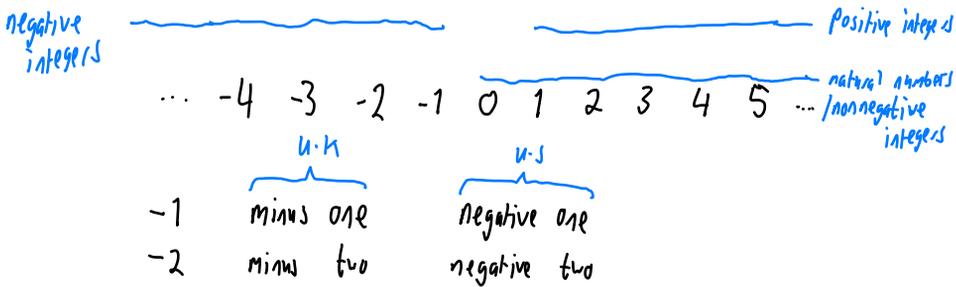


Last week: $(0) 1 2 3 4 \dots$

The natural numbers \mathbb{N}

This week we consider extensions of \mathbb{N} .

Firstly we add negative numbers



The whole numbers are in English called integers

\mathbb{Z} - the set of integers
↳ \mathbb{Z} from 'Zahlen' in German

Every integer x has an absolute value
(or modulus) $|x|$

e.g.
↑
for example

$$|-5| = 5$$

$$|3| = 3$$

$$|0| = 0$$

The operation of negation ^{noun}
^{verb} negates an integer.

$-x$ is the negation of x

e.g., $-(3) = -3$

$$-(-2) = 2$$

We can start to fill in gaps
between integers using fractions

Some very basic fractions

⊗ $\frac{1}{2}$ ^a one half

$\frac{1}{3}$ one third

⊗ $\frac{1}{4}$ one ^{u.u / u.s} quarter / ^{u.s} fourth

$\frac{1}{5}$ one fifth

In general use the ordinal number
except for ⊗ cases

The general form of a fraction is

$$\frac{m}{n}$$

m - numerator
 n - denominator

where $m \in \mathbb{Z}$

\uparrow is an element of
belongs to
is in

$$n \in \underbrace{\mathbb{Z} - \{0\}}$$

n is an integer
different from 0

removing 0 from the set \mathbb{Z}

To express a fraction:

Say the numerator as a cardinal number
and the denominator as an ordinal number
(or half / quarter)

(pluralising if there is more than one)

E.g.,

$$\frac{200}{250}$$

(the plural of "half"
is "halves")

"Two hundred two hundred and fiftieths"

Or

200 over 250

upon

on

using the cardinal number in both cases

The numbers we can form as fractions are called rational numbers

\mathbb{Q} - the set of rational numbers

from
"quotient"



$\frac{2}{3}$ is a proper fraction

$\frac{m}{n}$ is a proper fraction if
 $m \neq 0$ and $|m| < |n|$

$\frac{3}{2}$ is an improper fraction

Typically we prefer to write
improper fractions as mixed fractions

e.g. $\frac{3}{2} = 1 \frac{1}{2}$

integer part (pointing to 1) *fractional part* (pointing to $\frac{1}{2}$)

One and a half

One prefers to simplify fraction
to reduced fraction

e.g. $\frac{200}{250}$ simplifies to $\frac{4}{5}$

In general we simplify

$$\frac{m}{n} \quad \text{to} \quad \frac{m \div \text{gcd}(m,n)}{n \div \text{gcd}(m,n)}$$

$\text{gcd}(m,n)$ = the greatest common
divisor of m and n .

Euclid's algorithm for calculating the greatest common divisor

To compute $\text{gcd}(n_1, n_2)$

where we assume $n_1 > n_2$.

- calculate the remainder m when n_1 is divided by n_2
- if $m=0$ then return n_2 as the gcd
- otherwise, we repeat the process taking (n_2, m) in place of (n_1, n_2) .

Example $\text{gcd}(378, 105)$

Calculate: $378 \div 105 = 3$ rem 63

$105 \div 63 = 1$ rem 42

$63 \div 42 = 1$ rem 21

$42 \div 21 = 2$ rem 0

$\text{gcd} = 21$