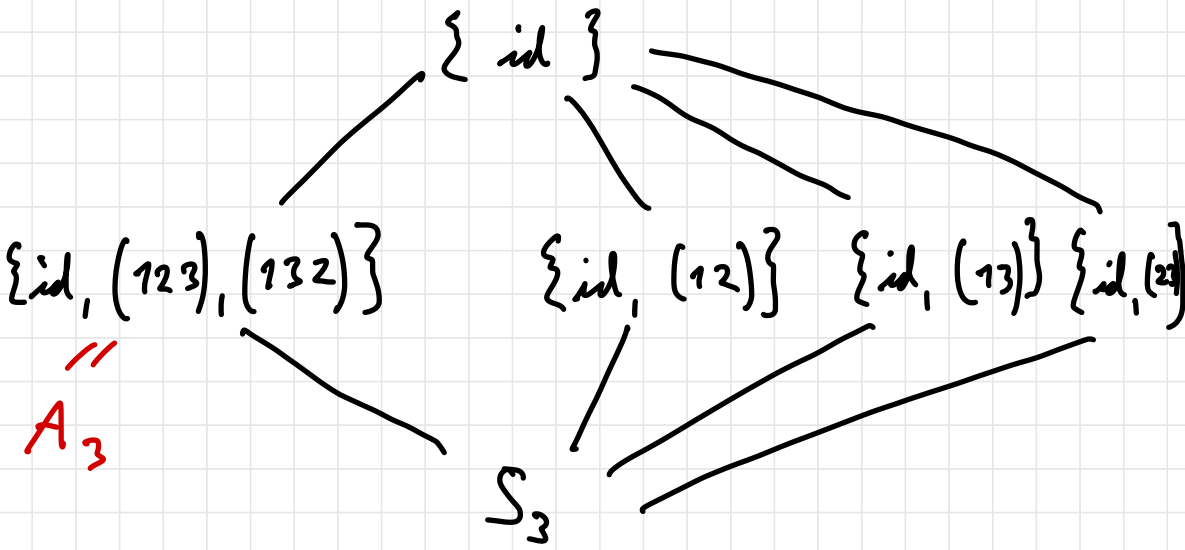
DS2 VAJE 12

1. POIŠČI VSE PODGRUPE S_3. KATERE SO EDINKE?



- ČE PODGRUPA VSEBUJE (12) IN (13)
  $\Rightarrow$ VSEBUJE $(12)(13) = (123)$ $\Rightarrow$ VSEBUJE (132)
  $\Rightarrow$ VSEBUJE $(123)(12) = (23)$ $\Rightarrow$ PODGRUPA S_3
  ...

- KATERE EDINKE?
  H JE EDINKA $\Leftrightarrow$ $xHx^{-1} = H$ $\forall x$

$H = \{ id, (12) \}$ $\quad x = (13)$

$$x H x^{-1} = \{ (13) id (13)^{-1}, (13)(12)(13)^{-1} \}$$

$$= \{ id, (23) \} \neq H$$

$\Rightarrow \{ id, (12) \}$ NI EDINKA

PODOBNO $\{ id, (13) \}$, $\{ id, (23) \}$ NISO EDINKE

A_3 EDINKA?

$$(13) A_3 (13) = \{ id, (13)(123)(13), (13)(132)(13) \}$$

$$= \{ id, (132), (123) \}$$

$$= A_3$$

$(12) A_3 (12) = A_3$

$(23) A_3 (23) = A_3$ $\Rightarrow A_3$ JE EDINKA

2. $H = \{\pi \in S_4 \mid \pi(4) = 4\}$.

ALI JE H EDINKA? POIŠČI LEVE ODSEKE PO H.

- H PODGRUPA ?

$a \cdot b^{-1} \in H \quad \forall a, b \in H$

$a \cdot b^{-1} \in H$ ✓

$4 \to 4 \to 4$

$a \in H$ KER $4 \to 4$

$b \in H$ KER $4 \to 4$

- H EDINKA ?

$x = (14) \qquad xHx^{-1}$

$(14)H(14) \stackrel{?}{=} H$

$h \in H \qquad (14)h(14) \stackrel{?}{\in} H$

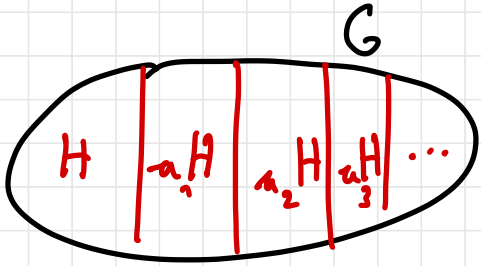
ZA $h = (12) \in H$

$(14)(12)(14)$

$4 \to 1 \to 2 \to 2 \Rightarrow \notin H$

$\Rightarrow$ H NI EDINKA

- ODSEKI PO H ?



ODSEK aH

$a_1 = (14)$

$a_1H = \{(14), (14)(12), (14)(13), (14)(23), (14)(123), (14)(132)\}$

$= \{(14), (142), (143), (14)(23), (1423), (1432)\}$

VSE PRESLIKAVE, KI PRESLIKAJO $1 \to 4$

$a_2 = (24)$ $\quad a_2H \leftarrow$ VSI $2 \to 4$

$a_2 = (34)$ $\quad a_3H \leftarrow$ VSI $3 \to 4$

ODSEKI $\{H, a_1H, a_2H, a_3H\}$

$a_1' = (743)$ $\quad a_1' H = \{(743), \dots\}$

$= a_1 H$

3. $G = \{ f : \mathbb{R} \to \mathbb{R} ; \ f(x) = ax + b, \ a, b \in \mathbb{R}, \ a > 0 \}$

$T = \{ f \in G \ ; \ f(x) = x + b \}$

a) PREVERI, DA G GRUPA.

$f_1(x) = a_1 x + b_1 \qquad f_2(x) = a_2 x + b_2$

$f_1 \cdot f_2 (x) = a_1 (a_2 x + b_2) + b_1$

$= a_1 a_2 x + (a_1 b_2 + b_1)$ ✓

f_1^{-1} :

$a_1 a_2 x + a_1 b_2 + b_1 = x$

$a_2 = \frac{1}{a_1} \qquad b_2 = -\frac{b_1}{a_1}$ ✓

b) T EDINKA ?

$$f_1 \cdot f_2^{-1}(x) = x + (-b_2 + b_1)$$

$\uparrow$ T $\quad$ $\uparrow$ T

$\Rightarrow \in T$

$\Rightarrow T$ PODGRUPA

$$f\, T\, f^{-1} \stackrel{?}{=} T$$

$$f \cdot \pi \cdot f^{-1} \stackrel{?}{\in} T$$

$f(x) = ax + b$

$\pi(x) = x + c$

$$f \cdot \pi \cdot f^{-1}(x) = f\left(\frac{1}{a}x + \left(-\frac{b}{a} + c\right)\right)$$

$$= x + \left(-b + ac\right) + b = x + ac$$

$\in T$

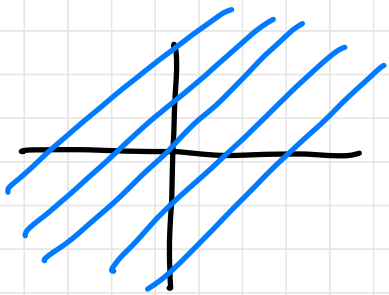
$\Rightarrow T$ EDINKA

- DESNI ODSEKI PO T

$$T \cdot f = \{ ax + b + c \mid c \in \mathbb{R} \}$$

$\{x+c\}$ $\quad ax+b$

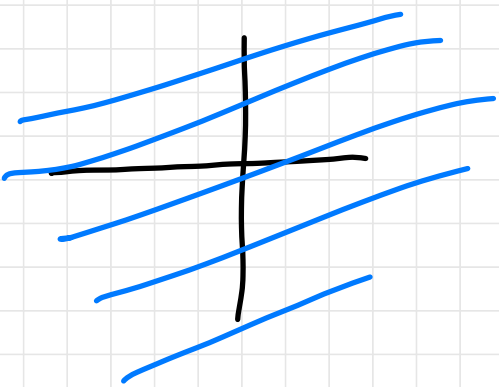
$$= \{ ax + d \mid d \in \mathbb{R} \}$$



T

T_{f_1}

$f_1(x) = 2x + b$

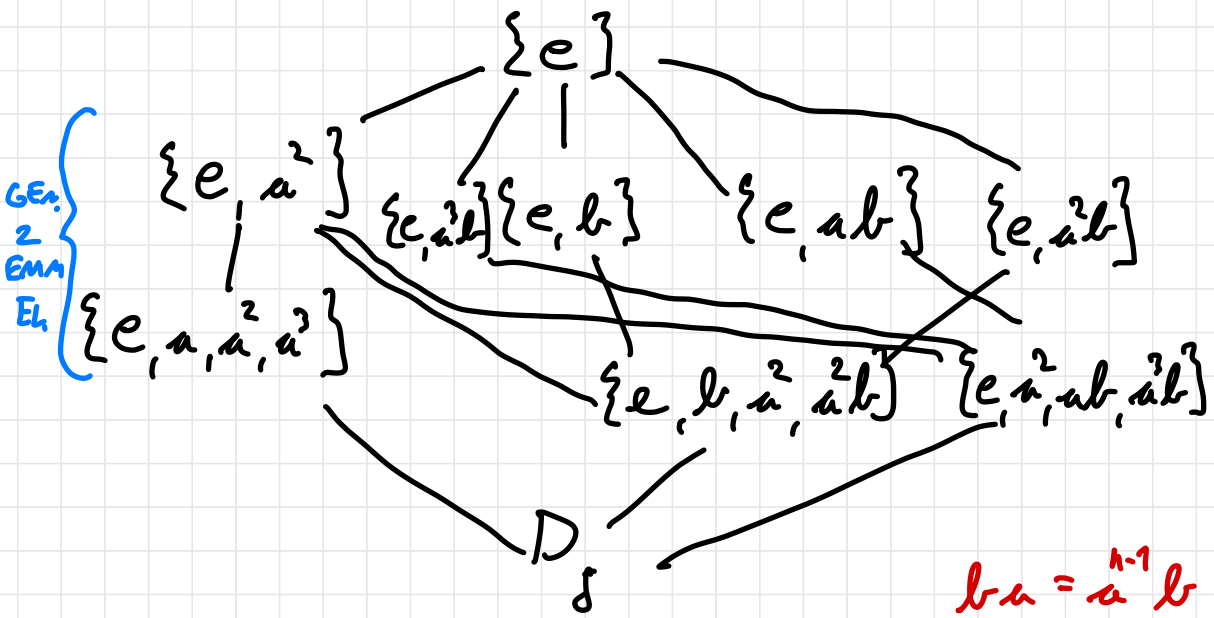


$f_2(x) = \frac{1}{10} x + c$

4. $G = D_{2n}$ $\quad 24$ $\quad n = 4$

$$D_8 = \langle a, b \mid a^4 = b^2 = (ab)^2 = e \rangle$$

a) POIŠČI VSE PODGRUPE EDINKE.



$ba = a^{n-1}b$

$= a^2 b$

$a^2b = a \cdot (ab)$

ZRCALJENJE

ROTACIJA

ZRCALJENJE

$(a^2 \cdot b)^2 = a^2 b a^2 b$

$= a^2 b a a b$

$= a^2 a^3 b a b$

$= a^2 a^3 a^3 b b$

$= a^8 b^2 = e$

$ba^2 = baa = a^3 ba = a^6 b$

$= a^2 b$

- $a\{e, b\}a^{-1} = \{e, aba^2\} =$

$= \{e, a^2b\} \neq \{e, b\}$

NISTA EDINKI

$a\{e, ab\}a^{-1} = \{e, a^2b\}$

NISTA EDINKI

- $a\{e, a^2\}a^{-1} = \{e, a^2\}$

$b\{e, a^2\}b^{-1} = \{e, bba^2b\} =$

$= \{e, a^2a^2bb\} =$

$= \{e, a^2\}$

$\Rightarrow$ JE EDINKA

DN PODGRUPE REDA 4 ...

(SO EDINKE)

b) KATERA JE CENTER GRUPE?

$Z(G)$

$a^2 \stackrel{?}{\in} Z(G)$

$a^2 \cdot a \stackrel{?}{=} a a^2$ ✓

$a^2 b \stackrel{?}{=} b a^2 = a^3 b a = a^3 a^3 b =$

$= a^2 b$ ✓

$\Rightarrow a^2 \in Z(G)$

$\Rightarrow Z(G) = \{e, a^2\}$

c) $D_8 / Z(D_8) =$

$= \{ \{e, a^2\}, \{a, a^3\}, \{b, a^2 b\},$

$Z(G)$ $a \cdot Z(G)$ $b \cdot Z(G)$

$ab\,Z(G) = \{ab, a^3 b\}\}$

KAKO MNOŽIMO ODSEKE?

$$xH \cdot yH = xy \cdot H$$

$$aZ(G) \cdot bZ(G) = abZ(G)$$

$$aZ(G) \cdot aZ(G) = a^2Z(G) = Z(G)$$

$$bZ(G) \cdot bZ(G) = b^2Z(G) = Z(G)$$

$$bZ(G) \cdot aZ(G) = ba\,Z(G) = a^3bZ(G) = abZ(G)$$

$b^2 = e$

$$D_8/Z(D_8) \cong ?$$

$$Ali \cong (\mathbb{Z}_4, +) ?$$

ALI VSEBUJE $D_8/Z(D_8)$ EL.

REDA 4? $aZ(G)$?

$$(aZ(G))^2 = aZ(G) \cdot aZ(G) =$$
$$= a^2 Z(G) = \underline{Z(G)}$$

$\parallel$ ENOTA v $D_8/Z(G)$

$$\text{red}\,(aZ(G)) = 2$$
$$\text{red}\,(bZ(G)) = 2$$
$$\text{red}\,(abZ(G)) = 2$$

DN

$$\mathbb{Z}_2 \times \mathbb{Z}_2$$

$$f : D_8/Z(D_8) \to \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$Z(D_8) \longmapsto (0,0)$$
$$aZ(D_8) \longmapsto (1,0)$$
$$bZ(D_8) \longmapsto (0,1)$$
$$abZ(D_8) \longmapsto (1,1)$$

DN PREVERI IZOMORFIZEM