DS 2 VAJE 9

1. a) $(A, \cdot)$ $A = \{a, b, c, d, e\}$

$\cdot$	a	b	c	d	e
a	a	b	c	d	e
b	a	b	c	d	e
c	a	b	c	d	e
d	a	b	c	d	e
e	a	b	c	d	e

CAYLEYEVA TABELA

- KOMUTATIVNA? NI $a \cdot b = b$, $b \cdot a = a$ (≠)

- ASOCIATIVNOST? $(x \cdot y) \cdot z \overset{?}{=} x \cdot (y \cdot z)$

V NAŠI STRUKTURI: $x \cdot y = y$

$(x \cdot y) \cdot z = y \cdot z = z$

$x \cdot (y \cdot z) = x \cdot z = z$

✓

- ENOTA? e ENOTA $\Leftrightarrow \forall x \quad e \cdot x = x \cdot e = x$

VSI ELEMENT LEVA ENOTA

NIMAMO DESNE ENOTE.

- NE MOREMO GOVORITI O INVERZIH.

b)

$\cdot$	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$(A, \cdot)$

$A = \{a, b, c\}$

- KOMUTATIVNOST: DA, TABELA SIM. ČEZ DIAGONALO

- ASOC.:

$$(a \cdot a) \cdot a = a \cdot (a \cdot a)$$
$$(a \cdot a) \cdot b = a \cdot (a \cdot b)$$
$$(a \cdot a) \cdot c = a \cdot (a \cdot c)$$
$$\vdots$$

PREVERIMO VSEH 3^3 MOŽNOSTI ✓

- ENOTA: a JE ENOTA

- INVERZ: x^{-1} TAK DA $x^{-1} \cdot x = x \cdot x^{-1} = e$ ↑ ENOTA

$a^{-1} = a$, $b^{-1} = c$ $\quad c^{-1} \circ b$

JE GRUPA

2. $(\mathbb{R}, *)$

$$a * b = 1 - (a+b) + 2 \cdot ab$$

- KOM.: ✓
- ASOC.: $(a * b) * c = (1 - (a+b) + 2ab) * c$

$$= 1 - \Big(\big(1 - (a+b) + 2ab\big) + c \Big) + 2 \big(1 - (a+b) + 2ab\big) \cdot c =$$

$$= a + b + c - 2ab - 2ac - 2bc + 4abc$$

$$a * (b * c) = a * (1 - (b+c) + 2bc) =$$

$$= 1 - \Big(a + \big(1 - (b+c) + 2bc\big)\Big) + 2 \cdot a \cdot \big(1 - (b+c) + 2bc\big)$$

$$= a + b + c - 2ab - 2ac - 2bc + 4abc$$

✓

- ENOTA :

$$a * e = a \quad \forall a$$

$$1 - (a+e) + 2ae = a \quad \forall a$$

$$-e + 2ae = 2a - 1 \quad \forall a$$

$$e(2a-1) = 2a-1 \quad \forall a$$

$$e = \frac{2a-1}{2a-1} = 1 \qquad \text{ČE } 2a-1 \neq 0$$

$$\Rightarrow e = 1$$

- INVERZ : $a^{-1} * a = e$

$$1 - (a + a^{-1*}) + 2a \cdot a^{-1*} = 1$$

$$a^{-1*}(-1 + 2a) = a$$

$$a^{-1*} = \frac{a}{2a-1} \qquad \text{ČE } 2a-1 \neq 0$$

$$a \neq \frac{1}{2}$$

IMA INVERZ

$$a^{-1*} \cdot \left(-1 + 2 \cdot \frac{1}{2}\right) = \frac{1}{2} \qquad \text{ČE } a = \frac{1}{2}$$

$$a^{-1*} \cdot 0 = \frac{1}{2} \quad \rightarrow\leftarrow$$

$\frac{1}{2}$ NIMA INVERZA

3.

	a	b	c
a	c	b	a
b	?	?	b
c	a	?	?

x y z w

DOPOLNI DO MONOIDA.

- c EDINA MOŽNOST ZA ENOTO:

$\Rightarrow \quad z = b \qquad w = c$

- $(a \cdot c) \cdot b = a \cdot (c \cdot b)$

$a \cdot b = a \cdot b$ NE POMAGA

$(a \cdot b) \cdot a = a \cdot (b \cdot a)$

$b \cdot a = a \cdot x$

$x = a \cdot x \quad \Rightarrow \quad x = a$ //

$x = b$ ✓

$x = c$ //

$a \neq a \cdot a$

$b = a \cdot b$

$c \neq a \cdot c$

$$(a \cdot b) \cdot b = a \cdot (b \cdot b)$$

$$b \cdot b = a \cdot y$$

$$y = a \cdot y \qquad \text{ISTA ENAČBA}$$

$$\Rightarrow y = b$$

PREVERI DA VELJA ASOCIATIVNOST, ČE

$x = b$, $y = b$, $w = c$, $z = b$

4. ANALIZIRAJ STRUKTURE:

- $(\mathbb{N}, *)$ $\quad a * b = a^b$

ASOC.: $\quad (2^2)^2 = 4^2 = 2^{2 \cdot 2}$

$$(2)^{2^2} = 2^4 \;//$$

$$(3^3)^3 = 3^{3 \cdot 3} = 3^9$$

$$3^{(3^3)} = 3^{27} \;\times$$

NI ASOC.

- $(\mathbb{N}, *)$ $\quad a * b = \gcd\{a, b\}$

ASOC.: $\quad \gcd\{a, \gcd\{b, c\}\}$

$$\gcd\{\gcd\{a,b\},c\}$$

$$\| \quad \|$$

$$\gcd\{a,b,c\}$$

KOM.: ✓

ENOTA: //

- $(\mathbb{N}_0, *)$ $a * b = \max\{a,b\}$

  ASOC: ✓

  KOM: ✓

  ENOTA: 0

  INVERZ: SAMO 0^{-1} OBSTAJA

- $(\mathbb{N}_0, *)$ $a * b = \min\{a,b\}$

  ASOC: ✓

  KOM: ✓

  ENOTA: //

- $\mathbb{R}_0^+$ $a * b = \sqrt{a^2 + b^2}$

  ASOC: $a * (b * c) = \sqrt{a^2 + \sqrt{b^2 + c^2}}$

$$(a * b) * c = \sqrt{\sqrt{a^2+b^2}^2 + c^2} \quad \checkmark$$

KOM: ✓

ENOTA: 0

INVERZ: $\sqrt{a^2 + (\dot{a}^{-1})^2} = 0$

SAMO 0^{-1}

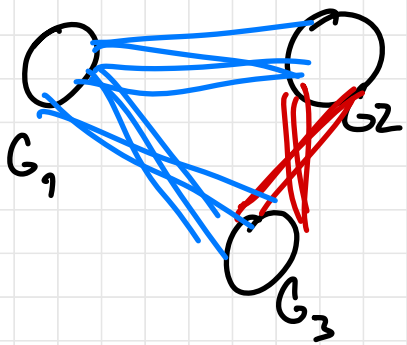
5. $(A, \circ)$

A = množica vseh končnih grafov

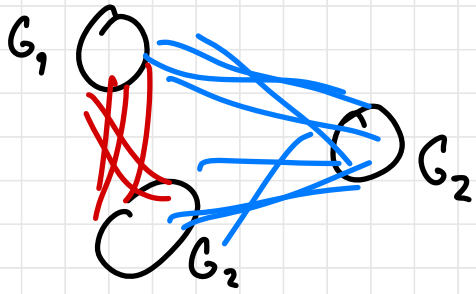
$V(G_1 \circ G_2) = V(G_1) \sqcup V(G_2)$

$E(G_1 \circ G_2) = E(G_1) \sqcup E(G_2) \cup$ {VSE POVEZAVE MED $V(G_1)$ IN $V(G_2)$}

• ASOC: $G_1 \circ (G_2 \circ G_3)$



$$(G_1 \circ G_2) \cdot G_3$$



JE ASOC.

KOM: ✓

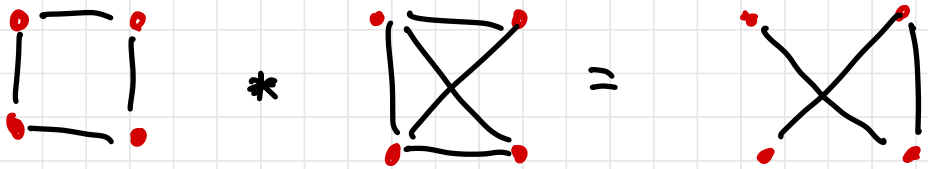
ENOTA: PRAZEN GRAF

6. $A = \{$ GRAFI NA $\{0,1,\ldots n-1\}\}$

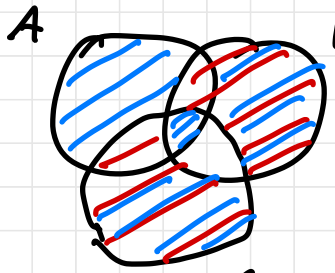
$V(G_1 * G_2) = \{0,1,\ldots,n-1\}$

$E(G_1 * G_2) = E(G_1) \triangle E(G_2)$

↑ SIMETRIČNA RAZLIKA



ASOC.: $A \triangle (B \triangle C) \stackrel{?}{=} (A \triangle B) \triangle C$



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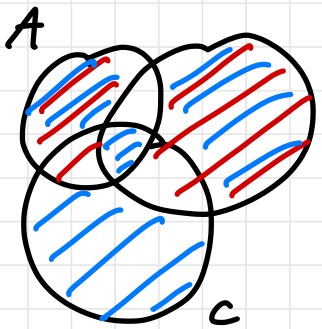


✓

POVEZAVA $(0,1)$ ↙ POVEZAVA $(0,2)$

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \dots \times \mathbb{Z}_2$ ↖ POVEZAVA $(n-2, n-1)$

$(0, 1, \dots, 1)$

↗

EN GRAF Z POVEZAVAMI, KJER 1

- KOM: ✓
- ENOTA: GRAF BREZ POVEZAV
- INVERZ: $G^{-1} = G$

GRUPA

7. $F = \{ f : \mathbb{Z} \to \mathbb{Z} \}$

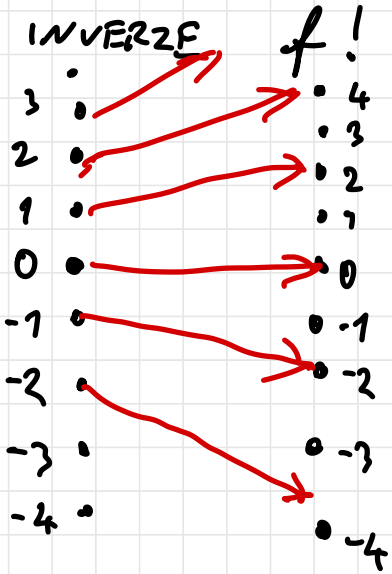
$(F, \circ)$ ← KOMPOZITUM

- ASOC. : ✓
- KOM. : //
- ENOTA : id
- INVERZI : $g(x) = 0$ NIMA INVERZA

← MONOID

$f : n \to 2n$

POIŠČI INVERZE f^{-1}



$f^{-1} : 2n \to n$

- DESNI INVERZ: $f \circ h = id$

$$f(h(1)) = 1$$

f NE SLIKA V 1

NIČESAR

$\Rightarrow$ NIMA DESNEGA INVERZA

- LEVI INVERZ:

$$h(x) = \begin{cases} \frac{x}{2} & x \text{ SODO} \\ x & x \text{ LIHO} \end{cases}$$

POLJUBNA IZBIRA

$\Rightarrow$ NESKONČNO LEVIH INVERZOV

8. $M = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix};\ a, b, d \in \mathbb{R},\ ad \neq 0 \right\}$

$(M, \cdot)$ GRUPA ?

$$\begin{bmatrix} a_1 & b_1 \\ 0 & d_1 \end{bmatrix} \begin{bmatrix} a_2 & b_2 \\ 0 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 a_2 & a_1 b_2 + b_1 d_2 \\ 0 & d_1 d_2 \end{bmatrix}$$

$a_1 \cdot a_2 \cdot d_1 \cdot d_2 \neq 0$

$\Rightarrow$ OPERACIJA NOTRANJA

- ASOC: ✓
- KOM: //
- ENOTA: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- INVERZ: $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}^{-1} = \frac{1}{ad} \begin{bmatrix} d & -b \\ 0 & a \end{bmatrix}$ ✓

$\Rightarrow$ JE GRUPA

9. $A = \mathbb{Z}_n \setminus \{0\}$

$(\mathbb{Z}_n, \cdot)$ NI GRUPA (0 NIMA INVERZA)

$(A, \cdot)$

ZA KATERE n JE OPERACIJA NOTRANJA?

$$x \cdot y \pmod n \in A \ ?$$

- ČE n NI PRAŠTEVILO:

$$n = p_1^{k_1} \cdot \ldots \cdot p_l^{k_l}$$

$$x = p_1^{k_1 - 1} \cdot p_2^{k_2} \cdot \ldots \cdot p_l^{k_l}$$

$$y = p_2 \cdot \ldots \cdot p_l$$

$$x \cdot y = 0 \notin A$$

- ČE n JE PRAŠTEVILO:

$$x \cdot y \stackrel{?}{=} 0 \pmod n$$

$$x \cdot y = k \cdot n \quad \text{ZA NEKI } k$$

$\Rightarrow$ PO PRAŠTEVILSKEM RAZCEPU

$x = n \cdot x'$

ALI

$y = n \cdot y'$

IN $0 < x, y < n$

$\rightarrow\leftarrow$

(A_1) GRUPA ? (ČE n PRAŠTEVILO)

ASOC: ✓

KOM: ✓

ENOTA: 1

INVERZ:

$$x^{-1} \cdot x = 1 \pmod{n}$$

$$x^{-1} \cdot x = 1 + k \cdot n$$

$$x^{-1} \cdot x + k' \cdot n = 1 \qquad k' = -k$$

DIOFANSKA ENAČB.

IŠČEMO

REŠITEV OBSTAJA KER x IN n TUJA.

n PRAŠTEVILO

$\leftarrow$

$(A_1, \cdot)$ JE GRUPA