Naj bo $L : U \to V$ linearna preslikava.

Naj bo $\mathcal{B} = \{u_1, \dots, u_m\}$ baza za U

in naj bo $\mathcal{C} = \{v_1, \dots, v_n\}$ baza za V

Predpostavljamo, da sta $\mathcal{B}$ in $\mathcal{C}$ ortonormirani

Najprej izračunajmo $[L]_{\mathcal{C} \leftarrow \mathcal{B}}$

Vektorje $Lu_1, \dots, Lu_m$ razvijemo po $v_1, \dots, v_n$

Pri tem upoštevamo formulo za razvoj vektorja po ortonormirani bazi $w = \sum \langle w, v_i \rangle v_i$

$$Lu_1 = \langle Lu_1, v_1 \rangle v_1 + \dots + \langle Lu_1, v_n \rangle v_n$$

$$\vdots$$

$$Lu_m = \langle Lu_m, v_1 \rangle v_1 + \dots + \langle Lu_m, v_n \rangle v_n$$

Torej matrika L glede na $\mathcal{B}$ in $\mathcal{C}$

$$[L]_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{bmatrix} \langle Lu_1, v_1 \rangle & \dots & \langle Lu_m, v_1 \rangle \\ \vdots & & \vdots \\ \langle Lu_1, v_n \rangle & \dots & \langle Lu_m, v_n \rangle \end{bmatrix}$$

Element $\langle Lu_i, v_j \rangle$ se nahaja na preseku i-tega stolpca in j-te vrstice

- Izračunajmo še matriko $L^* : V \longrightarrow U$ glede na bazi $\mathcal{C}$ in $\mathcal{B}$. Se pravi $L^*v_1, \dots, L^*v_n$ moramo razviti po bazi $u_1, \dots, u_m$. Upoštevamo formulo za razvoj vektorja po ortonormirani bazi $z = \langle z, u_1 \rangle u_1 + \dots + \langle z, u_m \rangle u_m$

$$L^*v_1 = \langle L^*v_1, u_1 \rangle u_1 + \dots + \langle L^*v_1, u_m \rangle u_m$$

$$\vdots$$

$$L^*v_n = \langle L^*v_n, u_1 \rangle u_1 + \dots + \langle L^*v_n, u_m \rangle u_m$$

Od tod sledi

$$[L^*]_{\mathcal{B} \leftarrow \mathcal{C}} = \begin{bmatrix} \langle L^*v_1, u_1 \rangle & \dots & \langle L^*v_n, u_1 \rangle \\ \vdots & & \vdots \\ \langle L^*v_1, u_m \rangle & \dots & \langle L^*v_n, u_m \rangle \end{bmatrix}$$

Primerjamo obe matriki. Na (i,j)-tem mestu matrike $[L^*]_{\mathcal{B} \leftarrow \mathcal{C}}$ se nahaja

$$\langle L^*v_j, u_i \rangle \overset{\text{konj. simetričnost}}{=} \overline{\langle u_i, L^*v_j \rangle} \overset{\text{def. } L^*}{=} \overline{\langle Lu_i, v_j \rangle}$$

Opazimo, da se $\langle Lu_i, v_j \rangle$ nahaja na (j,i)-tem mestu matrike $[L]_{\mathcal{C} \leftarrow \mathcal{B}}$

· Povzetek

(i,j)-ti element matrike $[L^*]_{B \leftarrow \mathcal{C}}$ je enak konjugiranemu (j,i)-temu elementu matrike $[L]_{\mathcal{C} \leftarrow B}$

Torej je $[L^*]_{B \leftarrow \mathcal{C}}$ matrika, ki jo dobimo tako, da matriko $[L]_{\mathcal{C} \leftarrow B}$ najprej transponiramo in potem vse elemente še konjugiramo.

Oznaka: Za vsako kompleksno matriko A definiramo adjungirano matriko takole

$$A^* = \overline{A^T} = (\overline{A})^T$$

$\overline{A}$ = vse elemente A konjugiramo

A^T = transponiranka matrike A

S pomočjo te oznake lahko naš trek zapišemo takole

$$\underset{\substack{\uparrow \\ \text{adj. lin. preslikava}}}{[L^*]_{B \leftarrow \mathcal{C}}} = \left([L]_{\mathcal{C} \leftarrow B}\right)^{\overset{*}{\underset{\text{adj. matrika}}{\uparrow}}}$$

$$(A^*)^* = \Big(\overline{(\underbrace{\overline{A}}_{B})^T}\Big)^T$$

$$= \left(\overline{B^T}\right)^T$$

$$= \left((\overline{B})^T\right)^T$$

$$= \overline{B}$$

$$= \overline{\overline{A}}$$

$$= A$$