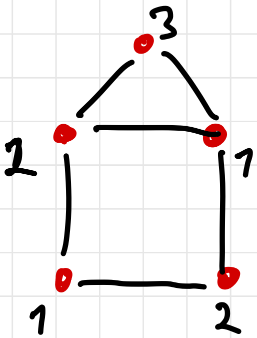
DS 2 VAJE 8

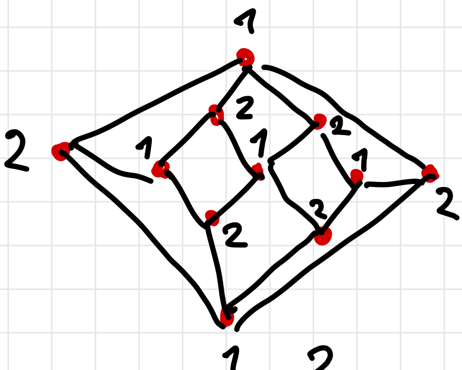
1. DOLOČI KROMATIČNO ŠTEVILO $\chi(G)$ IN KROMATIČNI INDEKS $\chi'(G)$



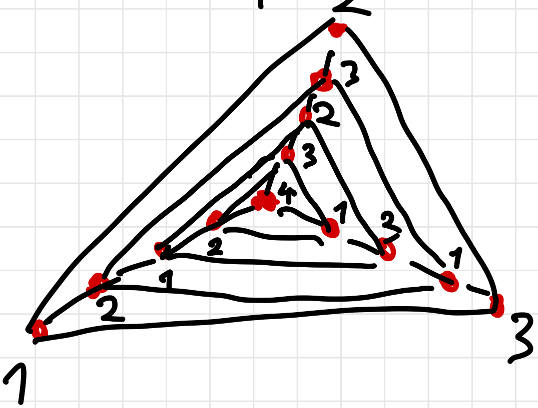
$\chi(G) \le 3$

KER IMA LIH CIKEL $\chi(G) > 2$

$\Rightarrow \chi(G) = 3$



$\chi(G) = 2$

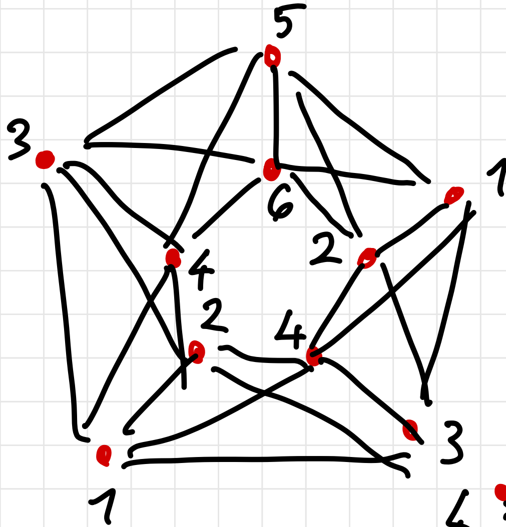


$\chi(G) \le 4$

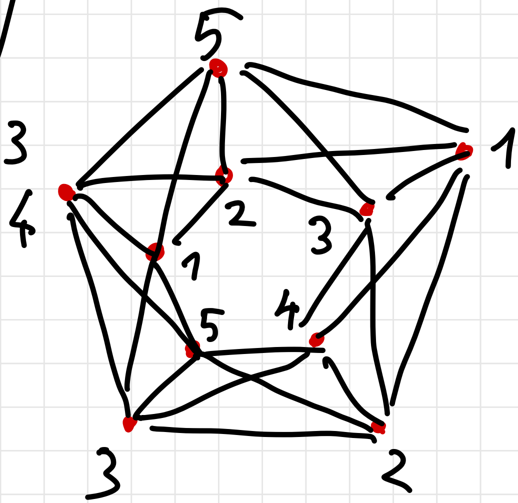
KER VSEBUJE K_4

$\Rightarrow \chi(G) \ge 4$

$\Rightarrow \chi(G) = 4$

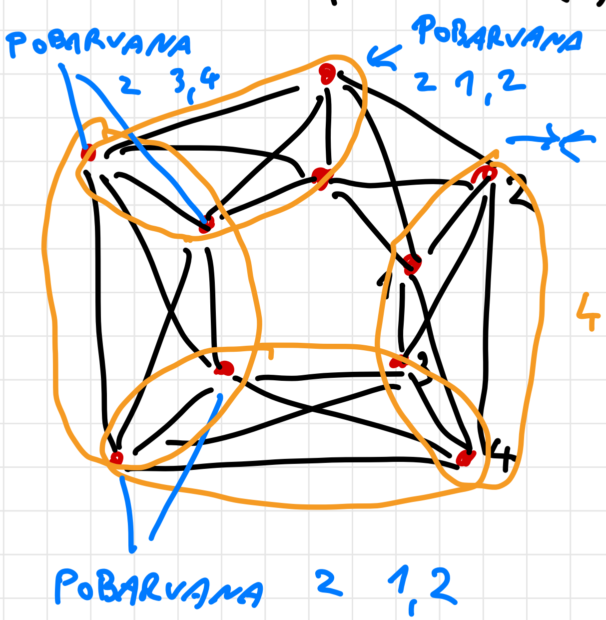


$\chi(G) \leq 6$



$\chi(G) \leq 5$

VSEBUJE $K_4 \Rightarrow \chi(G) \geq 4$



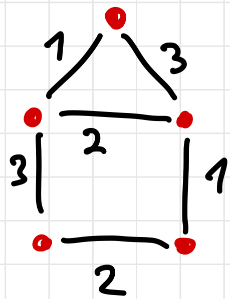
RECIMO, DA $\chi(G) = 4$

4 RAZLIČNE BARVE

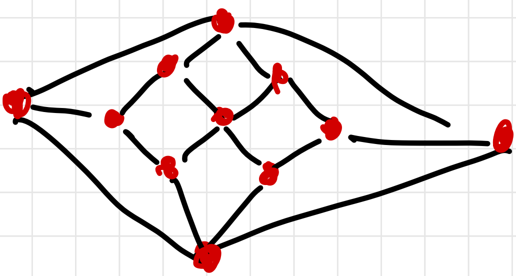
$\Rightarrow \chi(G) = 5$

VIZINGOV IZREK :

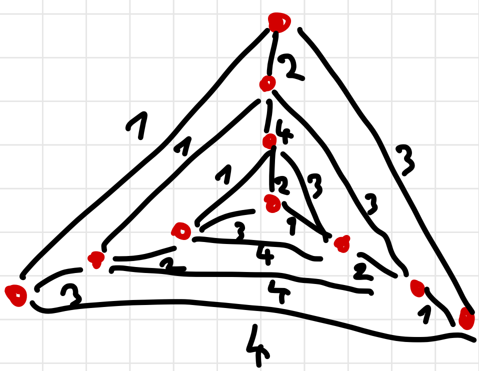
$$\chi'(G) = \begin{cases} \Delta(G) & \text{TIPA 1} \\ \Delta(G) + 1 & \text{TIPA 2} \end{cases}$$

- 

$\chi'(G) = 3$ TIPA 1

- 

KER DVODELEN $\Rightarrow$ TIPA 1

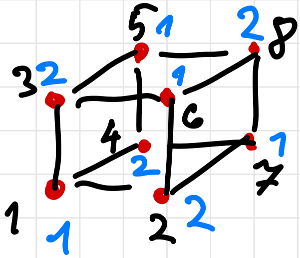
- 

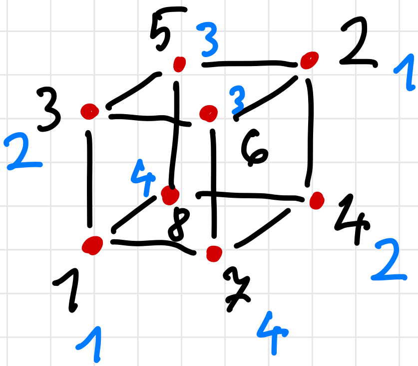
$\chi'(G) \in \{4, 5\}$

$\Rightarrow \chi'(G) = 4$

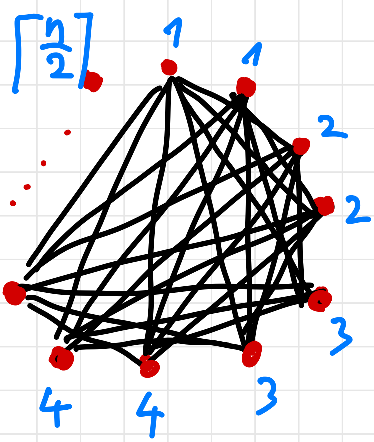
TIPA 1

2. POŽREŠNO POBARVAJ Q_3 TAKO, DA DOBIMO OPTIMALNI REZULTAT IN ČIM SLABŠI.

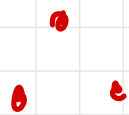




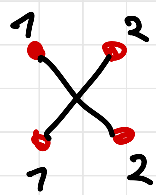
3. $\chi(\bar{C}_n)$ ZA $n \geq 3$



$n = 3$

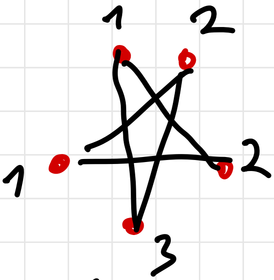


$n=4$



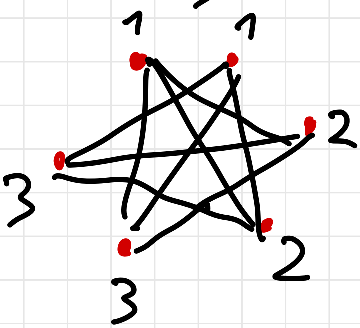
$\chi(\bar{C}_4) = 2$

$n=5$



$\chi(\overline{C_5}) = 3$

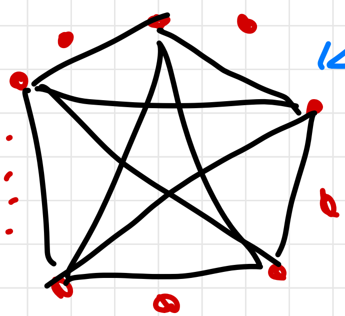
$n=6$



$\chi(\overline{C_6}) = 3$

$$\chi(\overline{C_n}) \leq \left\lceil \frac{n}{2} \right\rceil$$

- n SOD



VSAKO DRUGO VOZLIŠČE

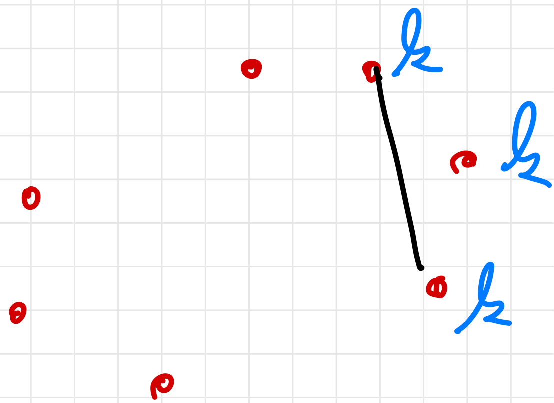
$$K_{\frac{n}{2}} \Rightarrow \chi(G) \geq \frac{n}{2}$$

$$\Rightarrow \chi(G) = \frac{n}{2}$$

• n LIHO

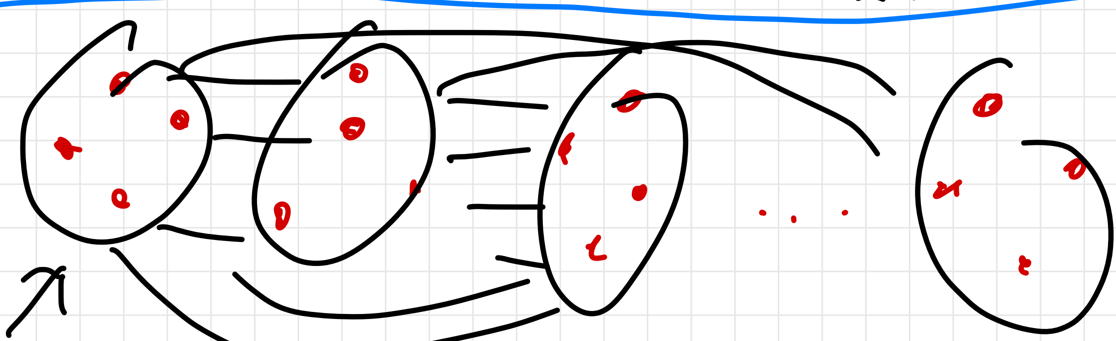
RECIMO, DA $\chi(\bar{C}_n) = \lfloor \frac{n}{2} \rfloor$

$\Rightarrow$ ENA BARVA 3X



VSAKE 3 VOZLIŠČA VSEBUJEJO POVEZAVO MED NJIMI

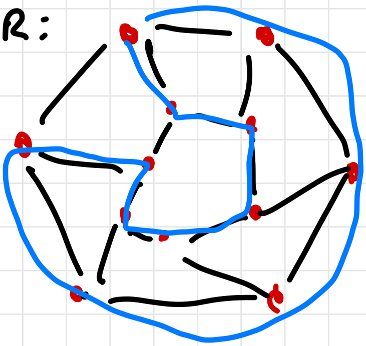
$$\Rightarrow \chi(\bar{C}_n) = \lceil \frac{n}{2} \rceil$$



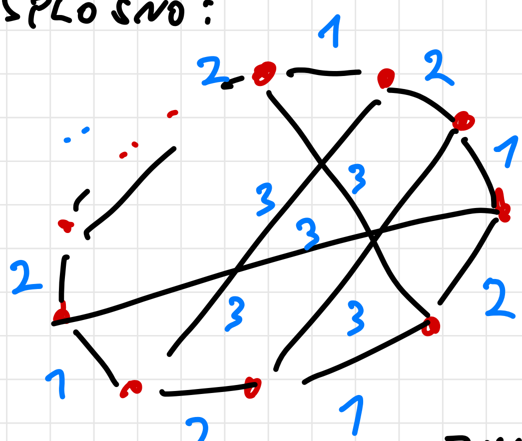
NEODVISNE MNOŽICE

4. POKAŽI, DA JE VSAK 3-REGULAREN HAMILTONOV GRAF TIPA 1.

PRIMER:



SPLOŠNO:



KER

$$2 \cdot |E(G)| = 3 \cdot |V(G)|$$

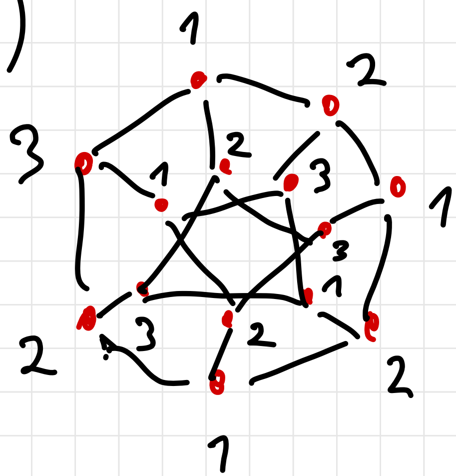
$$\Rightarrow |V(G)| \text{ SODO}$$

ZUNANJE POVEZAVE Z BARVAMA 1, 2, NOTRANJE 3

5. $\chi(P(n,k))$ ZA $n \geq 5$

$1 \leq k < \frac{n}{2}$

$P(7,2)$



$\chi(P(7,3)) = 3$

$\chi(P(n,k)) = 2 \iff P(n,k)$ DVODELEN

$\iff$ n SODO, k LIH

VAJE 2

POŽREŠNO BARVANJE (VRSTNI RED "SOSEDOV")

$\chi(P(n,k)) \leq 4 \Rightarrow \chi(P(n,k)) \leq 3$

BROOKSOV IZREK: $\chi(G) \leq \Delta(G)$

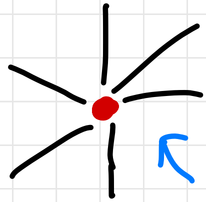
RAZEN C_n LIH, K_n

6. POKAŽI, DA JE d-REGULAREN GRAF NA LIHO MNOGO VOZLIŠČIH TIPA 2.

$$2\cdot\left(\text{ŠT. POVEZAV BARVE } i\right) = \sum_{v\in V}\left(\text{ŠT. POVEZAV IZ } v \text{ BARVE } i\right)$$

RECIMO, DA G TIPA 1:

d-REGULAREN



PORABIM VSEH d BARV

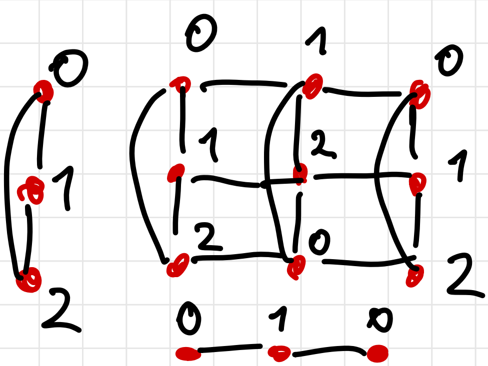
$$2\cdot\left(\text{ŠT. POVEZAV BARVE } i\right) = \sum_{v\in V} 1$$

$$= |V(G)|\cdot 1$$

SODO

LIHO

7. POKAŽI, DA $\chi(G \square H) = \max\{\chi(G), \chi(H)\}$



$$\chi_G : V(G) \to \{0, \dots, \chi(G)-1\}$$
$$\chi_H : V(H) \to \{0, \dots, \chi(H)-1\}$$

$$\chi_{G \square H}(v_1, v_2) = \chi_G(v_1) + \chi_H(v_2) \pmod{\max\{\chi(G), \chi(H)\}}$$

DOBRO BARVANJE:

$(v_1, v_2) \sim (u_1, u_2)$:

- $v_1 = u_1$ IN $v_2 \sim u_2$:

$$\chi_{G\square H}(v_1, v_2) = \chi_G(v_1) + \chi_H(v_2) \bmod \max$$
$$\chi_{G\square H}(u_1, u_2) = \chi_G(u_1) + \chi_H(u_2) \bmod \max$$

KER ISTA

KER SOSEDNA

$\Rightarrow \neq \Rightarrow$ BARVANJE DOBRO

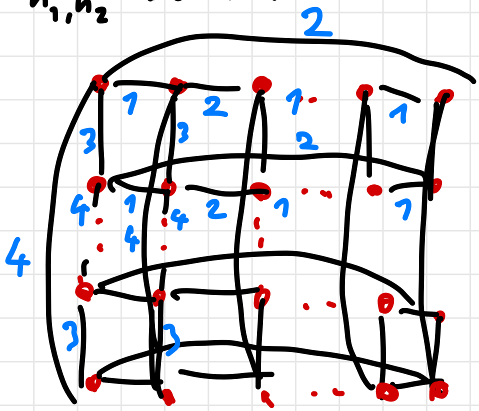
- $v_1 \sim u_1$ IN $v_2 = u_2$:

SIMETRIČNO ...

8. $\chi'(C_{n_1} \square C_{n_2}) = ?$

$\in \{4, 5\}$

ČE n_1, n_2 SODA:



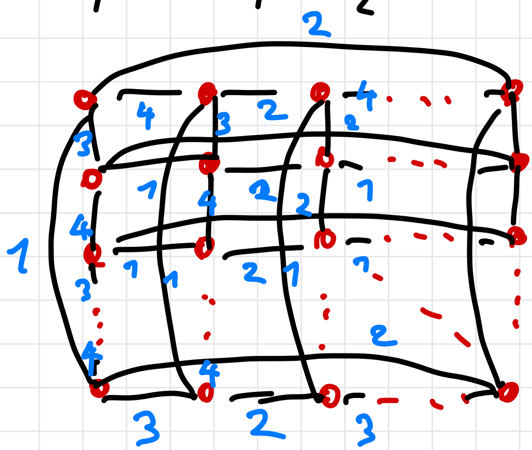
$\Rightarrow$ TIPA 1

ČE n_1, n_2 LIHA:

4-REGULAREN, LIHO MNOGO VOZLIŠČ

$\Rightarrow$ TIPA 2

ČE n_1 LIH, n_2 SOD:



$\chi(G)=4$

$\Rightarrow$ TIPA 1