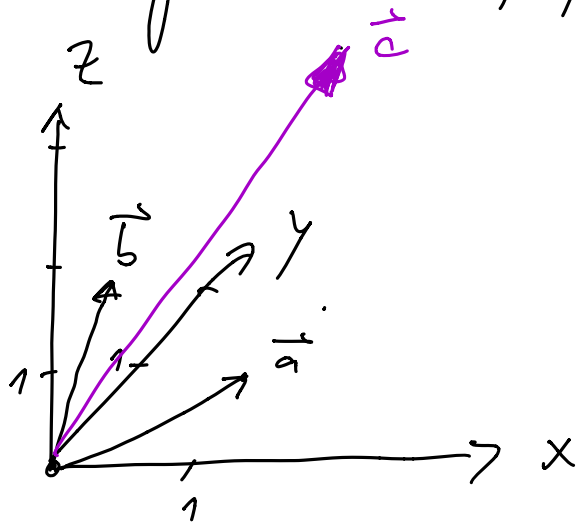
Naloga: Kateri vektor je v ravnini, ki jo razpenjata $\vec{a} = (1,1,0)^T$ in $\vec{b} = (0,1,1)^T$, najbližje vektorju $\vec{c} = (1,2,3)^T$?



a) Analitična rešitev:

Iščemo vektor v ravnini, ki jo določata $\vec{a}$ in $\vec{b}$ tako, da bo v $\|\cdot\|_2$ najbližje $\vec{c}$.

Iščemo $\alpha, \beta \in \mathbb{R}$: $\|\alpha \vec{a} + \beta \vec{b} - \vec{c}\|_2$ minimalno

$A = [\vec{a}, \vec{b}]$; $\vec{x} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ => iščemo minimum

$$\|A\vec{x} - \vec{c}\|_2$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}; \quad \vec{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \|A\vec{x} - \vec{c}\|_2$$

$$= \left\| (\alpha, \alpha+\beta, \beta)^T - (1,2,3)^T \right\|_2$$

$$= \sqrt{\underbrace{(\alpha-1)^2 + (\alpha+\beta-2)^2 + (\beta-3)^2}_{f(\alpha,\beta)}} \Rightarrow$$

i iščemo $\min\limits_{\alpha,\beta} \sqrt{f(\alpha,\beta)}$.

Minimum nastopi v isti točki, kot

$$\boxed{\min_{\alpha,\beta} f(\alpha,\beta)}$$

$$\frac{\partial f}{\partial \alpha}(\alpha,\beta) = \cancel{2}(\alpha-1) + \cancel{2}(\alpha+\beta-2) = 0$$

$$\frac{\partial f}{\partial \beta}(\alpha,\beta) = \cancel{2}(\alpha+\beta-2) + \cancel{2}(\beta-3) = 0$$

$$\Rightarrow \boxed{\begin{aligned} 2\alpha + \beta &= 3 \\ \alpha + 2\beta &= 5 \end{aligned}} \quad \Big/ \cdot (-2) \Big\} +$$

$$\Rightarrow -3\alpha = -1 \Rightarrow \alpha = \frac{1}{3} \Rightarrow \beta = \frac{7}{3}$$

je ravno normalni sistem enačb:

$$A^T A \vec{x} = A^T \vec{c}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^T \vec{c} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$ ✓

QR razcep (<u>osnovni</u>):

$A = QR$, $Q \in \mathbb{R}^{m \times n}$ matrika z ortonormiranimi stolpci, $R \in \mathbb{R}^{n \times n}$ zgornja trikotna.

$$A^T A x = A^T b$$

$$R^T \underbrace{Q^T Q}_{I} R x = R^T Q^T b \quad / (R^T)^{-1} \text{ z leve}$$

$$\boxed{Rx = Q^T b}$$ obratno notranj.