Jedra, kvarki, leptoni

1. izpit, 10. februar 2021

1. *(1.25 točke)* Oceni celotno razpadno širino hipotetično prostega kvarka b. Upoštevaj vse dovoljene trodelčne razpade na leptone in proste kvarke. Za parametre Wolfensteinove parametrizacije vzemi $A = 0.81$, $\lambda = 0.23$, $\rho = 0.14$, $\eta = 0.36$. Mase kvarkov so $m_b = 4.2\,\mathrm{GeV}$, $m_t = 172\,\mathrm{GeV}$, mase ostalih delcev v končnem stanju lahko zanemariš. Upoštevaj $G_F = 1.17 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$. Kolikšno je razmerje $\Gamma(b \to \text{kvarki})/\Gamma(b \to \text{kvarki} + \text{leptoni})$? Kolikšno je razmerje razpadnih širin $\Gamma(b \to u + \text{leptoni})/\Gamma(b \to c + \text{leptoni})$?

2. *(1.75 točke)* Opazujemo mase psevdoskalarnih (0^-) in vektorskih mezonov (1^-), ki so izospinski singleti ($T = 0$, $Y = 0$). Za primer psevdoskalarjev imamo stanji η_8 in η_1, kjer je $\eta_1 = (1/\sqrt{3})(u\bar{u} + d\bar{d} + s\bar{s})$ okusni singlet, η_8 pa je član mezonskega okteta. Fizikalni stanji sta mešanici

$$\eta = \eta_8 \cos\vartheta_P + \eta_1 \sin\vartheta_P,$$
$$\eta' = -\eta_8 \sin\vartheta_P + \eta_1 \cos\vartheta_P,$$

   kjer je kot $\vartheta_P = -14°$. Za vektorske mezone imamo analogno stanji ω_8 in ω_1, fizikalni stanji pa sta $\omega = \omega_8 \cos\vartheta_V + \omega_1 \sin\vartheta_V$ in $\varphi = -\omega_8 \sin\vartheta_V + \omega_1 \cos\vartheta_V$.

   (a) Izpelji okusno valovno funkcijo, ki opisuje stanji η_8 in ω_8! Upoštevaj, da je η_8 ortogonalna na η_1 in π^0.

   (b) Določi kot ϑ_V, če veš, da je okusna struktura vektorskega mezona $\varphi = s\bar{s}$.

   (c) Določi masi kvarkov $m_q = m_u = m_d$ in m_s če poznaš $m_\eta = 548\,\mathrm{MeV}$ in $m_{\eta'} = 958\,\mathrm{MeV}$. Upoštevaj, da je masa mezona $m_M = \langle M \,|\, H \,|\, M \rangle$, in da je npr. $\langle q\bar{q} \,|\, H \,|\, q\bar{q} \rangle = 2m_q$.

   (d) Z dobljenimi masami kvarkov sedaj v modelu hiperfine sklopitve določi maso mezona φ, če poznaš $m_\omega = 782\,\mathrm{MeV}$. V tem modelu k masi mezona prispeva dodatno člen $\alpha \frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{m_1 m_2}$, kjer 1 in 2 predstavljata okus kvarkov.

3. *(1.0 točke)* Pri globokem neelastičnem sipanju elektronov na protonih in nevtronih opazimo, da je pri $E = 15\,\mathrm{GeV}$, $E' = 13\,\mathrm{GeV}$ in sipalnem kotu elektronov $\vartheta = 6°$ razmerje sipalnih presekov $d\sigma^{ep\to eX} \approx \frac{3}{2} d\sigma^{en\to eX}$. Izračunaj x in oceni razmerje partonskih distribucijskih funkcij $u^p(x)/d^p(x)$ pri danem x. Upoštevaj le valenčne partone ter izospinsko simetrijo! Nato opazujemo sipanje pod kotom 3°. Kolikšna mora sedaj energija E', da se x ne spremeni? Za kolikšen faktor se je spremenil $d\sigma^{ep\to eX}$?

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1. *(1.25 točke)* Estimate the total width of a free b-quark Include all allowed 3-body decays to leptons and free quarks. For the Wolfenstein parameters you can take $A = 0.81$, $\lambda = 0.23$, $\rho = 0.14$, $\eta = 0.36$. Quark massses are $m_b = 4.2\,\mathrm{GeV}$, $m_t = 172\,\mathrm{GeV}$, whereas other particle masses may be neglected. The Fermi constant is $G_F = 1.17 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$. What is the ratio $\Gamma(b \to \text{quarks})/\Gamma(b \to \text{quarks} + \text{leptons})$? What is the ratio $\Gamma(b \to u + \text{leptons})/\Gamma(b \to c + \text{leptons})$?

2. *(1.75 točke)* We study masses of pseudoscalar (0^-) and vector mesons (1^-) which have are in addition isospin singlets ($T = 0$, $Y = 0$). For the pseudoscalar case we have states η_8 and η_1, where $\eta_1 = (1/\sqrt{3})(u\bar{u} + d\bar{d} + s\bar{s})$ is a flavour singlet, while η_8 is a member of flavour octet. Physical states are the following combinations:

$$\eta = \eta_8 \cos\vartheta_P + \eta_1 \sin\vartheta_P,$$
$$\eta' = -\eta_8 \sin\vartheta_P + \eta_1 \cos\vartheta_P,$$

   where angle $\vartheta_P = -14°$. For vector mesons we have analogously states ω_8 and ω_1, with corresponding physical states $\omega = \omega_8 \cos\vartheta_V + \omega_1 \sin\vartheta_V$ and $\varphi = -\omega_8 \sin\vartheta_V + \omega_1 \cos\vartheta_V$.

   (a) Derive the flavour wavefunction that describes states η_8 in ω_8! Take into account that η_8 is orthogonal to η_1 and π^0.

   (b) Determine the angle ϑ_V, if we know the flavour wavefunction of $\varphi = s\bar{s}$.

   (c) Determine the masses of quarks $m_q = m_u = m_d$ and m_s if you know $m_\eta = 548\,\mathrm{MeV}$ and $m_{\eta'} = 958\,\mathrm{MeV}$. Take into account that the mass of meson M is given by $m_M = \langle M \,|\, H \,|\, M \rangle$, and furthermore, e.g. $\langle q\bar{q} \,|\, H \,|\, q\bar{q} \rangle = 2m_q$.

   (d) Using the obtained quark massses determine the mass φ meson, if you know $m_\omega = 782\,\mathrm{MeV}$. This time, use the hyperfine splitting model, where there is an additional contribution to mass, $\alpha \frac{\mathbf{s}_1 \cdot \mathbf{s}_2}{m_1 m_2}$, where 1 and 2 stand for two quarks of the meson.

3. *(1.0 points)* For DIS reactions $en \to eX$ and $ep \to eX$ we observe that at $E = 15\,\mathrm{GeV}$, $E' = 13\,\mathrm{GeV}$ and scattering angle $\vartheta = 6°$ the ratio of differential cross sections is $d\sigma^{ep\to eX} \approx \frac{3}{2} d\sigma^{en\to eX}$. Calculate x in estimate the ratio of parton distribution functions $u^p(x)/d^p(x)$ at given x. Take into account only valence partons and isospin symmetry! Afterwards we put the detector at scattering angle $\vartheta = 3°$. At what energy E' should we measure cross section if we want to probe the same x? What is the relative change in $d\sigma^{ep\to eX}$?