DS 2 VAJE 7

1. PREVERI ALI SO GRAFI RAVNINSKI:

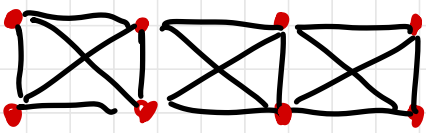
ČE GRAF RAVNINSKI

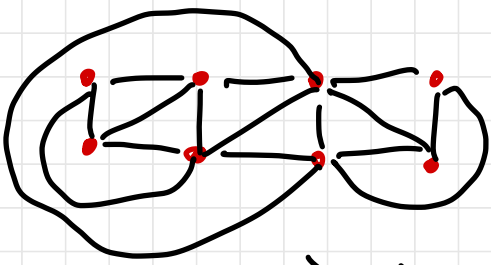
$|E(G)| \leq 3|V(G)| - 6$

$16 \leq 3 \cdot 8 - 6$

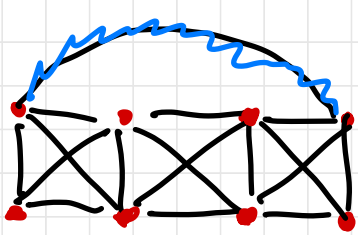
≤ 18

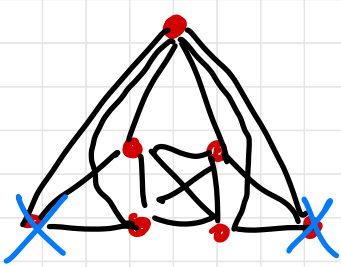
↑ NE POMAGA

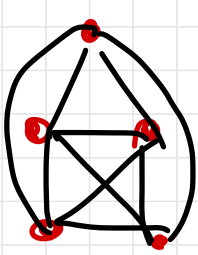
- 



⇒ JE RAVNINSKI

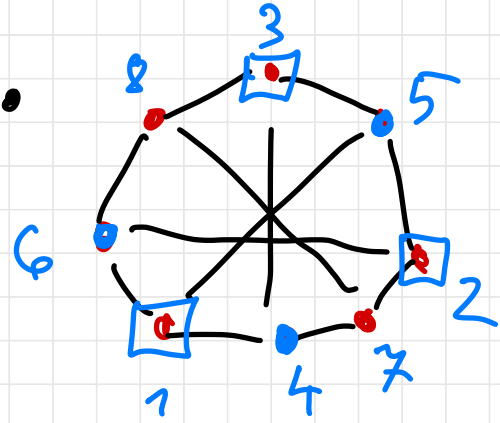
- 

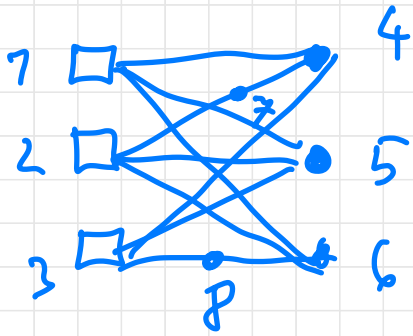




MINOR K_5

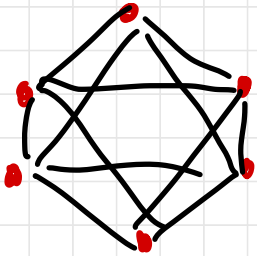
NI RAVNINSKI

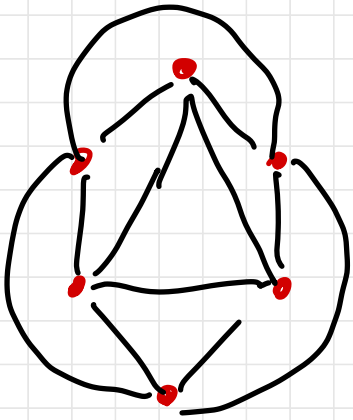




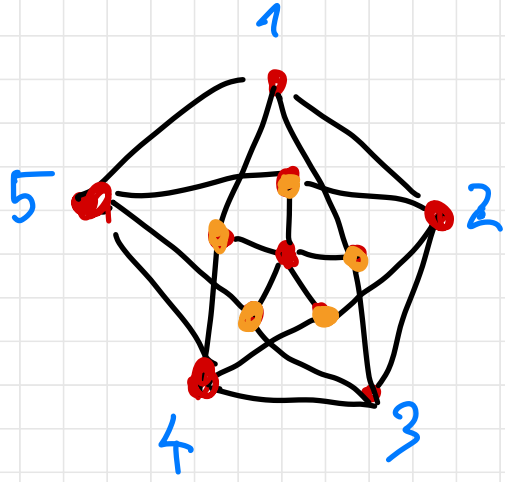
VSEBUJE SUBDIVIZIJO $K_{3,3}$

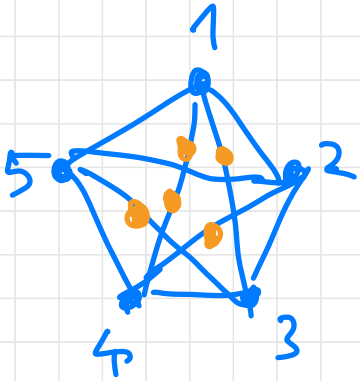
$\Rightarrow$ NI RAVNINSKI





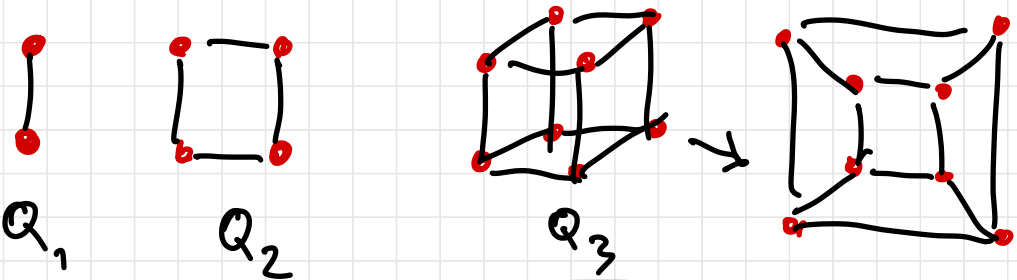
JE RAVNINSKI



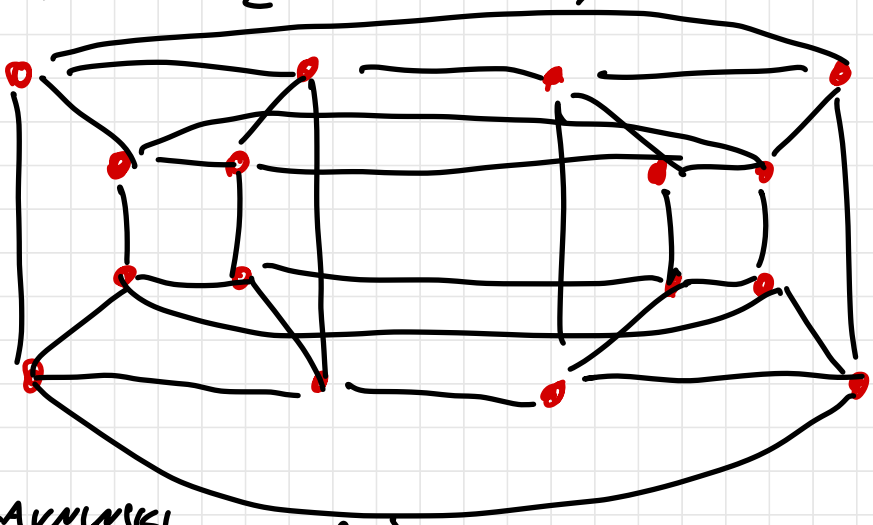


VSEBUJE SUBDIVIZIJO K_5 $\Rightarrow$ NI RAVNINSKI

2. ZA KATERE n JE Q_n RAVNINSKI GRAF?



Q_4:



ČE RAVNINSKI

$$|E(G)| \leq \frac{g(G)}{g(G)-2}\left(|V(G)|-2\right)$$

ČE Q_4 RAVNINSKI

$$|E(Q_4)| \leq \frac{4}{4-2}\cdot\left(|V(Q_4)|-2\right)$$

$$32 \leq 2 \cdot (16 - 2)$$

$$32 \leq 28 \quad \rightarrow\leftarrow$$

$\Rightarrow Q_4$ NI RAVNINSKI

Q_n $n \geq 4$ VELJA, DA $Q_4 \subseteq Q_n$

$\Rightarrow Q_n$ NI RAVNINSKI

3. POKAŽI, DA IMA VSAK POVEZAN RAVNINSKI GRAF VSAJ ENO VOZLIŠČE ST. NAJVEČ 5. ČE JE BREZ TRIKOTNIKOV, VSAJ ENO VOZLIŠČE ST. NAJVEČ 3.

ČE RAVNINSKI:

$$|E(G)| \leq 3 \cdot |V(G)| - 6$$

ČE SO VSE STOPNJE ≥ 6

$$2|E(G)| = \sum_{v \in V(G)} \deg(v) \geq 6 \cdot |V(G)|$$

$$|E(G)| \geq 3 \cdot |V(G)| \quad \rightarrow\leftarrow$$

ČE NI TRIKOTNIKOV:

RAVNAŠKI:

$$|E(G)| \leq 2 \cdot |V(G)| - 4$$

ČE $\delta(G) \geq 4$

$$2|E(G)| \geq 4 \cdot |V(G)|$$

$$|E(G)| \geq 2 \cdot |V(G)|$$

$\rightarrow \leftarrow$

4. POKAŽI, DA IMA POVEZAN 3-REGULAREN RAVNINSKI GRAF, PRI KATEREM SO VSA LICA 6-KOTNIKI IN 5-KOTNIKI NATANKO 12 5-KOTNIKOV.

$$\sum_{l \in F(G)} \text{dolžina}(l) = 2 \cdot |E(G)|$$

n_5 - ŠT. LIC DOLŽINE 5

n_6 - -"- 6

$$n_5 \cdot 5 + n_6 \cdot 6 = 2 \cdot |E(G)|$$

$$\sum_{v \in V(G)} \deg(v) = 2 \cdot |E(G)|$$

$$3 \cdot |V(G)| = 2 \cdot |E(G)|$$

$$|V(G)| - |E(G)| + |F(G)| = 2$$

$$\frac{2}{3} \cdot |E(G)| - |E(G)| + n_5 + n_6 = 2$$

$$-\frac{n_5 \cdot 5 + n_6 \cdot 6}{3 \cdot 2} + n_5 + n_6 = 2$$

$$\frac{1}{6} n_5 = 2$$

$$n_5 = 12$$

5. G POVEZAN REGULAREN GRAF STOPNJE $p \geq 3$ VLOŽEN V RAVNINO TAKO DA SO VSA LICA DOLŽINE $g \geq 3$. POIŠČI VSE TAKE G-JE.

EULERJEVA FORMULA:

$$|V(G)| - |E(G)| + |F(G)| = 2$$

LEMA O ROKOVANJU:

$$2 \cdot |E(G)| = p \cdot |V(G)|$$

PRESTEVANJE PO LICIH:

$$2 \cdot |E(G)| = q \cdot |F(G)|$$

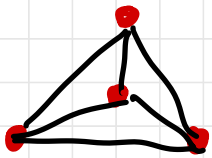
$$\Rightarrow 2 = \frac{2}{p} \cdot |E(G)| - |E(G)| + \frac{2}{q} \cdot |E(G)|$$

$$\frac{2}{p} + \frac{2}{q} = 1 + \frac{2}{|E(G)|}$$

$$\frac{2}{p} + \frac{2}{q} > 1$$

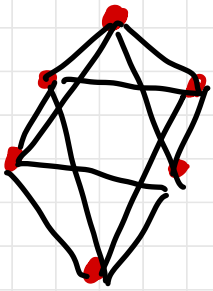
- $p = 3 \quad q = 3$:

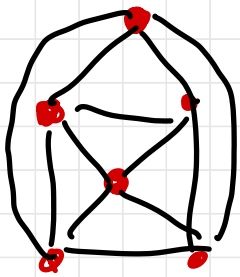
  $\frac{2}{3} + \frac{2}{3} > 1$ ✓



- $p = 4 \quad q = 3$

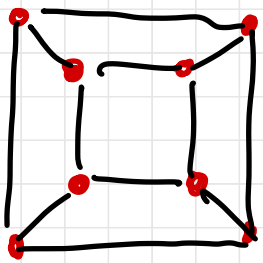
  $\frac{2}{4} + \frac{2}{3} > 1$ ✓





- $p = 3 \quad q = 4$

$$\frac{2}{3} + \frac{2}{4} > 1$$



- $p = 3 \quad q = 5$

$$\frac{2}{3} + \frac{2}{5} > 1$$

DODEKAEDER

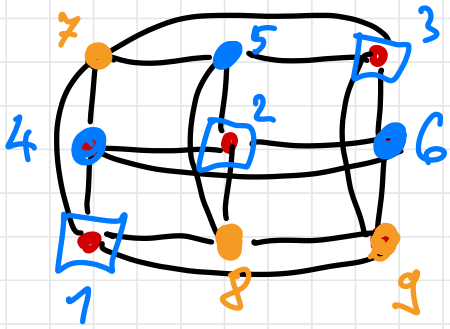
- $p = 5 \quad q = 3$

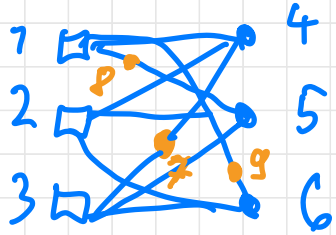
$$\frac{2}{5} + \frac{2}{3} > 1$$

IKOZAEDER

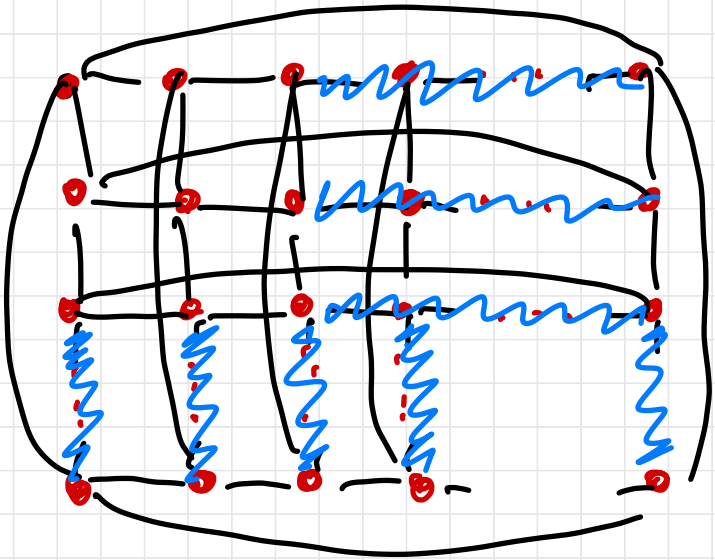
6. ALI JE $C_n \square C_m$ RAVNINSKI ?

$C_3 \square C_3$

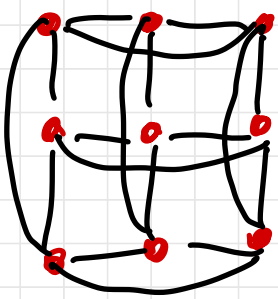




$\Rightarrow$ NI RAVNINSKI



MINOR



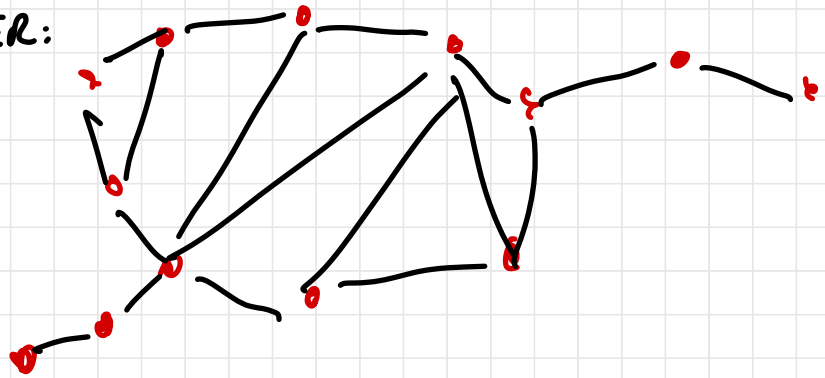
$C_3 \square C_3$

KER $C_3 \square C_3$ NI RAVNINSKI

TUDI $C_n \square C_m$ NI RAV.

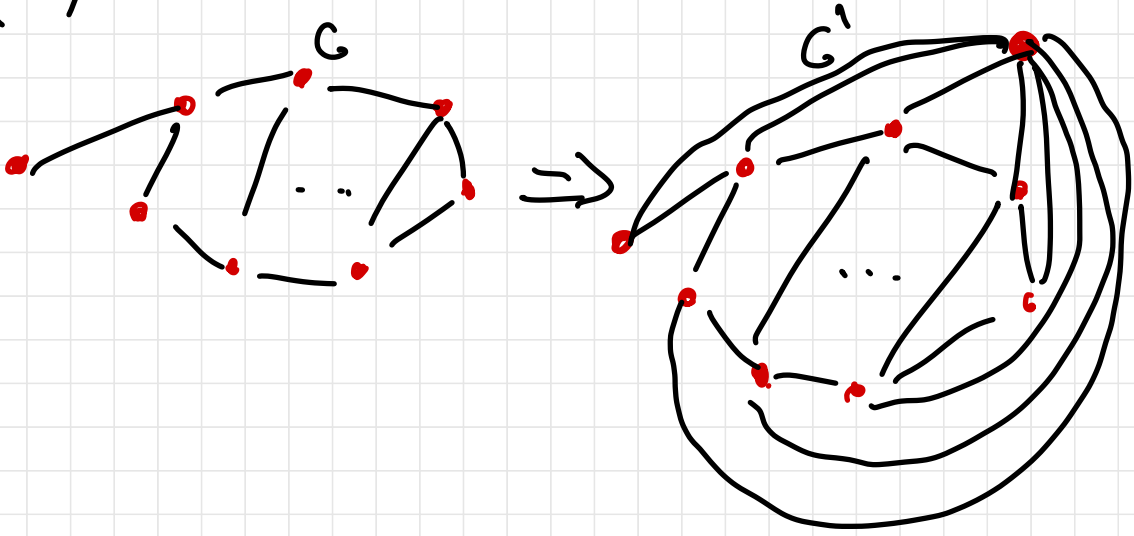
7. GRAF JE ZUNANJE RAVNINSKI ČE GA LAHKO NARIŠEMO BREZ SEKANJA TAKO, DA SO VSA VOZLIŠČA NA ZUNANJI LICU.

PRIMER:



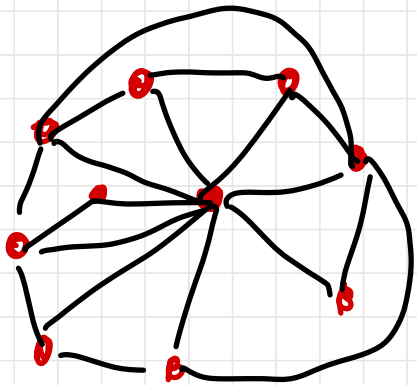
a) POKAŽI, DA G ZUNANJE RAVNINSKI $\Leftrightarrow$ RAVNINSKI G', KI GA DOBIMO TAKO DA G DODAMO VOZLIŠČE IN GA POVEŽEMO Z VSEMI.

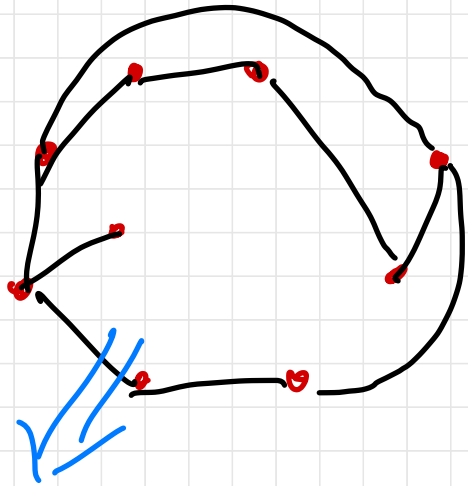
($\Rightarrow$) G JE ZUNANJE RAVNINSKI



DODAM V ZUNANJE LICE.

($\Leftarrow$) G' RAVNINSKI $\Rightarrow$ G



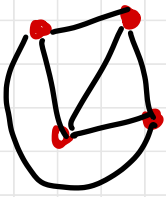


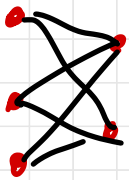
OBRAMO LICE NAVZVEN

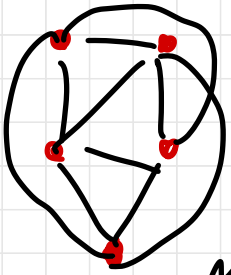
$\Rightarrow$ JE ZUNANJE RAVNINSKI

b) G ZUNANJE RAVNINSKI $\Leftrightarrow$ KO NE VSEBUJE SUBDIVIZIJE K_4 ALI $K_{3,2}$

($\Rightarrow$) G ZUNANJE RAVNINSKI IN RECIMO DA VSEBUJE SUB. K_4 ALI $K_{3,2}$







K_5

NI RAVNINSKI

NI RAVNINSKI

VSEBUJE

$K_{3,3}$

$\Rightarrow K_{2,3}$ NI ZUNANJE RAV.

$\Rightarrow$ Po a) K_4 NI ZUNANJE RAVNINSKI

$(\Leftarrow)$ G NE VSEBUJE SUBDIVIZIJE K_4 ALI $K_{2,3}$

RECIMO, DA G NI ZUNANJE RAV.

$\Rightarrow G'$ NI RAVNINSKI

$\Rightarrow G'$ VSEBUJE SUBDIVIZIJO K_5 ALI $K_{3,3}$

$\Rightarrow$ G VSEBUJE SUBDIVIZIJO $K_{2,3}$ ALI K_4