SPOMNIMO: UVEDBA NOVE SPREMENLJIVKE V NEDOLOČENI INTEGRAL:

$$x = g(t) \qquad t = h(x)$$

$$dx = g'(t)\,dt \qquad \text{ALI} \qquad dt = h'(x)\,dx$$

$$dx = \frac{dt}{h'(x)}$$

OSREDNJA IDEJA TE LEKCIJE:

ISKATI TAK $t = h(x)$, DA $h'(x)\,dx$ ŽE NASTOPA V INTEGRALU OZ. SE USTREZNO POKRAJŠA ALI PREOBLIKUJE.

NALOGA: Z UVEDBO NOVE SPREMENLJIVKE INTEGRIRAJ

(a) $\int tg\,x\,dx$ (b) $\int \frac{dx}{1-\cos x}$ (c) $\int \frac{dx}{\sin x}$

REŠITEV:

(a) $\int tg\,x\,dx = \int \frac{\sin x}{\cos x}\,dx$

SPLAČA SE UVESTI $t = \cos x$, SAJ $dt = -\sin x\,dx$.

$$\int tg\,x\,dx = -\int \frac{dt}{t} = -\ln|t| + C = -\ln|\cos x| + C$$

(b) ISTI TRIK KOT PRI LIMITAH:

$$\int \frac{1}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x}\,dx = \int \frac{1+\cos x}{1-\cos^2 x}\,dx =$$

$t=\sin x$

$dt=\cos x\,dx$

$$= \int \frac{dx}{\sin^2 x} + \int \frac{\cos x\,dx}{\sin^2 x} =$$

$$= -\mathrm{ctg}\,x + \int \frac{dt}{t^2} =$$

$$= -\mathrm{ctg}\,x - t^{-1} + C = -\mathrm{ctg}\,x - \frac{1}{\sin x} + C$$

(c) $$\int \frac{1}{\sin x} \cdot \frac{\sin x}{\sin x}\,dx = \int \frac{\sin x\,dx}{1-\cos^2 x} = \int \frac{dt}{t^2-1}$$

$t=\cos x$, $dt=-\sin x\,dx$

$$\frac{A}{t-1} + \frac{B}{t+1} = \frac{At+A+Bt-B}{t^2-1} = \frac{1}{t^2-1}$$

$$\left.\begin{array}{l} A+B=0 \\ A-B=1 \end{array}\right\} \quad A=\frac{1}{2} \quad B=-\frac{1}{2}$$

$$\int \frac{dx}{\sin x} = \frac{1}{2}\int \frac{dt}{t-1} - \frac{1}{2}\int \frac{dt}{t+1} = \frac{1}{2}\left(\ln|t-1| - \ln(t+1)\right) + C$$

$$= \ln\sqrt{\frac{\cos x - 1}{\cos x + 1}} + C$$

ALTERNATIVNA POT!

$$\int \frac{dx}{\sin x} = \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \int \frac{2\,dt}{2\cdot t\cdot \cos^2\frac{x}{2}} =$$

$$t = \sin\frac{x}{2} \Rightarrow dt = \cos\frac{x}{2}\cdot\frac{1}{2}\,dx$$

$$dx = \frac{2\,dt}{\cos\frac{x}{2}}$$

$$= \int \frac{dt}{t\cdot(1-\sin^2\frac{x}{2})} = \int \frac{dt}{t(1-t^2)}$$

DOMAČA NALOGA:

REŠI DO KONCA S PARCIALNIMI ULOMKI:

$$\frac{A}{t} + \frac{B}{1-t} + \frac{C}{1+t} = \frac{1}{t(1-t^2)}$$

NALOGA: Z UVEDBO NOVE SPREMENLJIVKE REŠI:

(a) $\int \frac{dx}{ch\,x}$ (b) $\int tg^4 x\,dx$ (c) $\int th^3 x\,dx$

REŠITEV:

(a) $$\int \frac{dx}{ch\,x} = \int \frac{2\,dx}{e^x + e^{-x}} = \int \frac{2\,dt}{t(t+\frac{1}{t})} = 2\int \frac{dt}{1+t^2}$$

$$t = e^x$$
$$dt = e^x\,dx$$
$$dx = \frac{dt}{t}$$

$$= 2\,\mathrm{arctg}\, t + C$$
$$= 2\,\mathrm{arctg}\, e^x + C$$

ALTERNATIVNA POT!

$$\int \frac{dx}{ch\,x} = \int \frac{ch\,x\; dx}{ch^2 x} = \int \frac{dt}{1+t^2} =$$

$t = sh\,x$

$dt = ch\,x\; dx$

$ch^2 x = 1 + sh^2 x = 1 + t^2$

$$= \operatorname{arctg} t + C = \operatorname{arctg}(sh\,x) + C$$

SKLEP: $2\operatorname{arctg} e^x = \operatorname{arctg}(sh\,x) + \text{konst.}$

ŠE VEČ, ČE VSTAVIMO $x = 0$, UGOTOVIMO:

$$2 \cdot \operatorname{arctg} 1 = \operatorname{arctg}(0) + \text{konst}$$

$$\text{konst} = \frac{\pi}{2}$$

(b) $\int tg^4 x\, dx$

NASVET: $t = tg\,x \Rightarrow dt = \frac{dx}{\cos^2 x} = (1 + tg^2 x)\, dx$

$dx = \frac{dt}{1+t^2}$

$$\int tg^4 x\, dx = \int \frac{t^4}{1+t^2}\, dt = \int \frac{t^2(1+t^2) - t^2}{1+t^2}\, dt =$$

$$= \int t^2\, dt - \int \frac{t^2+1-1}{1+t^2}\, dt = \frac{t^3}{3} - \int dt + \int \frac{dt}{1+t^2} =$$

$$= \frac{t^3}{3} - t + \operatorname{arctg} t = \frac{tg^3 x}{3} - tg\,x + \operatorname{arctg}(tg\,x) + C$$

OPOMBA: LAHKO BI TUDI DELILI POLINOMA t^4 IN t^2+1.

(c) $\int th^3 x\,dx = \int \frac{sh^3 x}{ch^3 x}dx = \int \frac{sh^2 x}{t^3}dt =$

SPET IZKORISTIMO: $t = chx$, $dt = shx\,dx$

$sh^2 x = ch^2 x - 1 = t^2 - 1$

$= \int \frac{t^2-1}{t^3}dt = \int (t^{-1} - t^{-3})dt = \frac{t^{-2}}{2} + \ln|t| + C$

$= \frac{1}{2}\frac{1}{ch^2 x} + \ln(chx) + C$