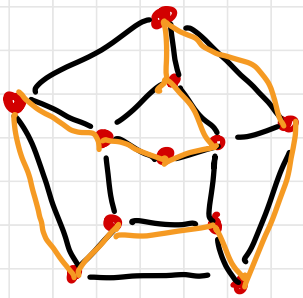


$C_3 \square K_5$

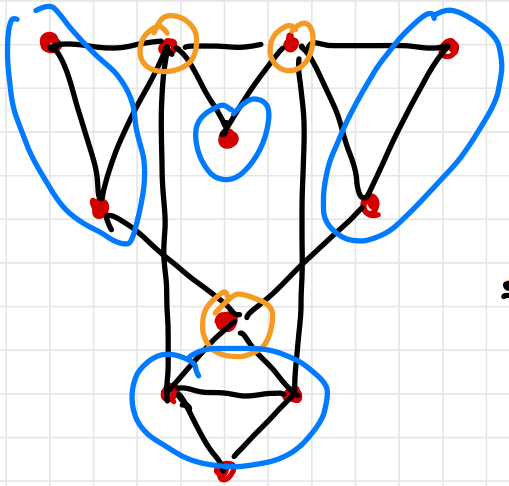
$$\deg_{G \square H}(u, v) = \deg_G(u) + \deg_H(v)$$

SODO + SOD = SODO

3. KATERI GRAFI SO HAMILTONOVI?



JE HAMILTONOV

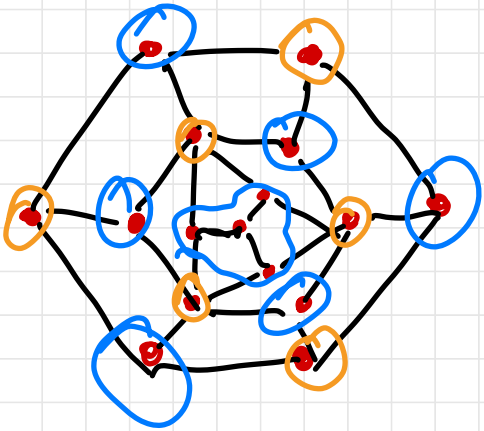


$|S| = 3$

↑ ORANŽNA VOZLIŠČA

$\Rightarrow$ $G \setminus S$ IMA 4 POVEZANE KOMPONENTE

$\Rightarrow$ NI HAMILTONOV



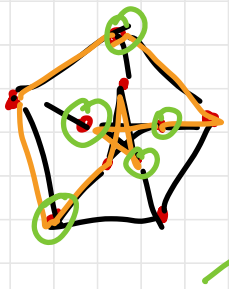
$|S| = 6$

$\Omega(G \setminus S) = 7$

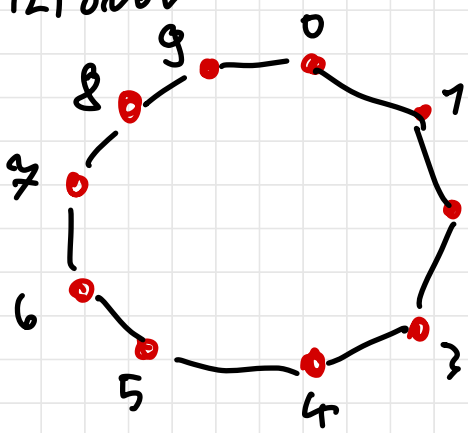
$\Rightarrow$ GRAF NI HAMILTONOV

4. ALI JE PETERSENOV GRAF HAMILTONOV?

$P(5,2)$

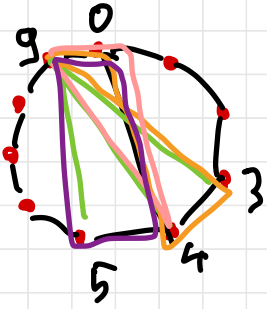


ČE HAMILTONOV



+ DIAGONALE

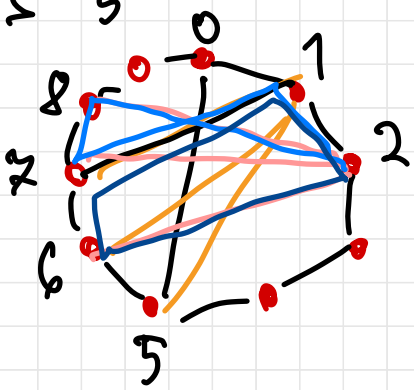
OŽINA $P(5,2) = 5$, 3-REGULAREN



- ČE 0 POVEŽEMO 4
  9 LAHKO POVEŽEMO S 3, 4 ALI 5

⇒ VEDNO DOBIMO CIKEL DOLŽINE 5 //

• ČE 0 POVEŽEMO S 5



⇒ 1 POVEŽEMO S 7

⇒ 2 POVEŽEMO S 6 ALI 8 6

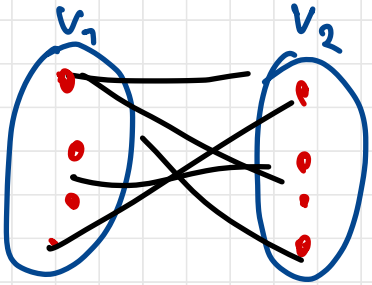
V OBEH PRIMERIH DOBIMO

↳ CIKEL

//

⇒ $P(5,2)$ NI HAMILTONOV

5. NAJ BO G DVODELEN GRAF IN V_1, V_2 BIPARTICIJA VOZLIŠČ G. POKAŽI, DA ČE $|V_1| \neq |V_2|$, GRAF NI HAMILTONOV.



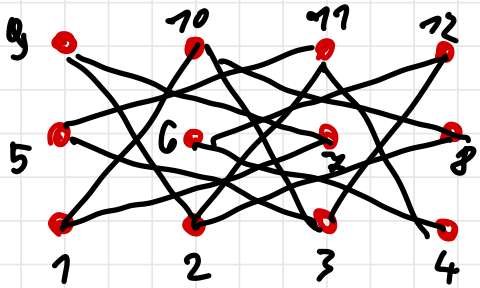
RECIMO $|V_1| < |V_2|$

⇒ $S = V_1$

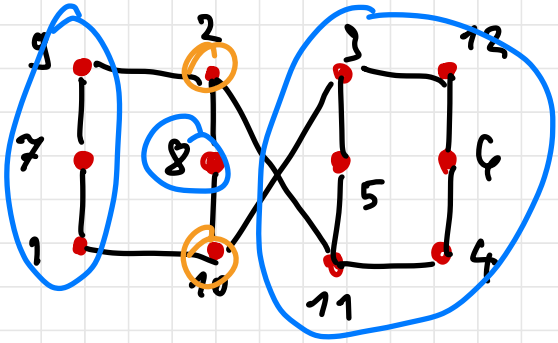
⇒ $|S| < |G \setminus S| = |V_2|$

⇒ NI HAMILTONOV

6. ALI SE LAHKO ŠAHOVSKI KONJIČEK SPREHODI ČEZ VSA POLJA 3×4 ŠAHOVNICE TAKO, DA GRE ČEZ VSAKO NATANKO ENKRAT IN ZAČNE IN KONČA V ISTEM.



ALI JE HAMILTONOV?



$|S| = 3 < 4 = \Omega(G \setminus S)$

$\Rightarrow$ NI HAM.

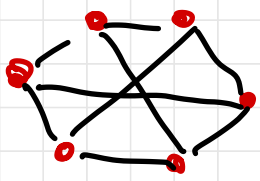
PREDSTAVITEV JE POMEMBNA:

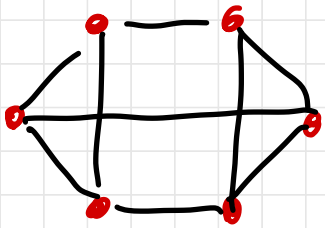
7. POIŠČI VSE 3-REGULARNE GRAFE NA 6 VOZLIŠČIH.

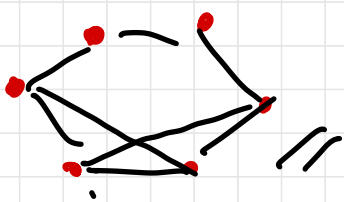
DIRAC: $\forall v \ \deg(v) \geq \frac{|V|}{2} \Rightarrow G$ HAMILTONOV

$3 \geq \frac{6}{2} \Rightarrow$ ✓

$\Rightarrow$ TAKI GRAFI HAMILTONOVI!



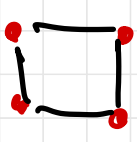




SAMO DVA NEIZOMORFNA

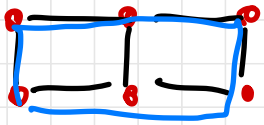
8. $P_{n_1} \square P_{n_2}$ HAMILTONOVI?

$P_2 \square P_2$

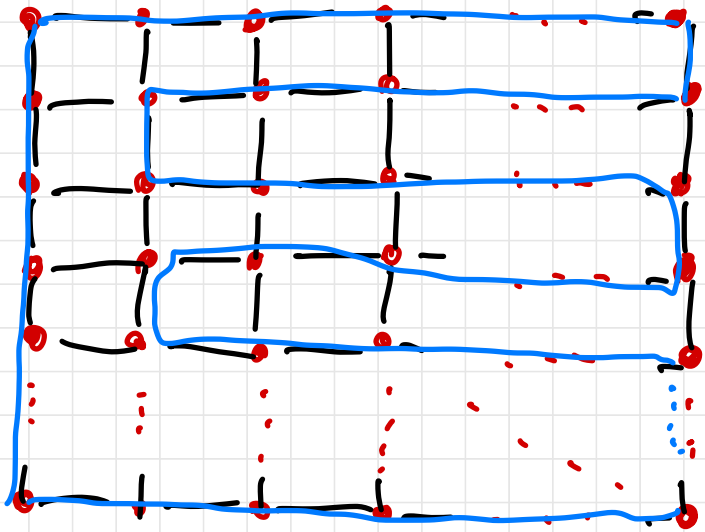


✓

$P_3 \square P_2$



P_{n_1}

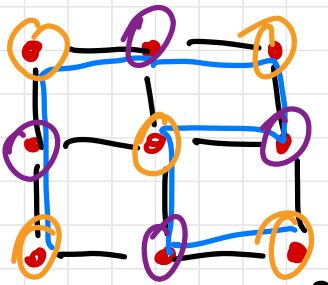


P_{n_2}

ČE n_1 SOD $\Rightarrow$ IMAMO HAM. CIKEL

SIMETRIČNO ČE n_2 SOD $\Rightarrow$ HAM.

ČE n_1, n_2 LIHA:



DVODELEN GRAF

2 9 VOZLIŠČI

$\Rightarrow$ LAHKO ODSTRANIMO BIPARTICIJO VELIKOSTI 4

V SPLOŠNEM:

DVODELEN GRAF z $n_1 \cdot n_2$ VOZLIŠČI
(↑ LIHO)

$\Rightarrow$ BIPARTICIJA $V_1, V_2 \Rightarrow |V_1| \neq |V_2|$

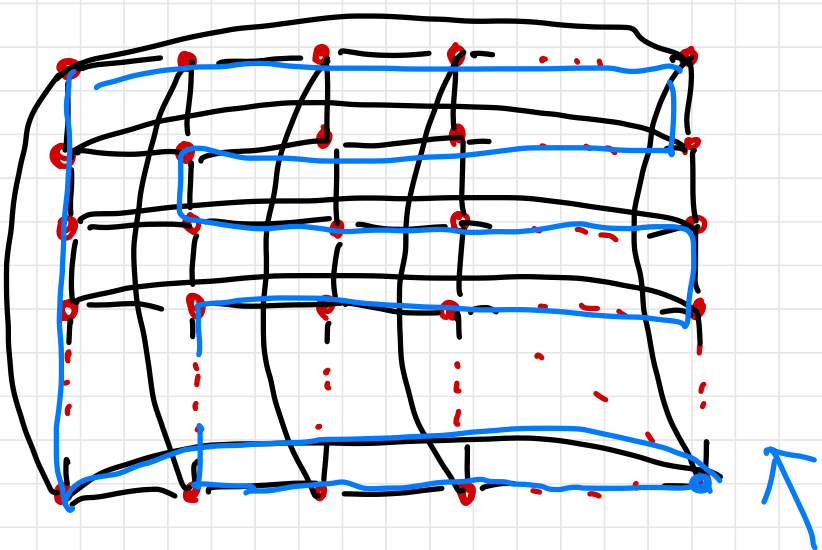
$\Rightarrow$ NI HAMILTONOV

9. POKAŽI, DA JE KARTEZIČNI PRODUKT HAM. GRAFOV HAMILTONOV.

G VSEBUJE VPET CIKEL

H -II-

$\Rightarrow$ $G \square H$ VSEBUJE VPET $C_{n_1} \square C_{n_2}$



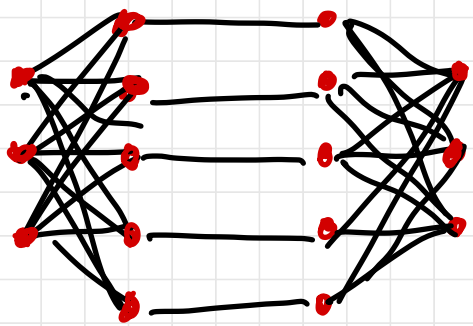
ČE n_1 ALI n_2 SOD, NAJDEMO CIKEL KOT PREJ.

CE OBA LIHA.

✓

10. G_n ZA $n \geq 3$:

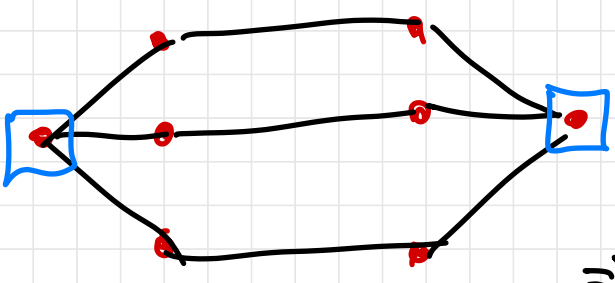
$K_{n,n-2}$



$K_{n,n-2}$

ZA KATERE n JE G_n HAMILTONOV?

$n=3$

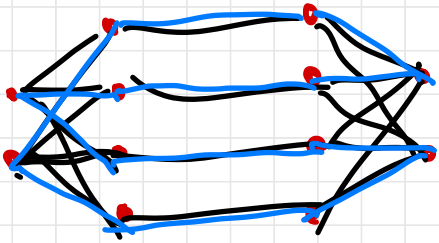


$|S|=2$

$\Omega(G \setminus S) = 3$

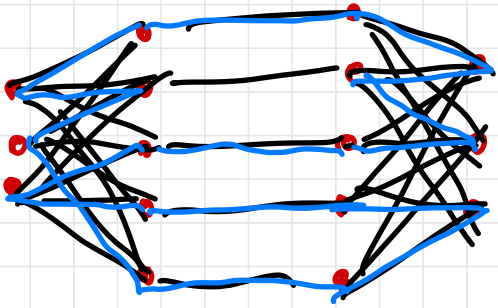
$\Rightarrow$ NI HAM.

$n=4$



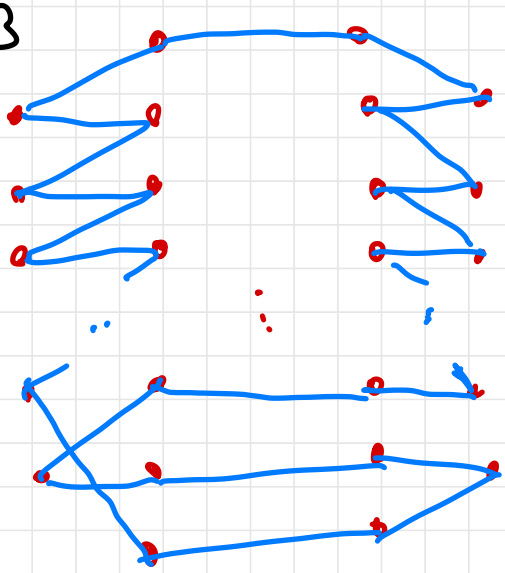
$\Rightarrow$ JE HAMILTONOV

$n=5$



$\Rightarrow$ JE HAM.

$n > 3$



$\Rightarrow$ JE HAM.