Nevilleov algoritem

Podatki: $(x_0, f_0), (x_1, f_1), \dots, (x_n, f_n)$

$x_i \neq x_j$ za $\forall i \neq j$

$x \in \mathbb{R}$

Rezultat: $p_n(x)$, vrednost interpol. polinoma na danih točkah pri x

Oznaka: p_{ij} polinom st. $\leq j-i$,

ki interpolira $(x_i, f_i), \dots, (x_j, f_j)$

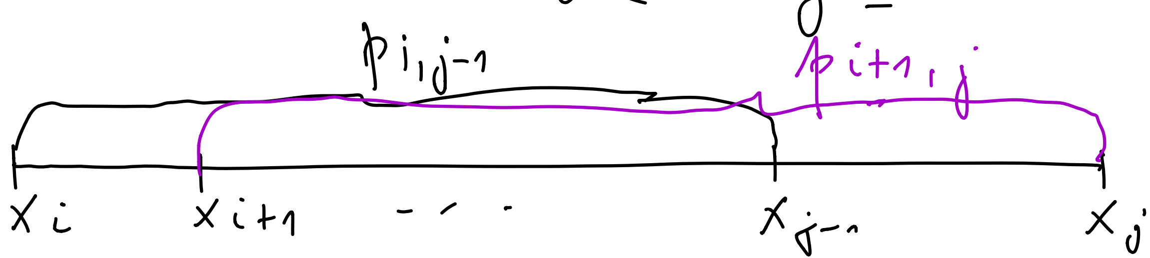
$j \geq i$

Lema: Polinom p_{ij} zadošča zvezam:

1. $p_{ii}(x) = f_i$, $0 \leq i \leq n$,

2. $$p_{ij}(x) = \frac{(x - x_j)\, p_{i,j-1}(x) - (x - x_i)\, p_{i+1,j}(x)}{x_i - x_j}$$

$0 \leq i < j \leq n$



Dokaz: ① je očitno

② Ideja: $i+1 \leq k \leq j-1 \Rightarrow$

$$p_{i,j-1}(x_k) = p_{i+1,j}(x_k) = f_k$$

Opazimo: Če je $\gamma \in \mathbb{R} \Rightarrow$

$$\gamma f_k + (1-\gamma) f_k = f_k$$

Poskusimo p_{ij} zapisati kot:

$$p_{ij}(x) = (\alpha x + \beta) p_{i,j-1}(x) + (1 - (\alpha x + \beta)) p_{i+1,j}(x)$$

$$p_{ij}(x_k) = f_k, \quad i+1 \leq k \leq j-1$$

Za x_i in x_j pa mora veljati:

$$p_{ij}(x_i) = (\alpha x_i + \beta) \overbrace{p_{i,j-1}(x_i)}^{f_i} + (1 - (\alpha x_i + \beta)) p_{i+1,j}(x_i) = f_i$$

$$p_{ij}(x_j) = (\alpha x_j + \beta) p_{i,j-1}(x_j)$$

$$+\left(1-(\alpha x_j+\beta)\right)\underbrace{p_{i+1,j}(x_j)}_{f_j} = f_j$$

$$\Rightarrow \quad \left.\begin{aligned} 1-\alpha x_i-\beta &= 0 \\ \alpha x_j+\beta &= 0 \end{aligned}\right\} \Rightarrow$$

$$1-\alpha x_i+\alpha x_j = 0$$

$$\alpha = \frac{1}{x_i-x_j}$$

$$\beta = -\alpha x_j = -\frac{x_j}{x_i-x_j}$$

<u>Primer</u>: Izračunajte vrednost interpolacijskega polinoma st. ≤ 2, ki interpolira

x_i	-1	0	2
y_i	-2	-1	7

pri $x=1$.

x_i	y_i	
-1	-2	
		0
0	-1	
		3
2	7	

$$p_{01}(1) = \frac{(1-0)(-2) - (1+1)(-1)}{-1-0} = \frac{-2+2}{-1} = 0$$

$$p_{12}(1) = \frac{(1-2)(-1) - (1-0)\,7}{0-2} = \frac{1-7}{-2} = 3$$

$$p_{02}(1) = \frac{(1-2)\cdot 0 - (1+1)\cdot 3}{-1-2} = \frac{-6}{-3} = 2$$

Sami preverite, da je

$$p_{02}(x) = x^2 + 2x - 1$$

$$p_{02}(1) = 1 + 2 - 1 = 2$$