Last week: (0) 1 2 3 4 ....

The natural numbers $\mathbb{N}$

This week we consider extensions of $\mathbb{N}$.

Firstly we add negative numbers

negative integers

Positive integers

natural numbers /nonnegative integers

$$\cdots\ -4\quad -3\quad -2\quad -1\quad 0\quad 1\quad 2\quad 3\quad 4\quad 5\ \cdots$$

	U.K	U.S
-1	minus one	negative one
-2	minus two	negative two

The whole numbers are in English called integers

$\mathbb{Z}$ – the set of integers

Z from 'Zahlen' in German

Every integer x has an absolute value (or modulus) $|x|$

e.g.

for example

$$|-5| = 5$$

$$|3| = 3$$

$$|0| = 0$$

The operation of negation ← noun

verb → negates an integer.

$-x$ is the negation of x

e.g.,

$$-(3) = -3$$

$$-(-2) = 2$$

We can start to fill in gaps between integers using fractions

Some very basic fractions

⊛ $1/2$ $\overset{\text{a}}{\text{one}}$ half

$1/3$ one third

⊛ $1/4$ one $\overset{\text{u.k / u.s}}{\text{quarter}}$ / $\overset{\text{u.s}}{\text{fourth}}$

$1/5$ one fifth

In general use the ordinal number except for ⊛ cases

The general form of a fraction is

$$\frac{m}{n} \quad \begin{matrix} - \ \underline{\text{numerator}} \\[1em] - \ \underline{\text{denominator}} \end{matrix}$$

where $m \in \mathbb{Z}$

$\uparrow$ is an element of
belongs to
is in

$$\underbrace{n \in \mathbb{Z} - \{0\}}_{\text{removing 0 from the set } \mathbb{Z}}$$

n is an integer
different from 0

To express a fraction:

Say the numerator as a cardinal number and the denominator as an ordinal number (or half / quarter) (pluralising if there is more than one)

E.g., $\frac{200}{250}$

(the plural of "half" is "halves")

"Two hundred two hundred and fiftieths"

<u>Or</u>

200 <u>over</u> 250

<u>upon</u>

<u>on</u>

using the cardinal number in both cases

The numbers we can form as fractions are called rational numbers

$\mathbb{Q}$ – the set of rational numbers

↑

from "quotient"

$\frac{2}{3}$ is a proper fraction

$\frac{m}{n}$ is a proper fraction if
$m \neq 0$ and $|m| < |n|$

$\frac{3}{2}$ is an improper fraction

Typically we prefer to write improper fractions as mixed fractions

e.g. $\frac{3}{2} = \overset{\text{integer part}}{1}\ \frac{1}{2}$ — fractional part

One and a half

One prefers to simplify fractions to reduced fractions

e.g. $\frac{200}{250}$ simplifies to $\frac{4}{5}$

In general we simplify

$$\frac{m}{n} \quad \text{to} \quad \frac{m \div \gcd(m,n)}{n \div \gcd(m,n)}$$

$\gcd(m,n)$ = the greatest common divisor of m and n.

Euclid's algorithm for calculating the greatest common divisor

To compute $\gcd(n_1, n_2)$
where we assume $n_1 > n_2$.

- calculate the remainder m when n_1 is divided by n_2
- if $m = 0$ then return n_2 as the gcd
- otherwise, we repeat the process taking (n_2, m) in place of (n_1, n_2).

Example $\gcd(378, 105)$

remainder

Calculate: $378 \div 105 = 3$ rem 63

$105 \div 63 = 1$ rem 42

$63 \div 42 = 1$ rem 21

$42 \div 21 = 2$ rem 0

$$\boxed{\gcd = 21}$$