# Mat 1 K3 osnutek rešitev

**1** (a) $\lim\limits_{x\to3} \dfrac{x^3+x^2-5x-21}{x-3} \underset{L'HP}{\overset{\frac00}{=}} \lim \dfrac{3x^2+2x-5}{1} = 27+6-5 = \underline{\underline{28}}$

$27+9-15-21 = 36-36 = 0$

(b) $\lim\limits_{x\to\infty} \dfrac{\sin(x^2)}{x} = \underline{\underline{0}}$ ker (izrek o sendviču): $-\dfrac{1}{x} \le \dfrac{\sin(x^2)}{x} \le \dfrac{1}{x}$

$\searrow \quad \downarrow \quad \swarrow$

$0$

(c) $\lim\limits_{x\to0} \dfrac{e^{2x}-2\sin x-\cos 2x}{\frac{x}{1-x}-\frac{x}{1+x}} \overset{\frac{1-0-1}{0}}{=} \lim \dfrac{-\|-}{\frac{x(1+x)-x(1-x)}{(1-x)(1+x)}} = \lim \dfrac{(1-x^2)(-\|-)}{2x^2}$

$(1-x^2) \to 1$

$= \lim\limits_{x\to0} \dfrac{e^{2x}-2\sin x-\cos 2x}{2x^2} \underset{L'HP}{\overset{\frac00}{=}} \lim \dfrac{2e^{2x}-2\cos x+2\sin 2x}{4x} \underset{L'HP}{\overset{\frac{2-2}{0}}{=}}$

$= \lim\limits_{x\to0} \dfrac{4e^{2x}+2\sin x+4\cos 2x}{4} = \dfrac{4+0+4}{4} = \underline{\underline{2}}$

**2** $f(x) = \begin{cases} x+a & ; x<0 \\ b\,e^{x(x-1)} & ; 0\le x\le 1 \\ x^2+x+1 & ; 1<x \end{cases}$

$f|_{(-\infty,0)}, f|_{[0,1]}, f|_{(1,\infty)}$ so elementarne, torej odvedljive (in notr. vemo).

Problem je edino v $x=0,1$.

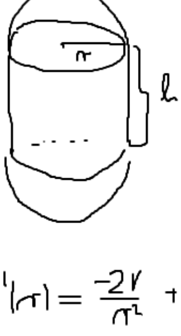
$\lim\limits_{x\uparrow0} f(x) = a = \lim\limits_{x\downarrow0} f(x) = b\cdot e^0 \Rightarrow a=b$

$\lim\limits_{x\uparrow1} f(x) = b\cdot e^0 = \lim\limits_{x\downarrow1} f(x) = 3 \Rightarrow \underline{\underline{b=3=a}}$

$f$ odvedljiva? $\lim\limits_{x\uparrow0} f'(x) = \lim\limits_{x\uparrow0} 1 = 1$

$\lim\limits_{x\downarrow0} f'(x) = \lim\limits_{x\downarrow0} b\,e^{x^2-x}\cdot(2x-1) = b\,e^0\cdot(-1) = -b = -3$ $\Rightarrow$ $f$ ni odvedljiva

**3**



$P = 2\pi r\cdot h + 4\pi r^2 = f(r)$ ... minimiziramo

$\underbrace{V = \pi r^2\cdot h + \frac{4\pi r^3}{3}}$ ... fiksen

$h = \dfrac{V-4\pi r^3/3}{\pi r^2} \Rightarrow f(r) = 2\pi r\,\dfrac{V-4\pi r^3/3}{\pi r^2} + 4\pi r^2 =$

$= \dfrac{2V}{r} - \dfrac{8\pi r^2}{3} + 4\pi r^2 = \dfrac{2V}{r} + \dfrac{4\pi r^2}{3}$

$0 = f'(r) = \dfrac{-2V}{r^2} + \dfrac{8\pi r}{3} \Rightarrow \dfrac{2V}{r^2} = \dfrac{8\pi r}{3} \Rightarrow \dfrac{3V}{4\pi} = r^3 \Rightarrow \underline{\underline{r = \sqrt[3]{\dfrac{3V}{4\pi}}}},\ h = \dfrac{V-4\pi\cdot\frac{3V}{4\pi}/3}{\pi r^2} = \underline{\underline{0}}$

$\boxed{4}$ $f(x) = -\frac{2x+2}{e^{\sqrt{x}}} = -2(x+1)\,e^{-\sqrt{x}}$

$D_f = [0,\infty)$

ničle: $f(x)=0 \iff x=-1$

$\lim_{x\downarrow 0} -2(x+1)\,e^{-\sqrt{x}} = -2\cdot e^{0} = \underline{\underline{-2}}$

$\lim_{x\to\infty} -2\,\frac{x+1}{e^{\sqrt{x}}} \underset{\substack{t=\sqrt{x}\\ t^2=x}}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} -2\,\frac{t^2+1}{e^{t}} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim -2\,\frac{2t}{e^{t}} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{t\to\infty} -2\,\frac{2}{e^{t}} = -2\cdot\frac{2}{\infty} = 0$

$f' = \left(-2(x+1)e^{-\sqrt{x}}\right)' = -2\left(e^{-\sqrt{x}} + (x+1)\,e^{-\sqrt{x}}\cdot\frac{-1}{2\sqrt{x}}\right) = -2\,e^{-\sqrt{x}}\,\frac{2\sqrt{x}-x-1}{2\sqrt{x}} = e^{-\sqrt{x}}\,\frac{x+1-2\sqrt{x}}{\sqrt{x}}$

$f'=0 \iff \underset{t=\sqrt{x}>0}{x+1-2\sqrt{x}=0} \iff t^2+1-2t = (t-1)^2 = 0 \iff t=1 \iff \sqrt{x}=1 \iff x=1$

$f' = e^{-\sqrt{x}}\,\frac{(\sqrt{x}-1)^2}{\sqrt{x}} \geq 0 \Rightarrow f$ povsod narašča

$f'' = e^{-\sqrt{x}}\cdot\frac{-1}{2\sqrt{x}}\cdot\frac{(\sqrt{x}-1)^2}{\sqrt{x}} + e^{-\sqrt{x}}\cdot\frac{2(\sqrt{x}-1)\frac{1}{\cancel{2\sqrt{x}}}\cdot\cancel{\sqrt{x}} - (\sqrt{x}-1)^2\cdot\frac{1}{2\sqrt{x}}}{x}$ $\qquad -x+\cancel{\sqrt{x}}+2\sqrt{x}-\cancel{\sqrt{x}}+1$

$= e^{-\sqrt{x}}\left(\frac{-(\sqrt{x}-1)^2}{2x} + \frac{2\sqrt{x}(\sqrt{x}-1)-(\sqrt{x}-1)^2}{2x\sqrt{x}}\right) = e^{-\sqrt{x}}\,\frac{(\sqrt{x}-1)\overbrace{\left(-\sqrt{x}(\sqrt{x}-1)+2\sqrt{x}-(\sqrt{x}-1)\right)}}{2x\sqrt{x}}$

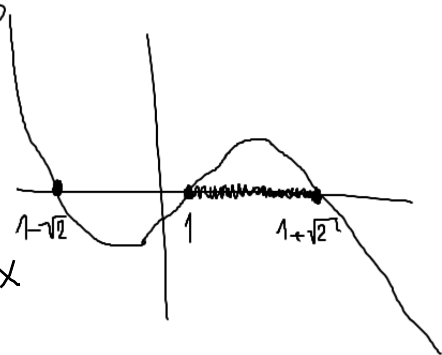
$= e^{-\sqrt{x}}\,\frac{(\sqrt{x}-1)(-x+2\sqrt{x}+1)}{2x\sqrt{x}}$

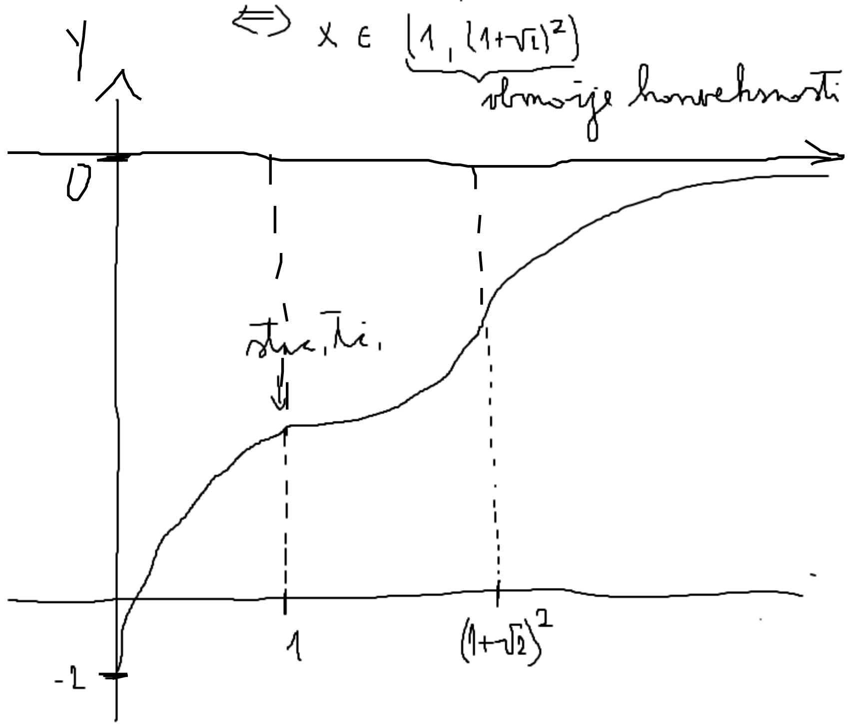
$t=\sqrt{x} \Rightarrow -t^2+2t+1=0 \iff t_{1,2} = \frac{-2\pm\sqrt{4+4}}{-2} = 1\mp\sqrt{2}$

$f''>0 \iff -(\sqrt{x}-1)\left(\sqrt{x}-(1-\sqrt{2})\right)\left(\sqrt{x}-(1+\sqrt{2})\right)>0$

$\iff \sqrt{x}\in(-\infty,1-\sqrt{2})\cup(1,1+\sqrt{2})$

$\iff x\in\underbrace{(1,(1+\sqrt{2})^2)}_{\text{območje konveksnosti}}$




$f$ na $(1,(1+\sqrt{2})^2)$:

$f$ na $(0,1)\cup((1+\sqrt{2})^2,\infty)$: