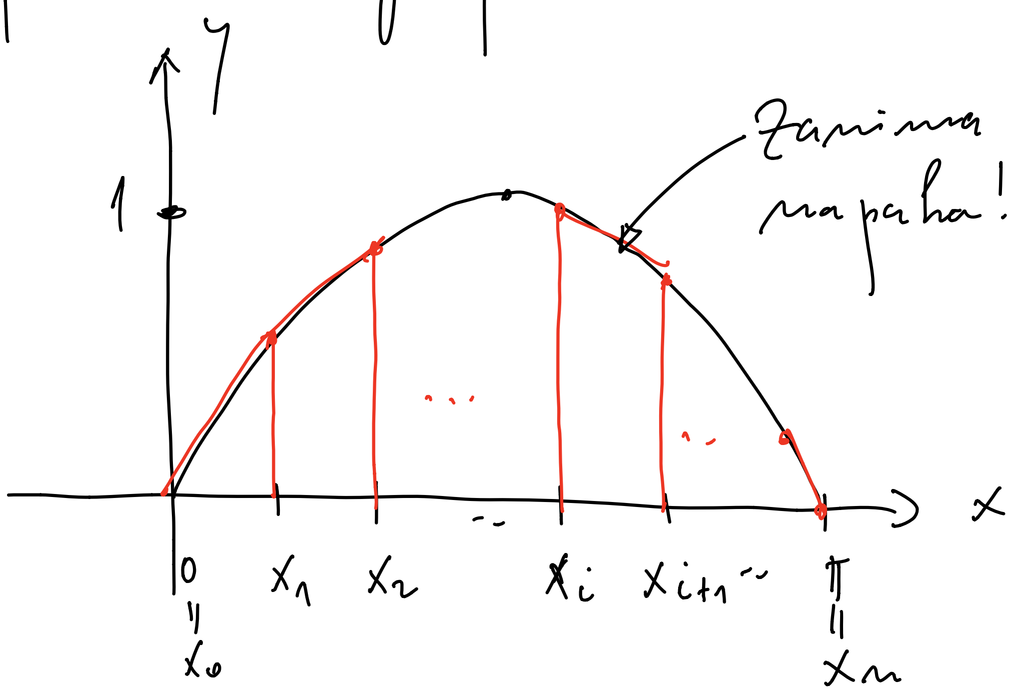
Naloga: Funkcijo $f(x) = \sin x$ aproksimiramo z odsekoma linearno funkcijo l na $n+1$ ekvidistantnih točkah

$$x_0 = 0 < x_1 < \dots < x_n = \pi$$ tako,

da l interpolira dane točke, na vsakem podintervalu $[x_i, x_{i+1}]$, $i = 0, \dots, n-1$, pa ji l daljica med interp. točkama.
Kolikšna mora biti n, da bo napaka aproksimacije pod 10^{-3}.



$$l : [\overset{0}{x_0}, \overset{\pi}{x_n}] \longrightarrow \mathbb{R}, \quad l\big|_{[x_i, x_{i+1}]} =: l_i \text{ ji } \underset{\text{premica}}{}$$

premica

Zanima nas

$$\max_{x_0 \le x \le x_n} |f(x) - l(x)| = ?$$

Vemo, da je $x_{i+1} - x_i =: h$.

$$\max_{x_0 \le x \le x_n} |f(x) - l(x)| = \max_{0 \le i \le n-1} \max_{x_i \le x \le x_{i+1}} |f(x) - l_i(x)|$$

Naj bo $x \in [x_i, x_{i+1}]$:

$$|f(x) - l_i(x)| = \left| \frac{f^{(2)}(\xi_i)}{2!} w_i(x) \right| ;$$

$$w_i(x) = (x - x_i)(x - x_{i+1}) ; \quad \xi_i \in [x_i, x_{i+1}].$$

$$\left| \frac{f^{(2)}(\xi_i)}{2!} \right| \le \frac{1}{2} \overbrace{\max_{x_i \le \xi \le x_{i+1}} |-\sin(\xi)|}^{\Delta_i}$$

$$= \frac{1}{2} \Delta_i$$

$$\max_{x_i \le x \le x_{i+1}} |w_i(x)| = ?$$

$$w_i(x) = (x - x_i)(x - x_{i+1}) \quad ; x_{i+1} = x_i + h$$

$$= (x - x_i)(x - x_i - h)$$

$$w_i'(x) = (x - x_i - h) + (x - x_i)$$

$$= 2x - 2x_i - h = 0$$

$$x = x_i + \frac{h}{2}$$

$$w_i'\left(x_i+\frac{h}{2}\right)=\left(\cancel{x_i}+\frac{h}{2}-\cancel{x_i}\right)\left(\cancel{x_i}+\frac{h}{2}-\cancel{x_i}-h\right)$$

$$=\frac{h}{2}\left(-\frac{h}{2}\right)=-\frac{h^2}{4}$$

Toref ji:

$$\max_{x_i\le x\le x_{i+1}}\left|\frac{f^{(2)}(\xi_i)}{2!}w_i(x)\right|$$

$$\le\frac{1}{2}\gamma_i\cdot\frac{h^2}{4}$$

Oziroma:

$$\max_{0\le i\le n-1}\ \max_{x_i\le x\le x_{i+1}}\left|f(x)-l_i(x)\right|$$

$$\le\max_{0\le i\le n-1}\frac{1}{8}h^2\gamma_i\le\frac{1}{8}\cdot h^2\cdot 1\overset{!!}{<}10^{-3}$$

$$h^2<8\cdot 10^{-3}=0{,}008$$

$$h<0{,}0894$$

$$n\cdot h=\pi\Rightarrow h=\frac{\pi}{n}$$

$$\frac{\pi}{n} < 0{,}0894$$

$$n > \frac{\pi}{0{,}0894} \doteq 35.12$$

$$\Rightarrow \qquad n \geq 36$$