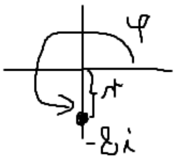
Mat 1 I1 osnutek rešitev

1 $z^3 = -8i \overset{\text{POLARNI ZAPIS}}{=} 8\cdot e^{i\frac{3\pi}{2}} \Rightarrow z = \underline{\underline{\sqrt[3]{8}\cdot e^{i\left(\frac{3\pi/2+2k\pi}{3}\right)}}}$ za $k = 0,1,2$



$\varphi = \frac{3\pi}{2}$

$r = |-8i| = 8$

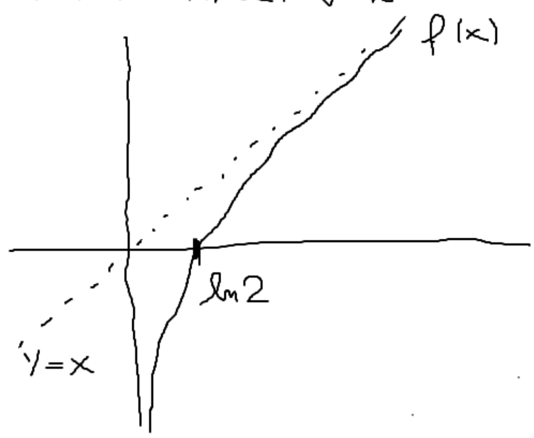
$= 2\left(\cos\left(\frac{\pi}{2}+\frac{2k\pi}{3}\right)+i\sin\left(\frac{\pi}{2}+\frac{2k\pi}{3}\right)\right)$

2 $f(x) = \ln(e^x-1)$, $D_f: e^x-1>0 \Leftrightarrow e^x>1 \Leftrightarrow x>0$, torej $D_f = (0,\infty)$

- ničle: $f(x)=0 \Leftrightarrow e^x-1=1 \Leftrightarrow x = \ln 2$
- $\lim\limits_{x\downarrow 0} \ln(\underbrace{e^x-1}_{\to 0}) = -\infty$, $\lim\limits_{x\to\infty} \ln(\underbrace{e^x-1}_{\to\infty}) = \infty$
- asimptota $y = ax+b$? $a = \lim\limits_{x\to\infty}\frac{f(x)}{x} \overset{\text{L'HP}}{=} \lim\limits_{x\to\infty}\frac{\frac{e^x}{e^x-1}}{1} = \lim \frac{1}{1-e^{-x}} = 1$

$b = \lim\limits_{x\to\infty} f(x)-ax = \lim\limits_{x\to\infty}\ln(e^x-1)-x \overset{\infty-\infty}{=} \lim\limits_{x\to\infty}\ln(e^x-1)-\ln(e^x) = \lim\limits_{x\to\infty}\ln\underbrace{\left(\frac{e^x-1}{e^x}\right)}_{\to 1} = \ln 1 = 0$

- $f' = \frac{e^x}{e^x-1} > 0$ na $x>0$, torej f povsod naraščajoča in nima stac. točk



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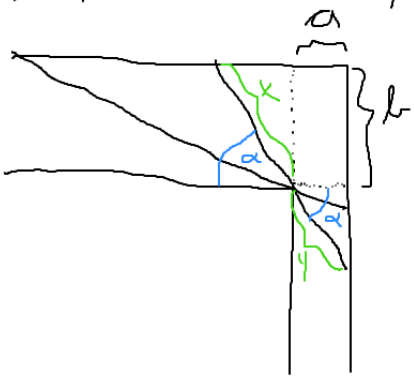

$\sin\alpha = \frac{b}{x}$

$\cos\alpha = \frac{a}{y}$

minimiziramo $f(\alpha)$ = dolžina daljice pri kotu α
$= x + y = \frac{b}{\sin\alpha} + \frac{a}{\cos\alpha}$

$f' = \frac{-b\cos\alpha}{\sin^2\alpha} + \frac{+a\sin\alpha}{\cos^2\alpha} = \frac{a\sin^3\alpha - b\cos^3\alpha}{\sin^2\alpha\cdot\cos^2\alpha}$, $f' = 0 \Leftrightarrow a\sin^3\alpha = b\cos^3\alpha$

$\Leftrightarrow \operatorname{tg}^3\alpha = \frac{b}{a}$

$\Leftrightarrow \underline{\alpha_0 = \operatorname{arctg}\sqrt[3]{\frac{b}{a}}}$

$\Rightarrow f(\alpha_0) = \underline{\underline{\frac{b}{\sin\operatorname{arctg}\sqrt[3]{b/a}} + \frac{a}{\cos\operatorname{arctg}\sqrt[3]{b/a}}}}$

4 $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\qquad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$$\lim_{x\to 0}\frac{\cos(x^2)-e^{(x^4)}}{\sin(x^4)} = \lim_{x\to 0}\frac{\left(\cancel{1} - \frac{x^4}{2!} + \frac{x^8}{4!} - \dots\right) - \left(\cancel{1} + x^4 + \frac{x^8}{2!} + \frac{x^{12}}{3!} + \dots\right)}{\left(x^4 - \underbrace{\frac{x^{12}}{3!} + \frac{x^{20}}{5!} - \dots}_{\to 0}\right)}$$

$$= \lim_{x\to 0}\frac{\cancel{x^4}\left(-\frac{3}{2} + x^4\left(\frac{1}{4!}+\frac{1}{2!}\right) + \dots\right)}{\cancel{x^4}\left(1 - \underbrace{\frac{x^8}{3!}}_{\to 0} + \underbrace{\frac{x^{16}}{5!}}_{\to 0} - \dots\right)} = \underline{\underline{\frac{-3/2}{1}}}$$