Naloga: Rešite sistem enačb

$$\begin{bmatrix} 2 & 2 & 6 \\ 2 & 1 & -2 \\ 1 & 6 & -2 \end{bmatrix} x = \begin{bmatrix} 6 \\ -1 \\ -7 \end{bmatrix}$$

s QR razcepom s Householderjevimi zrcaljenji.

$$A = \begin{bmatrix} 2 & 2 & 6 \\ 2 & 1 & -2 \\ 1 & 6 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ -1 \\ -7 \end{bmatrix}.$$

Želimo $A = QR$; $Q \in \mathbb{R}^{3\times 3}$ ortogonalna,

$R = \begin{bmatrix} \ast & \ast & \ast \\ 0 & \ast & \ast \\ 0 & 0 & \ast \end{bmatrix} \in \mathbb{R}^{3\times 3}$.

$Ax = b \iff QRx = b \iff Rx = Q^T b \Rightarrow$ rešimo zgornji trikotni sistem.

1. korak: ničemo $P^{(1)} A x = P^{(1)} b$

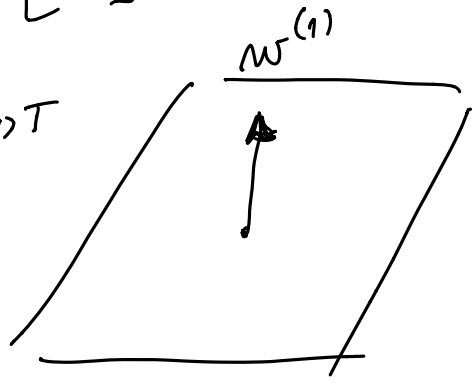
$$\begin{bmatrix} \ast & \ast & \ast \\ 0 & \ast & \ast \\ 0 & \ast & \ast \end{bmatrix} x = \begin{bmatrix} \ast \\ \ast \\ \ast \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 6 & 6 \\ 2 & 1 & -2 & -1 \\ 1 & 6 & -2 & -7 \end{bmatrix}$$

$$A =: A^{(1)} = [a_1^{(1)} \; a_2^{(1)} \; a_3^{(1)}]$$

$P^{(1)}$ mora preslikati $a_1^{(1)}$ v $\begin{bmatrix} * \\ 0 \\ 0 \end{bmatrix}$.

$$P^{(1)} = I - \frac{2}{w^{(1)T} w^{(1)}} w^{(1)} w^{(1)T}$$



$$a_1^{(1)} = [2 \; 2 \; 1]^T$$

$$w^{(1)} = a_1^{(1)} \pm \begin{bmatrix} \| a_1^{(1)} \|_2 \\ 0 \\ 0 \end{bmatrix}$$

izberemo tako, da ji $\|w^{(1)}\|_2$ čim večja

$$= \sqrt{2^2+2^2+1^2} = 3$$

$$w^{(1)} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} \| a^{(1)} \|_2 \\ \\ \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

$$\text{in } w^{(1)} w^{(1)T} = 30$$

$$P^{(1)} a_1^{(1)} = \left(I - \frac{2}{w^{(1)T} w^{(1)}} w^{(1)} w^{(1)T} \right) a_1^{(1)}$$

$$= a_1^{(1)} - \frac{2}{w^{(1)T} w^{(1)}} w^{(1)} \left(w^{(1)T} a_1^{(1)} \right)$$

$$= \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} - \frac{2}{30} \cdot 15 \cdot \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$$

$$P^{(1)} a_2^{(1)} = a_2^{(1)} - \frac{1}{30} \left(w^{(1)T} a_2^{(1)} \right) w^{(1)}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix} - \frac{2}{30} \cdot \frac{3}{5} 18 \cdot \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -7/5 \\ 24/5 \end{bmatrix}$$

$$P^{(1)} a_3^{(1)} = \begin{bmatrix} 6 \\ -2 \\ -2 \end{bmatrix} - \frac{2}{30} \cdot \frac{4}{5} 24 \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -26/5 \\ -18/5 \end{bmatrix}$$

$$P^{(1)} b = \begin{bmatrix} 6 \\ -1 \\ -7 \end{bmatrix} - \frac{2}{30} \cdot \frac{7}{10} 21 \cdot \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -19/5 \\ -42/5 \end{bmatrix} \} b_2$$

$$\begin{bmatrix} -3 & -4 & -2 \\ 0 & -7/5 & -26/5 \\ 0 & 24/5 & -18/5 \end{bmatrix} x = \begin{bmatrix} -1 \\ -19/5 \\ -42/5 \end{bmatrix}$$

$$P^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & & P_2 \end{bmatrix} \quad : \quad P_2 A_2 = \begin{bmatrix} * & * \\ 0 & * \end{bmatrix}$$

$$P_2 = I_2 - \frac{2}{w_2^T w_2} w_2 w_2^T \quad ; \quad w_2 = \begin{bmatrix} -7/5 - \left\| \begin{matrix} -7/5 \\ 24/5 \end{matrix} \right\|_2 \\ 24/5 \end{bmatrix}$$

$$\left\| \begin{matrix} -7/5 \\ 24/5 \end{matrix} \right\| = \frac{1}{5} 25 = 5 \Rightarrow w_2 = \begin{bmatrix} -7/5 - 5 \\ 24/5 \end{bmatrix}$$

$$= \begin{bmatrix} -32/5 \\ 24/5 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -32 \\ 24 \end{bmatrix} \Rightarrow w_2^T w_2 = 64$$

$$A_2 = \begin{bmatrix} a_1^{(2)} & a_2^{(2)} \end{bmatrix}$$

$$P_2 a_1^{(2)} = a_1^{(2)} - \frac{2}{64} \cdot \frac{1}{5} [-32, 24] \cdot \begin{bmatrix} -7/5 \\ 24/5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$P_2 a_2^{(2)} = \ldots = \begin{bmatrix} -2 \\ -6 \end{bmatrix} \qquad P_2 b_2 = \ldots = \begin{bmatrix} -7 \\ -6 \end{bmatrix}$$

$$P^{(1)} P^{(2)} A x = \begin{bmatrix} -1 \\ -7 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -4 & -2 \\ 0 & 5 & -2 \\ 0 & 0 & -6 \end{bmatrix} x = \begin{bmatrix} -1 \\ -7 \\ -6 \end{bmatrix}$$

obvatno

$\Rightarrow \quad x_3 = 1 \Rightarrow 5x_2 - 2 = -7 \Rightarrow x_2 = -1$

$\Rightarrow x_1 = 1$

$$x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$