Mat 1 K3 vzorek řešení

1

$$\int \frac{dx}{\sin x+\cos x-1} = \int \frac{\frac{2\,dt}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}-1} = \int \frac{2\,dt}{2t+1-t^2-1-t^2} = \int\frac{dt}{t-t^2} = \int\frac{-dt}{t(t-1)} =$$

$t=\operatorname{tg}\frac{x}{2} \xrightarrow{\text{VASE}} \sin x = \frac{2t}{1+t^2}$, $dx = \frac{2\,dt}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$$\frac{-1}{t(t-1)} = \frac{A}{t}+\frac{B}{t-1} = \frac{At-A+Bt}{t(t-1)}$$

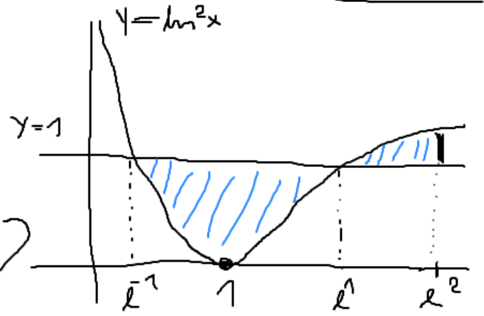
$$\left.\begin{aligned} 1&: -1 = -A \\ t&: \ 0 = A+B \end{aligned}\right] \Rightarrow \begin{aligned} A&=1 \\ B&=-1 \end{aligned}$$

$$= \int\left(\frac{1}{t}-\frac{1}{t-1}\right)dt = \ln|t| - \ln|t-1| + C$$

$$= \ln\left|\frac{t}{t-1}\right| + C = \underline{\underline{\ln\left|\frac{\operatorname{tg}\frac{x}{2}}{\operatorname{tg}\frac{x}{2}-1}\right| + C}}$$

2

$y=\ln^2 x$, $y=1$, $x=e^2$



$$\ln^2 x = 1 \iff \ln x = \pm 1 \iff x = e^{\pm 1}$$

$$P = \int_{e^{-1}}^{e}(1-\ln^2 x) + \int_{e}^{e^2}(\ln^2 x - 1) = (e-e^{-1}) + \int_{e}^{e^{-1}}\ln^2 x + \int_{e}^{e^2}\ln^2 x + (e-e^2)$$

$$\int \ln^2 x \overset{\text{P.P.}}{=} x\cdot\ln^2 x - \int x\cdot\overbrace{2\ln x\cdot\frac{1}{x}}^{2\ln x} \overset{\text{P.P.}}{=} x\ln^2 x - 2x\ln x + \int 2x\cdot\frac{1}{x} = x\left(\ln^2 x - 2\ln x + 2\right)$$

$$= 2e - e^{-1} - e^2 + x\left(\ln^2 x - 2\ln x + 2\right)\Big|_{e}^{e^{-1}} + \Big|_{e}^{e^2} =$$

$$= \cancel{2e} - e^{-1} - e^2 + e^{-1}(1+2+2) + e^2(4-4+2) - \cancel{2e}(1-2+2) = \underline{\underline{4e^{-1}+e^2}}$$

3

$y=\sqrt{x}\,e^{x^2}$, $x\in[0,3]$

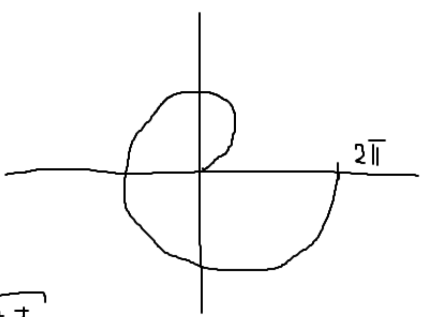
$$\Rightarrow V = \pi\int_{x_0}^{x_1} y^2(x)\,dx = \pi\int_0^3 x e^{2x^2}dx = \frac{\pi}{4}\int_0^{18} e^t\,dt = \underline{\underline{\frac{\pi}{4}\left(e^{18}-1\right)}}$$

$t=2x^2$, $dt=4x\,dx$

4

$$\left.\begin{aligned} x&=t\cos t \\ y&=t\sin t \end{aligned}\right] \xrightarrow{\text{POLARNE}} r = \sqrt{x^2+y^2} = \sqrt{t^2(\cos^2 t+\sin^2 t)} = t$$

$t\in[0,2\pi]$ $\qquad r(t) = t$



$$l = \int_{t_0}^{t_1}\sqrt{\dot x^2+\dot y^2+\dot z^2}\,dt = \int_0^{2\pi}\sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2}$$

$$= \int_0^{2\pi}\sqrt{\cos^2 t - \cancel{2t\sin t\cos t} + t^2\sin^2 t + \sin^2 t + \cancel{2t\sin t\cos t} + t^2\cos^2 t}$$

$$= \int_0^{2\pi}\sqrt{1+t^2} = \int_{\operatorname{argsh}0}^{\operatorname{argsh}2\pi}\sqrt{1+\operatorname{sh}^2 s}\,\operatorname{ch} s\,ds = \int_{\operatorname{argsh}0}^{\operatorname{argsh}(2\pi)}\operatorname{ch}^2 s\,ds =$$

$t=\operatorname{sh} s$, $dt = \operatorname{ch} s\,ds$

$$= \int_0^{\operatorname{argsh}(2\pi)}\frac{e^{2s}+2+e^{-2s}}{4} = \frac{1}{4}\left(\frac{e^{2s}}{2}+2s+\frac{e^{-2s}}{-2}\right)\Big|_0^{\operatorname{argsh}(2\pi)}$$

$$= \underline{\underline{\frac{1}{4}\left(2\operatorname{argsh}(2\pi) + \operatorname{sh}(2\operatorname{argsh}(2\pi))\right)}}$$

$\dot x = \cos t - t\sin t$

$\dot y = \sin t + t\cos t$

$\operatorname{ch} s = \frac{e^s+e^{-s}}{2}$