# Jedra, kvarki, leptoni

1. kolokvij, 9.december 2020

1. *(1.5 točke)* Obravnavamo razpade mezona $\Sigma^{*0}$ iz barionskega dekupleta, ki razpada preko močne interakcije v stanja $\Lambda\pi$ in $\Sigma\pi$ (barion iz okteta in pion).

   (a) Uporabi izospinsko simetrijo za napoved razmerja razpadnih širin

   $$\frac{\Gamma(\Sigma^{*0} \to \Lambda\pi)}{\Gamma(\Sigma^{*0} \to \Sigma\pi)}.$$

   Upoštevaj vsa možna končna stanja $\Sigma\pi$ v imenovalcu! Upoštevaj, da imata $\Lambda$ in $\Sigma$ iz barionskega okteta enaki masi.

   (b) Popravi napoved iz točke (a) z upoštevanjem različnih mas $\Lambda$ in $\Sigma$. Razpadna širina za dvodelčni razpad v tem primeru je

   $$\Gamma = \frac{1}{8\pi} |\mathcal{M}|^2 \frac{|\boldsymbol{p}_f|}{M^2},$$

   kjer je $M$ masa začetnega delca, $\boldsymbol{p}_f$ pa je gibalna količina delcev v končnem stanju (v težiščnem sistemu).

   (c) Popravi napoved iz točk (a) in (b) z upoštevanjem števila možnih končnih stanj barve. Zamisli si eno barvno kombinacijo začetnega stanja $\Sigma^{*0}$, npr. $u^R d^G s^B$, ter preštej, koliko je možnosti za barve v končnem stanju. Predpostavi, da je par kvarkov $q\bar{q}$, ki se rodi iz vakuuma, barvni singlet (npr. $q^i \bar{q}^{\bar{i}}$). S tem faktorjem pomnoži posamezne razpadne širine (b).

   Podatki: $m_{\Sigma^{*0}} = 1385\,\text{MeV}$, $m_\Sigma = 1190\,\text{MeV}$, $m_\Lambda = 1116\,\text{MeV}$, $m_\pi = 140\,\text{MeV}$.

2. *(1.5 točke)* Izračunaj parameter $\kappa$ za model hiperfine sklopitve med spini kvarkov:

   $$M_{\text{barion}} = m_1 + m_2 + m_3 + \kappa \left( \frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{m_1 m_2} + \frac{\mathbf{s}_2 \cdot \mathbf{s}_3}{m_2 m_3} + \frac{\mathbf{s}_3 \cdot \mathbf{s}_1}{m_3 m_1} \right),$$

   kjer upoštevaj $m_q = m_u = m_d = 300\,\text{MeV}$ in $m_s = 500\,\text{MeV}$. Poznamo masno razliko $\Delta M = M_{\Sigma^*} - M_\Sigma = 195\,\text{MeV}$ med stanji barionskega dekupleta ($J^P = (3/2)^+$) in okteta ($J^P = (1/2)^+$). (Namig: brez škode za splošnost lahko pogledaš le eno permutacijo okusov kvarkov.)

3. *(1.0 točke)* Na mirujočem protonu sipamo elektron z energijo 15 MeV. Kolikšen $Q^2 = -q^2$ ustreza sipalnemu kotu $\vartheta = 2°$? Iz splošne formule za $d\sigma/d\Omega$ poišči najpomembnejši člen v razvoju za majhne $q^2$. Z uporabo tega izraza izračunaj vrednost sipalnega preseka $\sigma(ep \to ep, 2° < \vartheta < 3°)$.

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1. *(1.5 points)* Consider decays of a $\Sigma^{*0}$ from the baryon decuplet, which decays via the strong interaction to states $\Lambda\pi$ and $\Sigma\pi$ (baryon of octet and a pion).

   (a) Employ the isospin symmetry to predict the ratio of decay widths:

   $$\frac{\Gamma(\Sigma^{*0} \to \Lambda\pi)}{\Gamma(\Sigma^{*0} \to \Sigma\pi)} .$$

   Take into account all possible final states $\Sigma\pi$ in the denominator! Take into account that $\Lambda$ and $\Sigma$ from the octet have same mass.

   (b) Correct the prediction from (a) by taking into account the mass difference between $\Lambda$ and $\Sigma$. The decay width for a two-body decay in this case reads

   $$\Gamma = \frac{1}{8\pi} |\mathcal{M}|^2 \frac{|\boldsymbol{p}_f|}{M^2},$$

   where $M$ is the mass of the decaying particle, whereas $\boldsymbol{p}_f$ is the momentum of final state particles (in center of mass system).

   (c) Correct the prediction from (a) and (b) by taking into account the number of possible final state colours. Consider one color combination of the initial state of $\Sigma^{*0}$, e.g. $u^R d^G s^B$, and count the number of possible final state colours. Assume that the pair $q\bar{q}$ born from the vacuum is a color singlet (e.g. $q^i \bar{q}^{\bar{i}}$). Multiply the widths obtained in (b) by this factor.

   Numerical values: $m_{\Sigma^{*0}} = 1385\,\text{MeV}$, $m_\Sigma = 1190\,\text{MeV}$, $m_\Lambda = 1116\,\text{MeV}$, $m_\pi = 140\,\text{MeV}$.

2. *(1.5 points)* Calculate the parameter $\kappa$ for a model of hyperfine coupling between quark spins:

   $$M_{\text{barion}} = m_1 + m_2 + m_3 + \kappa \left( \frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{m_1 m_2} + \frac{\mathbf{s}_2 \cdot \mathbf{s}_3}{m_2 m_3} + \frac{\mathbf{s}_3 \cdot \mathbf{s}_1}{m_3 m_1} \right),$$

   and take into account $m_q = m_u = m_d = 300\,\text{MeV}$ and $m_s = 500\,\text{MeV}$. We know the mass difference $\Delta M = M_{\Sigma^*} - M_\Sigma = 195\,\text{MeV}$ among the states of baryon decuplet ($J^P = (3/2)^+$) and octet ($J^P = (1/2)^+$). (Hint: you may consider only one flavor permutation.)

3. *(1.0 points)* On a proton at rest we scatter an electron with energy 15 MeV. What $Q^2 = -q^2$ corresponds to a scattering angle of $\vartheta = 2°$? Using a general formula for $d\sigma/d\Omega$ find the leading term in the small $q^2$ expansion. Use this expression to calculate the scattering cross section $\sigma(ep \to ep, 2° < \vartheta < 3°)$.

# 34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND $d$ FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

| | | $J$ | $J$ | ... |
|---|---|---|---|---|
| | | $M$ | $M$ | ... |
| $m_1$ | $m_2$ | Coefficients | | |
| $m_1$ | $m_2$ | | | |
| . | . | | | |

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

## 1/2×1/2

| $m_1$ $m_2$ | 1, +1 |
|---|---|
| +1/2 +1/2 | 1 |

| $m_1$ $m_2$ | 1, 0 | 0, 0 |
|---|---|---|
| +1/2 −1/2 | 1/2 | 1/2 |
| −1/2 +1/2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 1, −1 |
|---|---|
| −1/2 −1/2 | 1 |

## 1×1/2

| $m_1$ $m_2$ | 3/2, +3/2 |
|---|---|
| +1 +1/2 | 1 |

| $m_1$ $m_2$ | 3/2, +1/2 | 1/2, +1/2 |
|---|---|---|
| +1 −1/2 | 1/3 | 2/3 |
| 0 +1/2 | 2/3 | −1/3 |

| $m_1$ $m_2$ | 3/2, −1/2 | 1/2, −1/2 |
|---|---|---|
| 0 −1/2 | 2/3 | 1/3 |
| −1 +1/2 | 1/3 | −2/3 |

| $m_1$ $m_2$ | 3/2, −3/2 |
|---|---|
| −1 −1/2 | 1 |

## 2×1

| $m_1$ $m_2$ | 3, +3 |
|---|---|
| +2 +1 | 1 |

| $m_1$ $m_2$ | 3, +2 | 2, +2 |
|---|---|---|
| +2 0 | 1/3 | 2/3 |
| +1 +1 | 2/3 | −1/3 |

| $m_1$ $m_2$ | 3, +1 | 2, +1 | 1, +1 |
|---|---|---|---|
| +2 −1 | 1/15 | 1/3 | 3/5 |
| +1 0 | 8/15 | 1/6 | −3/10 |
| 0 +1 | 2/5 | −1/2 | 1/10 |

| $m_1$ $m_2$ | 3, 0 | 2, 0 | 1, 0 |
|---|---|---|---|
| +1 −1 | 1/5 | 1/2 | 3/10 |
| 0 0 | 3/5 | 0 | −2/5 |
| −1 +1 | 1/5 | −1/2 | 3/10 |

| $m_1$ $m_2$ | 3, −1 | 2, −1 | 1, −1 |
|---|---|---|---|
| 0 −1 | 2/5 | 1/2 | 1/10 |
| −1 0 | 8/15 | −1/6 | −3/10 |
| −2 +1 | 1/15 | −1/3 | 3/5 |

| $m_1$ $m_2$ | 3, −2 | 2, −2 |
|---|---|---|
| −1 −1 | 2/3 | 1/3 |
| −2 0 | 1/3 | −2/3 |

| $m_1$ $m_2$ | 3, −3 |
|---|---|
| −2 −1 | 1 |

## 1×1

| $m_1$ $m_2$ | 2, +2 |
|---|---|
| +1 +1 | 1 |

| $m_1$ $m_2$ | 2, +1 | 1, +1 |
|---|---|---|
| +1 0 | 1/2 | 1/2 |
| 0 +1 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 2, 0 | 1, 0 | 0, 0 |
|---|---|---|---|
| +1 −1 | 1/6 | 1/2 | 1/3 |
| 0 0 | 2/3 | 0 | −1/3 |
| −1 +1 | 1/6 | −1/2 | 1/3 |

| $m_1$ $m_2$ | 2, −1 | 1, −1 |
|---|---|---|
| 0 −1 | 1/2 | 1/2 |
| −1 0 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 2, −2 |
|---|---|
| −1 −1 | 1 |

## 2×1/2

| $m_1$ $m_2$ | 5/2, +5/2 |
|---|---|
| +2 +1/2 | 1 |

| $m_1$ $m_2$ | 5/2, +3/2 | 3/2, +3/2 |
|---|---|---|
| +2 −1/2 | 1/5 | 4/5 |
| +1 +1/2 | 4/5 | −1/5 |

| $m_1$ $m_2$ | 5/2, +1/2 | 3/2, +1/2 |
|---|---|---|
| +1 −1/2 | 2/5 | 3/5 |
| 0 +1/2 | 3/5 | −2/5 |

| $m_1$ $m_2$ | 5/2, −1/2 | 3/2, −1/2 |
|---|---|---|
| 0 −1/2 | 3/5 | 2/5 |
| −1 +1/2 | 2/5 | −3/5 |

| $m_1$ $m_2$ | 5/2, −3/2 | 3/2, −3/2 |
|---|---|---|
| −1 −1/2 | 4/5 | 1/5 |
| −2 +1/2 | 1/5 | −4/5 |

| $m_1$ $m_2$ | 5/2, −5/2 |
|---|---|
| −2 −1/2 | 1 |

## 3/2×1/2

| $m_1$ $m_2$ | 2, +2 |
|---|---|
| +3/2 +1/2 | 1 |

| $m_1$ $m_2$ | 2, +1 | 1, +1 |
|---|---|---|
| +3/2 −1/2 | 1/4 | 3/4 |
| +1/2 +1/2 | 3/4 | −1/4 |

| $m_1$ $m_2$ | 2, 0 | 1, 0 |
|---|---|---|
| +1/2 −1/2 | 1/2 | 1/2 |
| −1/2 +1/2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 2, −1 | 1, −1 |
|---|---|---|
| −1/2 −1/2 | 3/4 | 1/4 |
| −3/2 +1/2 | 1/4 | −3/4 |

| $m_1$ $m_2$ | 2, −2 |
|---|---|
| −3/2 −1/2 | 1 |

## 3/2×1

| $m_1$ $m_2$ | 5/2, +5/2 |
|---|---|
| +3/2 +1 | 1 |

| $m_1$ $m_2$ | 5/2, +3/2 | 3/2, +3/2 |
|---|---|---|
| +3/2 0 | 2/5 | 3/5 |
| +1/2 +1 | 3/5 | −2/5 |

| $m_1$ $m_2$ | 5/2, +1/2 | 3/2, +1/2 | 1/2, +1/2 |
|---|---|---|---|
| +3/2 −1 | 1/10 | 2/5 | 1/2 |
| +1/2 0 | 3/5 | 1/15 | −1/3 |
| −1/2 +1 | 3/10 | −8/15 | 1/6 |

| $m_1$ $m_2$ | 5/2, −1/2 | 3/2, −1/2 | 1/2, −1/2 |
|---|---|---|---|
| +1/2 −1 | 3/10 | 8/15 | 1/6 |
| −1/2 0 | 3/5 | −1/15 | −1/3 |
| −3/2 +1 | 1/10 | −2/5 | 1/2 |

| $m_1$ $m_2$ | 5/2, −3/2 | 3/2, −3/2 |
|---|---|---|
| −1/2 −1 | 3/5 | 2/5 |
| −3/2 0 | 2/5 | −3/5 |

| $m_1$ $m_2$ | 5/2, −5/2 |
|---|---|
| −3/2 −1 | 1 |

## 3/2×3/2

| $m_1$ $m_2$ | 3, +3 |
|---|---|
| +3/2 +3/2 | 1 |

| $m_1$ $m_2$ | 3, +2 | 2, +2 |
|---|---|---|
| +3/2 +1/2 | 1/2 | 1/2 |
| +1/2 +3/2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 3, +1 | 2, +1 | 1, +1 |
|---|---|---|---|
| +3/2 −1/2 | 1/5 | 1/2 | 3/10 |
| +1/2 +1/2 | 3/5 | 0 | −2/5 |
| −1/2 +3/2 | 1/5 | −1/2 | 3/10 |

| $m_1$ $m_2$ | 3, 0 | 2, 0 | 1, 0 | 0, 0 |
|---|---|---|---|---|
| +3/2 −3/2 | 1/20 | 1/4 | 9/20 | 1/4 |
| +1/2 −1/2 | 9/20 | 1/4 | −1/20 | −1/4 |
| −1/2 +1/2 | 9/20 | −1/4 | −1/20 | 1/4 |
| −3/2 +3/2 | 1/20 | −1/4 | 9/20 | −1/4 |

| $m_1$ $m_2$ | 3, −1 | 2, −1 | 1, −1 |
|---|---|---|---|
| +1/2 −3/2 | 1/5 | 1/2 | 3/10 |
| −1/2 −1/2 | 3/5 | 0 | −2/5 |
| −3/2 +1/2 | 1/5 | −1/2 | 3/10 |

| $m_1$ $m_2$ | 3, −2 | 2, −2 |
|---|---|---|
| −1/2 −3/2 | 1/2 | 1/2 |
| −3/2 −1/2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 3, −3 |
|---|---|
| −3/2 −3/2 | 1 |

## 2×3/2

| $m_1$ $m_2$ | 7/2, +7/2 |
|---|---|
| +2 +3/2 | 1 |

| $m_1$ $m_2$ | 7/2, +5/2 | 5/2, +5/2 |
|---|---|---|
| +2 +1/2 | 3/7 | 4/7 |
| +1 +3/2 | 4/7 | −3/7 |

| $m_1$ $m_2$ | 7/2, +3/2 | 5/2, +3/2 | 3/2, +3/2 |
|---|---|---|---|
| +2 −1/2 | 1/7 | 16/35 | 2/5 |
| +1 +1/2 | 4/7 | 1/35 | −2/5 |
| 0 +3/2 | 2/7 | −18/35 | 1/5 |

| $m_1$ $m_2$ | 7/2, +1/2 | 5/2, +1/2 | 3/2, +1/2 | 1/2, +1/2 |
|---|---|---|---|---|
| +2 −3/2 | 1/35 | 6/35 | 2/5 | 2/5 |
| +1 −1/2 | 12/35 | 5/14 | 0 | −3/10 |
| 0 +1/2 | 18/35 | −3/35 | −1/5 | 1/5 |
| −1 +3/2 | 4/35 | −27/70 | 2/5 | −1/10 |

| $m_1$ $m_2$ | 7/2, −1/2 | 5/2, −1/2 | 3/2, −1/2 | 1/2, −1/2 |
|---|---|---|---|---|
| +1 −3/2 | 4/35 | 27/70 | 2/5 | 1/10 |
| 0 −1/2 | 18/35 | 3/35 | −1/5 | −1/5 |
| −1 +1/2 | 12/35 | −5/14 | 0 | 3/10 |
| −2 +3/2 | 1/35 | −6/35 | 2/5 | −2/5 |

| $m_1$ $m_2$ | 7/2, −3/2 | 5/2, −3/2 | 3/2, −3/2 |
|---|---|---|---|
| 0 −3/2 | 2/7 | 18/35 | 1/5 |
| −1 −1/2 | 4/7 | −1/35 | −2/5 |
| −2 +1/2 | 1/7 | −16/35 | 2/5 |

| $m_1$ $m_2$ | 7/2, −5/2 | 5/2, −5/2 |
|---|---|---|
| −1 −3/2 | 4/7 | 3/7 |
| −2 −1/2 | 3/7 | −4/7 |

| $m_1$ $m_2$ | 7/2, −7/2 |
|---|---|
| −2 −3/2 | 1 |

## 2×2

| $m_1$ $m_2$ | 4, +4 |
|---|---|
| +2 +2 | 1 |

| $m_1$ $m_2$ | 4, +3 | 3, +3 |
|---|---|---|
| +2 +1 | 1/2 | 1/2 |
| +1 +2 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 4, +2 | 3, +2 | 2, +2 |
|---|---|---|---|
| +2 0 | 3/14 | 1/2 | 2/7 |
| +1 +1 | 4/7 | 0 | −3/7 |
| 0 +2 | 3/14 | −1/2 | 2/7 |

| $m_1$ $m_2$ | 4, +1 | 3, +1 | 2, +1 | 1, +1 |
|---|---|---|---|---|
| +2 −1 | 1/14 | 3/10 | 3/7 | 1/5 |
| +1 0 | 3/7 | 1/5 | −1/14 | −3/10 |
| 0 +1 | 3/7 | −1/5 | −1/14 | 3/10 |
| −1 +2 | 1/14 | −3/10 | 3/7 | −1/5 |

| $m_1$ $m_2$ | 4, 0 | 3, 0 | 2, 0 | 1, 0 | 0, 0 |
|---|---|---|---|---|---|
| +2 −2 | 1/70 | 1/10 | 2/7 | 2/5 | 1/5 |
| +1 −1 | 8/35 | 2/5 | 1/14 | −1/10 | −1/5 |
| 0 0 | 18/35 | 0 | −2/7 | 0 | 1/5 |
| −1 +1 | 8/35 | −2/5 | 1/14 | 1/10 | −1/5 |
| −2 +2 | 1/70 | −1/10 | 2/7 | −2/5 | 1/5 |

| $m_1$ $m_2$ | 4, −1 | 3, −1 | 2, −1 | 1, −1 |
|---|---|---|---|---|
| +1 −2 | 1/14 | 3/10 | 3/7 | 1/5 |
| 0 −1 | 3/7 | 1/5 | −1/14 | −3/10 |
| −1 0 | 3/7 | −1/5 | −1/14 | 3/10 |
| −2 +1 | 1/14 | −3/10 | 3/7 | −1/5 |

| $m_1$ $m_2$ | 4, −2 | 3, −2 | 2, −2 |
|---|---|---|---|
| 0 −2 | 3/14 | 1/2 | 2/7 |
| −1 −1 | 4/7 | 0 | −3/7 |
| −2 0 | 3/14 | −1/2 | 2/7 |

| $m_1$ $m_2$ | 4, −3 | 3, −3 |
|---|---|---|
| −1 −2 | 1/2 | 1/2 |
| −2 −1 | 1/2 | −1/2 |

| $m_1$ $m_2$ | 4, −4 |
|---|---|
| −2 −2 | 1 |

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

**Figure 34.1**: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.